

where

$$\langle \alpha_k, \phi(t) \rangle = \sum_{j=1}^n \bar{\alpha}_j^k \phi_j(t), \quad \|\alpha_k\|^2 = \sum_{j=1}^n |\alpha_j^k|^2.$$

If  $E = \bigcap_{k=1}^{\infty} E_k$ , then  $E$  consists of those  $t \in [0, 1]$  for which

$$|\langle \alpha_k, \phi(t) \rangle| \leq K \|\alpha_k\|, \quad k = 1, 2, \dots$$

Using the density of the  $\alpha_k$  this means that  $t \in E$  if and only if

$$(*) \quad \|\phi(t)\|_2^2 = \sum_{j=1}^n |\phi_j(t)|^2 \leq K^2.$$

Now for each fixed  $k$  the vector  $\sum_{j=1}^n \bar{\alpha}_j^k \phi_j$  belongs to  $M$  and hence for almost all  $t \in [0, 1]$  we have

$$\left| \sum_{j=1}^n \bar{\alpha}_j^k \phi_j(t) \right|^2 \leq K^2 \left\| \sum_{j=1}^n \bar{\alpha}_j^k \phi_j \right\|_2^2 = K^2 \cdot \sum_{j=1}^n |\alpha_j^k|^2.$$

Thus the sets  $E_k$  have measure 1 and consequently  $m(E) = 1$ , i.e., (\*) holds for almost all  $t$ . Hence

$$n = \sum_{k=1}^n \|\phi_k\|_2^2 = \sum_{k=1}^n \int_0^1 |\phi_k(t)|^2 dt \leq K^2 \cdot 1.$$

Also solved by Harry Furstenberg and Charles McCarthy, M. A. Malik, M. Rajagopalan and A. Wilansky, and the proposer.

**Notes.** See also S. Banach, *Theorie des Opérations Linéaires*, pp. 203–204 for the application of lacunary trigonometric series above.

The proposer settles the case  $p < \infty$  with the closed subspaces generated by the Rademacher functions; see Zygmund, *Trigonometric Series*, 1st ed., pp. 122, ff.

Furstenberg shows that  $M$  (in  $L^\infty$ ) is contained in the eigenspace of a Hilbert-Schmidt operator on  $L^2(0, 1)$  corresponding to the eigenvalue 1 and is, therefore, finite dimensional. He observes, too, that a closed subspace of  $l^2$  consisting of sequences in  $l^1$  is finite dimensional.

### A New Method of Catching a Lion

In this note a definitive procedure will be provided for catching a lion in a desert (see [1]).

Let  $Q$  be the operator that encloses a word in quotation marks. Its square  $Q^2$  encloses a word in double quotes. The operator clearly satisfies the law of indices,  $Q^m Q^n = Q^{m+n}$ . Write down the word 'lion,' without quotation marks. Apply to it the operator  $Q^{-1}$ . Then a lion will appear on the page. It is advisable to enclose the page in a cage before applying the operator.

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1. H. Petard, A contribution to the mathematical theory of big game hunting, this MONTHLY 45 (1938) 446–447.