

STOKES'S PROBLEM WITH MELTING

STEVEN H. EMERMAN and D. L. TURCOTTE

Department of Geological Sciences, Cornell University, Ithaca, NY 14853, U.S.A.

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Abstract—In this paper we solve for the drag experienced by a hot rigid sphere which melts its way through a cold medium. The temperature of the sphere is maintained by internal heat generation. The cold medium is solid and deforms only when the hot sphere heats it above its melting point. We find that the flow is confined to a thin layer about the forward hemisphere when the Péclet number is much greater than a known function of the Stefan number. We apply our results to the China Syndrome problem and show that in about 2000 years a nuclear reactor core could melt its way through the solid earth to the earth's core.

NOMENCLATURE

c_p	specific heat capacity of melt
F_b	buoyancy force
F_d	drag force
g	acceleration due to gravity
H	internal heat generation rate per unit mass
H_0	initial internal heat generation rate per unit mass
k	thermal conductivity of melt
L	latent heat of fusion
L'	reduced latent heat of fusion, $L + c_p(T_m - T_\infty)$
p	pressure
p_0	pressure at equator
Pe	Péclet number, $u_0 R / \kappa$
Q	total heat flux out of sphere
R	radius of sphere
Re	Reynolds number, $\rho_m u_0 R / \eta$
Ste	Stefan number, $c_p(T_0 - T_m) / L'$
t	time
T	temperature
T_m	melting point of medium
T_0	temperature on surface of sphere
T_∞	ambient temperature of medium
u	tangential velocity
\bar{u}	tangential velocity averaged over the molten layer
u_0	velocity of sphere
v	normal velocity
x	distance tangential to sphere
y	distance normal to sphere

Greek symbols

δ	molten layer thickness
$\Delta\rho$	density difference between sphere and molten medium
η	dynamic viscosity of melt
θ	colatitude
θ^*	colatitude at which lubrication approximation becomes invalid
κ	thermal diffusivity of melt
ρ_m	density of melt
ρ_s	density of sphere

1. INTRODUCTION

STOKES'S problem is the calculation of the drag experienced by a rigid sphere which moves at constant velocity through a viscous medium. In this paper we consider Stokes's problem with melting. We calculate the drag experienced by a hot rigid sphere which melts its way through a cold rigid solid. Heat transfer from the sphere melts the medium and the flow of the viscous melt allows the sphere to move through the solid medium. The problem has a number of possible applications to geophysics. One example is magma migration [1]. A magma body may melt its way through the solid mantle and crust of the earth. Another is core formation [2]. Bodies of iron may have melted their way through the earth's mantle during the formation of the earth's core. Detached lithospheric slabs may also fall into the mantle by this mechanism [3].

A related problem is the so-called China Syndrome [4]. For a given size and rate of heat release, how fast will a nuclear reactor core melt its way through the earth? In this paper we will emphasize the problem in which the required heat is generated within the sphere. An alternative source of heat is the gravitational potential energy of a heavy sphere which melts its way through a solid medium. The potential energy is converted into heat by viscous dissipation.

In order to carry out the analysis we assume that the sphere of radius R is fixed and that the solid medium moves toward the sphere at a velocity u_0 (see Fig. 1). We assume that the flow of melt is confined to a thin layer of thickness δ in front of the sphere. We will see that $(\delta/R) \ll 1$ whenever the Péclet number is much greater than a known function of the Stefan number. Behind the sphere is a cylindrical molten wake. When $(\delta/R) \ll 1$, we can use the lubrication approximation to simplify the Navier-Stokes equation. In Stokes's problem the inertial effects are negligible if $Re \ll 1$. However, if the flow is confined to a thin layer, then the inertial terms can be neglected if $(\delta/R) Re \ll 1$ [5]. We assume that drag and heat flux are negligible on the base of the sphere. The energy equation is solved using an approximate integral method. The related problem

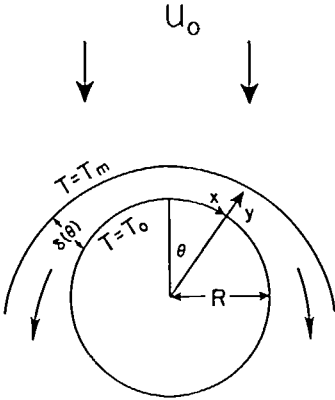


FIG. 1. Geometry for Stokes's problem with melting.

of the melting of a block of ice by a hot plate has been studied experimentally and theoretically by Emmons [6].

2. ANALYSIS

We first consider conservation of momentum. Upon application of the lubrication approximation, the Navier-Stokes equation becomes [5]

$$\eta \frac{\partial^2 u}{\partial y^2} = \frac{dp}{dx} \tag{1}$$

Assuming a solid spherical body, the boundary conditions on tangential velocity are

$$\begin{aligned} y = 0: & \quad u = 0, \\ y = \delta: & \quad u = 0, \end{aligned} \tag{2}$$

where δ is a function of θ . We assume $\delta/R \ll 1$ so that $u/u_0 \gg 1$. Thus it is a consistent approximation to require $u = 0$ at $y = \delta$. The solution of equations (1) and (2) is

$$u = \frac{1}{2\eta} \frac{dp}{dx} y(y - \delta). \tag{3}$$

We next use conservation of mass to find the relationship between the pressure gradient and the molten layer thickness. We find the pressure gradient by equating the mass flux ahead of the sphere to the mass flux about the sphere, and then show that the answer we obtain is consistent with the continuity equation. From Fig. 1, we see that

$$\pi(R \sin \theta)^2 u_0 = 2\pi R \delta \bar{u} \sin \theta, \tag{4}$$

or

$$\bar{u} = \frac{u_0 R}{2\delta} \sin \theta. \tag{5}$$

From equation (3) we obtain

$$\bar{u} = \frac{1}{\delta} \int_0^\delta u \, dy = \frac{-1}{12\eta} \frac{dp}{dx} \delta^2. \tag{6}$$

Combining equations (5) and (6) yields

$$\frac{dp}{dx} = \frac{-6\eta u_0 R}{\delta^3} \sin \theta. \tag{7}$$

The continuity equation for boundary layer flow or lubrication flow over a sphere is [5]

$$\frac{\partial}{\partial x} (u \sin \theta) + \sin \theta \frac{\partial v}{\partial y} = 0. \tag{8}$$

Inserting equation (3) into equation (8) and integrating with respect to y yields

$$\begin{aligned} v = & \frac{-1}{2\eta} \frac{d^2 p}{dx^2} \left(\frac{1}{3} y^3 - \frac{1}{2} y^2 \delta \right) - \frac{1}{2\eta R} \frac{dp}{dx} \\ & \times \cot \theta \left(\frac{1}{3} y^3 - \frac{1}{2} y^2 \delta \right) + \frac{1}{4\eta} y^2 \frac{dp}{dx} \frac{d\delta}{dx}, \end{aligned} \tag{9}$$

where we have used $x = R\theta$ and $v(y = 0) = 0$. Then inserting equation (7) into equation (9) and evaluating at $y = \delta$, we obtain

$$v(y = \delta) = -u_0 \cos \theta. \tag{10}$$

Finally, we use conservation of energy to determine the variation of the molten layer thickness with colatitude. When $(\delta/R) \ll 1$, the steady-state energy equation becomes [5]

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \kappa \frac{\partial^2 T}{\partial y^2}. \tag{11}$$

Instead of solving the partial differential equation (11), we integrate equation (11) over the molten layer to obtain an ordinary differential equation in x . Related approximate methods are the momentum integral method of boundary layer theory [5] and the heat balance integral method for the heat conduction equation [7]. Multiplying equation (11) by $\sin \theta$ and integrating with respect to y yields

$$\begin{aligned} \int_0^\delta u \frac{\partial T}{\partial x} \sin \theta \, dy + \int_0^\delta v \frac{\partial T}{\partial y} \sin \theta \, dy \\ = \kappa \int_0^\delta \frac{\partial^2 T}{\partial y^2} \sin \theta \, dy. \end{aligned} \tag{12}$$

Integrating the second term on the LHS by parts and evaluating the RHS yields

$$\begin{aligned} \int_0^\delta \frac{\partial}{\partial x} (uT \sin \theta) \, dy - u_0 T_m \cos \theta \sin \theta \\ = \kappa \sin \theta \left[\frac{\partial T}{\partial y} (y = \delta) - \frac{\partial T}{\partial y} (y = 0) \right]. \end{aligned} \tag{13}$$

The first term on the LHS of equation (13) is equal to

$$\frac{d}{dx} \left[\sin \theta \int_0^\delta uT \, dy \right], \tag{14}$$

since $u(y = \delta) = 0$. Hence equation (13) becomes

$$\begin{aligned} \frac{d}{dx} \left[\sin \theta \int_0^\delta uT \, dy \right] = u_0 T_m \sin \theta \cos \theta \\ + \kappa \sin \theta \left[\frac{\partial T}{\partial y} (y = \delta) - \frac{\partial T}{\partial y} (y = 0) \right]. \end{aligned} \tag{15}$$

The required boundary conditions on the temperature are

$$\begin{aligned}
 y = 0: & \quad T = T_0, \\
 y = \delta: & \quad T = T_m, \\
 y = \delta: & \quad \frac{\partial T}{\partial y} = -\frac{\rho_m u_0 L'}{k} \cos \theta. \quad (16)
 \end{aligned}$$

The reduced latent heat of fusion L' takes account of both the latent heat of fusion L and the internal energy $c_p(T_m - T_\infty)$ of the solid medium as it is heated upstream of the melting front. We further assume that the temperature profile can be adequately represented by a quadratic polynomial in y . The quadratic polynomial that satisfies the boundary conditions (16) is

$$\begin{aligned}
 T = T_0 + y \left[\frac{-2(T_0 - T_m)}{\delta} + \frac{\rho_m u_0 L'}{k} \cos \theta \right] \\
 + y^2 \left[\frac{T_0 - T_m}{\delta^2} - \frac{\rho_m u_0 L'}{\delta k} \cos \theta \right]. \quad (17)
 \end{aligned}$$

Inserting equation (17) into equation (15), we obtain

$$\begin{aligned}
 \frac{d\delta}{d\theta} = \frac{\kappa}{u_0 \sin \theta} \left[-3 Ste - \frac{u_0 \delta}{\kappa} \right. \\
 \left. \times \left(2 \cos \theta - \frac{\sin^2 \theta}{\cos \theta} \right) - 20 + 20 \frac{\kappa Ste}{\delta u_0 \cos \theta} \right], \quad (18)
 \end{aligned}$$

where we have used $x = R\theta$. With the boundary condition $d\delta/d\theta = 0$ at $\theta = 0$, the solution to equation (18) is

$$\delta = \frac{\kappa f(Ste)}{u_0 \cos \theta}, \quad (19)$$

where

$$f(a) = \frac{1}{2} \left\{ \frac{-3}{2} a - 10 + \left[\frac{9}{4} a^2 + 70a + 100 \right]^{1/2} \right\}. \quad (20)$$

For small Ste , we find

$$\delta = \frac{\kappa Ste}{u_0 \cos \theta}. \quad (21)$$

Hence from equations (17) and (21) we see that the temperature profile becomes linear in y as the $Ste \rightarrow 0$. We see moreover that $(\delta/R) \ll 1$ whenever $Pe \gg f(Ste)$. The dependence of $f(Ste)$ on Ste from equation (20) is given in Fig. 2.

As mentioned above, we neglect the heat flux to the base of the sphere. Thus the total heat loss from the sphere is given by

$$Q = -2\pi R^2 \int_0^{\pi/2} k \frac{\partial T}{\partial y} (y = 0) \sin \theta \, d\theta. \quad (22)$$

Substituting equations (17) and (19) into equation (22), we obtain

$$Q = \pi R^2 \kappa u_0 \left[\frac{2(T_0 - T_m)}{\kappa f(Ste)} - \frac{\rho_m L'}{k} \right]. \quad (23)$$

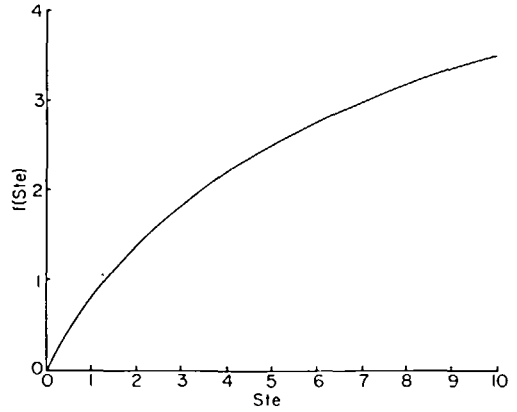


FIG. 2. For a solid sphere, $\delta = \kappa f(Ste)/u_0 \cos \theta$, where δ is the molten layer thickness, u_0 is the velocity of the sphere, κ is the thermal diffusivity of the melt and θ is colatitude.

And in the limit $Ste \rightarrow 0$ this reduces to

$$Q = \pi R^2 \rho_m u_0 L'. \quad (24)$$

In this limit the heating required per unit time is the latent heat of the volume $\pi R^2 u_0$.

We next determine the drag on the spherical body. From equations (3) and (7) we see that the shear stress is of the order of $(\eta u_0 R/\delta^2)$, while the pressure is of the order of $(\eta u_0 R^2/\delta^3)$. If $(\delta/R) \ll 1$, then the drag due to shear stress is negligible compared to the drag due to pressure. Hence we evaluate the integral

$$F_d = 2\pi R^2 \int_0^{\pi/2} (p - p_0) \cos \theta \sin \theta \, d\theta, \quad (25)$$

where p_0 is the pressure at $\theta = \pi/2$ which we assume is the pressure on the base of the sphere. Inserting equation (19) into equation (7), replacing x with $R\theta$, and integrating, we obtain

$$p - p_0 = \frac{3\eta R^2 u_0^4}{2\kappa^3 f^3(Ste)} \cos^4 \theta. \quad (26)$$

Substitution of equation (26) into equation (25) and integrating yields

$$F_d = \frac{\pi \eta R^4 u_0^4}{2\kappa^3 f^3(Ste)}. \quad (27)$$

And for small values of Ste , we obtain

$$F_d = \frac{\pi \eta R^4 u_0^4}{2\kappa^3 (Ste)^3}. \quad (28)$$

From equation (19) we see that the lubrication approximation, $(\delta/R) \ll 1$, is invalid for colatitude $\theta \geq \theta^*$, where

$$\theta^* = (1/10) \cos^{-1} [f(Ste)/Pe]. \quad (29)$$

However, we can easily show that the contribution to the heat flux and drag from the region $\theta^* \leq \theta \leq \pi/2$ goes to zero as $f(Ste)/Pe$ goes to zero. Evaluating the integral in equation (22) between θ^* and $\pi/2$ yields equation (23) multiplied by $[f(Ste)/Pe]^2$. Evaluating

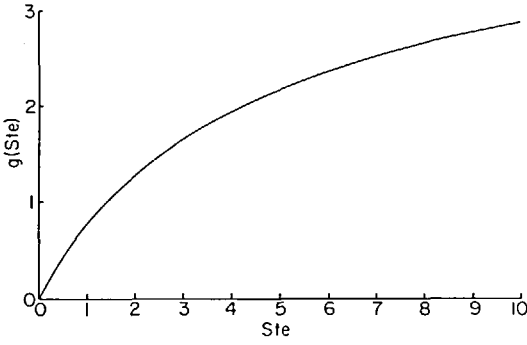


FIG. 3. For a fluid sphere, $\delta = \kappa g(Ste)/u_0 \cos \theta$, where δ is the molten layer thickness, κ is the thermal diffusivity of the melt and θ is colatitude.

the integral in equation (25) between θ^* and $\pi/2$ yields equation (27) multiplied by $[f(Ste)/Pe]^6$.

We also consider the drag and heat flux for a fluid sphere for which the viscosity of the molten medium is much greater than the viscosity of the sphere and for which the surface tension is sufficiently great to maintain the spherical shape. In this case the thermal boundary conditions are the same as before and the velocity boundary conditions become

$$y = 0: \quad v = 0, \quad \frac{\partial u}{\partial y} = 0. \tag{30}$$

We follow the same procedure as above to obtain

$$\frac{dp}{dx} = \frac{-3\eta u_0 R}{2\delta^3} \sin \theta, \tag{31}$$

and

$$\delta = \frac{\kappa g(Ste)}{u_0 \cos \theta}, \tag{32}$$

where

$$g(a) = \frac{1}{7} \{ -40 - 9 Ste + [1600 + 1280 Ste + 81(Ste)^2]^{1/2} \}. \tag{33}$$

The results for heat flux for a fluid sphere are identical to those for a solid sphere with $f(Ste)$ replaced by $g(Ste)$. The drag on a fluid sphere is a quarter the drag on a solid sphere with $f(Ste)$ replaced by $g(Ste)$. The dependence of $g(Ste)$ on Ste from equation (33) is given in Fig. 3.

Results (31)–(33) assume that the fluid sphere is inviscid. If the viscosity of the fluid sphere and the viscosity of the molten medium were comparable, then it would be necessary to take into account the internal dynamics of the fluid sphere. There would be an additional drag due to the flow within the sphere and the net drag would be increased accordingly.

3. DISCUSSION

We now use the above analysis to find the velocity of a hot solid sphere which rises or falls through a cold

medium due to the density difference between the sphere and the medium. We assume that heat is generated within the sphere at a rate H per unit mass. The total heat loss is related to the heat generation by

$$Q = \frac{4}{3} \pi R^3 \rho_s H, \tag{34}$$

where ρ_s is the density of the spherical body. The isothermal boundary condition on the surface of the sphere is still applicable if the thermal conductivity of the sphere is assumed to be much greater than the thermal conductivity of the melt. For a fluid sphere, vigorous internal convection could maintain the isothermal boundary condition. Combining equations (23) and (34) we obtain

$$u_0 = \frac{(4/3)(R\rho_s H/k)}{[2(T_0 - T_m)/\kappa f(Ste)] - (\rho_m L/k)}, \tag{35}$$

which is a balance between the heat generated within the spherical body and the heat lost to the molten layer. If H is sufficiently small, the Stefan number is also small and equations (24) and (34) combine to give

$$u_0 = \frac{4}{3} \frac{R\rho_s H}{\rho_m L}. \tag{36}$$

Thus for small rates of heat production the velocity of the sphere is linearly dependent on R and H and is independent of the buoyancy force and melt viscosity. The buoyancy force and melt viscosity are contained in equation (35) through their relation to T_0 .

The buoyancy force on the spherical body is given by

$$F_b = \frac{4}{3} \pi g \Delta \rho R^3, \tag{37}$$

where $\Delta \rho = \rho_s - \rho_m$. Equating this buoyancy force to the drag given by equation (27) yields

$$u_0 = \left[\frac{8g\Delta\rho\kappa^3 f^3(Ste)}{3\eta R} \right]^{1/4}. \tag{38}$$

Since $(\delta/R) \ll 1$, we can neglect the density difference between the solid medium and the molten medium. For small Ste , equations (28) and (37) combine to give

$$T_0 = T_m + \frac{L}{\kappa c_p} \left(\frac{3R\eta u_0^4}{8g\Delta\rho} \right)^{1/3}. \tag{39}$$

Substitution of equation (35) gives

$$T_0 = T_m + \frac{4}{3} \frac{1}{\kappa c_p} \left(\frac{\eta \rho_s^4 H^4 R^5}{2g\Delta\rho L \rho_m^4} \right)^{1/3}. \tag{40}$$

If R , H , and the physical properties are prescribed, it is in general necessary to solve equations (35) and (38) numerically for T_0 and u_0 . However, if H is sufficiently small, u_0 can be obtained from equation (36) and T_0 from equation (40).

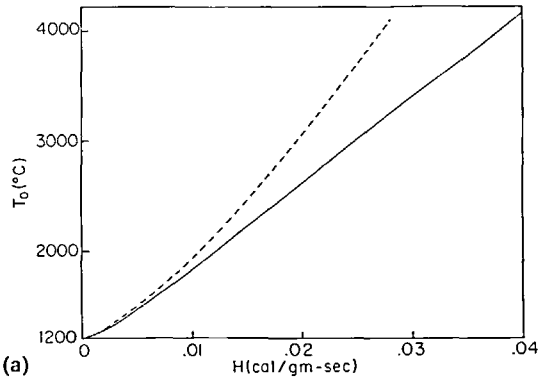
As a specific example we consider the China Syndrome problem. We assume that after meltdown a nuclear reactor core can be modeled as a spherical body with internal heat generation. Our object is to determine how fast the core will melt its way into the

interior of the earth. For rock properties we take $k = 0.01 \text{ cal cm}^{-1} \text{ s}^{-1} \text{ } ^\circ\text{C}^{-1}$, $\rho_m = 2.7 \text{ g cm}^{-3}$, $\kappa = 0.01 \text{ cm}^2 \text{ s}^{-1}$, $c_p = 0.25 \text{ cal g}^{-1} \text{ s}^{-1}$, $\eta = 100 \text{ poise}$, $L = 100 \text{ cal g}^{-1}$, $T_\infty = 0^\circ\text{C}$ and $T_m = 1200^\circ\text{C}$ [8]. For the reactor core we take $\rho_s = 9 \text{ g cm}^{-3}$, $R = 150 \text{ cm}$, and use the Wigner-Way empirical correlation for heat production

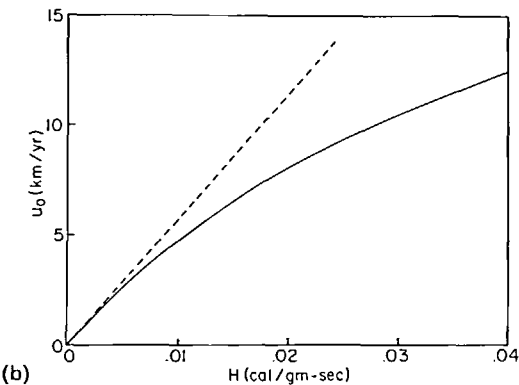
$$H = 0.0622H_0t^{-0.2}, \quad (41)$$

with $H_0 = 5.3 \text{ cal g}^{-1} \text{ s}^{-1}$ and t in seconds (R. J. Miller, personal communication). The dependence of T_0 and u_0 on H from equations (35) and (38) is given in Fig. 4. Also included are the small H approximations from equations (36) and (40).

With the heat production prescribed as a function of time by equation (41), the temperature and velocity of the sinking reactor core are obtained from equations (35) and (38). The velocity is then integrated to give the depth of the reactor as a function of time. In Fig. 5(a) we have assumed $H_0 = 5.3 \text{ cal g}^{-1} \text{ s}^{-1}$, in Fig. 5(b) $H_0 = 0.53 \text{ cal g}^{-1} \text{ s}^{-1}$, and in Fig. 5(c) $H_0 = 0.053 \text{ cal g}^{-1} \text{ s}^{-1}$. If the reactor core has its full heat productivity it will melt its way to the earth's core in about 2000 years. If this heat productivity is reduced by a factor of 10 due to dilution or other effects it will

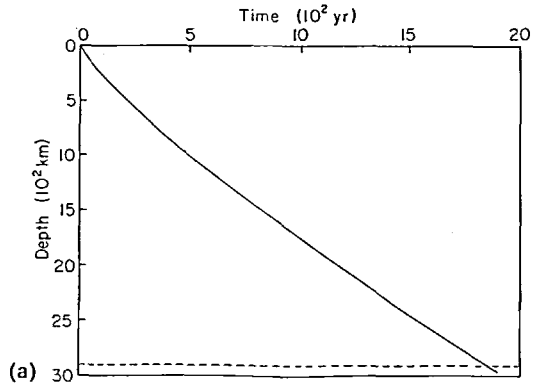


(a)

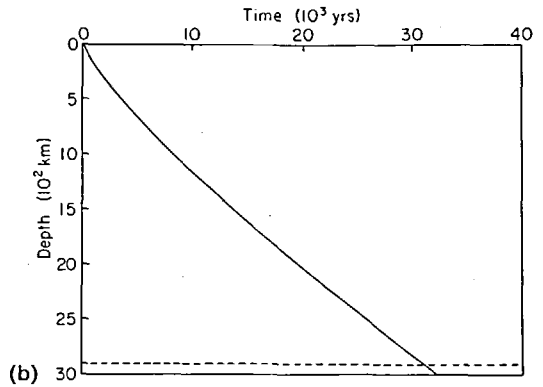


(b)

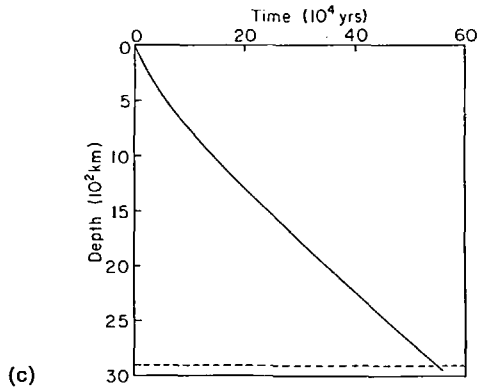
FIG. 4. The solid curves are obtained from numerical integrations of equations (35) and (38) which give (a) the temperature and (b) the velocity of the reactor core as a function of the internal heat production rate. The dashed lines are the small H approximations from equations (36) and (40).



(a)



(b)



(c)

FIG. 5. The depths of a reactor core as a function of time as it melts its way into the interior of the earth for (a) $H = 5.3 \text{ cal g}^{-1} \text{ s}^{-1}$, (b) $0.53 \text{ cal g}^{-1} \text{ s}^{-1}$, and (c) $0.053 \text{ cal g}^{-1} \text{ s}^{-1}$.

take about 30 000 years, and if the dilution is by a factor of 100 it will take about 500 000 years.

It should be emphasized that many approximations are involved in the application of our analysis to the China Syndrome problem. Probably the most serious is our assumption that, after meltdown, it is appropriate to consider the reactor core as a coherent body. It may be very substantially diluted by mixing with other structural units. Although we have attempted to model this effect by reducing H by factors of 10 and 100, this is certainly only an approximation. Also the reactor core may not be spherical, but this is unlikely to introduce large errors.

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PROBLEME DE STOKES AVEC FUSION

Résumé—On traite la trainée d'une sphère rigide chaude qui se déplace en fondant sur son chemin un milieu froid. La température de la sphère est maintenue par une source interne de chaleur. Le milieu froid est solide et ne déforme que lorsque la sphère le rechauffe au dessus de son point de fusion. On trouve que l'écoulement est confiné à une couche mince sur l'hémisphère amont quand le nombre de Péclet est supérieur à une fonction connue du nombre de Stefan. On applique les résultats au syndrome chinois et on montre qu'il faut deux mille ans à un coeur de réacteur nucléaire pour arriver au coeur de la Terre à travers la couche solide.

DAS PROBLEM VON STOKES MIT SCHMELZVORGÄNGEN

Zusammenfassung—In diesem Aufsatz wird der Widerstand ermittelt, den eine heiße, starre Kugel erfährt, welche sich ihren Weg durch ein kaltes Medium schmilzt. Die Temperatur der Kugel wird durch innere Wärmequellen aufrechterhalten. Das kalte Medium ist fest und deformiert sich nur dann, wenn es durch die heiße Kugel über seinen Schmelzpunkt erwärmt worden ist. Es zeigt sich, daß die Strömung auf eine dünne Schicht an der vorderen Hälfte der Kugel begrenzt ist, so lange die Peclet-Zahl wesentlich größer ist als eine bekannte Funktion der Stefan-Zahl. Durch Anwendung der Ergebnisse auf das Problem des "China-Syndroms" wird gezeigt, daß sich der Kern eines nuklearen Reaktors innerhalb von ungefähr zweitausend Jahren einen Weg durch die feste Erdkruste hindurch bis ins Zentrum der Erde schmelzen kann.

ЗАДАЧА СТОКСА В СЛУЧАЕ ПЛАВЛЕНИЯ

Аннотация—Дано решение для определения лобового сопротивления, испытываемого нагретой упругой сферой, проплавляющей холодную среду. Температура сферы поддерживается за счет внутреннего источника тепла. Холодная среда представляет собой твердый материал, который деформируется только от действия сферы, нагретой выше точки плавления. Найдено, что течение материала ограничено тонким слоем у передней полусферы, когда число Пекле намного превышает известную функцию числа Стефана. Полученные результаты использованы для решения задачи "китайского синдрома" и показано, что примерно через две тысячи лет активная зона ядерного реактора могла бы проплавить твердую мантию земли и достичь ее ядра.