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THE "STINGER GAP": THE DIFFERENCE BETWEEN THE "COST OF SUBSISTENCE" AND THAT OF A MINIMUM-COST NONINSTITUTIONAL DIET WITH PALATABILITY†

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Abstract—This study presents two different 1976 diets for a moderately active male, age 23–50, buying foods at retail. The first of these is a "subsistence" diet just adequate for nutrition at minimum cost, with standard serving sizes for all foods, and an upper limit of 90 servings per month. There are upper and lower limits for each of 11 nutrients such as calories, calcium, phosphorus, etc. The solution was obtained by Mixed-Integer Programming (MIP) from a candidate list of 392 foods available in Stillwater, Oklahoma supermarkets in January, 1976. The diet costs $15.55 per month or 51.8¢ a day and includes 20 different foods.

The second diet "nutrition plus palatability" has a wider variety of foods obtained by incorporating into the MIP model additional constraints reflecting tastes. This diet costs $34.51 a month or $1.15 a day and includes 68 different foods. The difference between these two costs $1.15 − $0.52 = $0.63 is an estimate of the 1976 "Stinger gap," the cost of palatability in an optimum diet.

INTRODUCTION

Nutrition and dietetics are the twin sciences governing the selection of foods. Nutrition defines standards for nutrients that will satisfy energy needs, and simultaneously provide for adequate growth and disease prevention. Dietetics shows how these needs can be met with foods that not only are nutritious, but also satisfy regional tastes.

Menu planning is usually done by nutritionists for an institution such as a hospital, dormitory, cafeteria, etc. A specified group of consumers is served in such a way that each individual has a limited number of choices for each type of menu item (salad, entree, etc.) from a candidate list prepared by the dietitian, different for each meal. For the most part, foods are purchased at wholesale rather than retail.

Scientific menu planning with the objective of satisfying local tastes at minimum cost by a programmed decision model is relatively new. Solutions are frequently obtained by mathematical programming, especially Dantzig's simplex method of linear programming.

This report presents the results of research into the problem of providing a lowest cost diet for a moderately active male age 23–50 in a noninstitutional environment, for foods purchased at retail, in Stillwater, Oklahoma supermarkets in January 1976. Upper limits were set to the number of standard serving sizes of each food.

Two different diets were generated using Mixed-Integer Programming (MIP), which is a variant of the Dantzig "simplex method" designed to provide optimum solutions when some or all of the variables are integer-valued. In this case, the number of servings of each food selected is integer valued. The first of these, Model 1, is a "subsistence diet" with an upper limit of 90 servings per month. The second diet, Model 2, "nutrition plus palatability," incorporates added constraints to prevent the diet from becoming too monotonous. These include: "separations," the minimum number of meals between successive servings; lower limits on certain categories of food; upper limits on other categories of foods; and complementary requirements for foods served together (e.g. bread and butter). Each diet consists of an integer number of servings in a consecutive 30-day period. The difference in cost between these two diets was called by Smith[13] the "Stinger gap."

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†This paper is based upon a linear programming (LP) version of the diet problem developed and documented in three different research reports, by Wing[23] using conventional LP and by Nikzad and Samimi[10, 12] (Mr. and Mrs. Nikzad respectively), using Mixed-Integer Programming (MIP). Anand Desai obtained the actual integer solutions reported upon herein using the data sets generated by Nikzad and Samimi. We are indebted to Helen West, Dietitian, Hillcrest Medical Center, Tulsa, OK and Mary Leidigh, retired, both formerly of the faculty of Food, Nutrition and Institution Administration at Oklahoma State University, who provided consultation on nutritional requirements and standards.

RELEVANT LITERATURE SUMMARY

Among many researchers who have considered the nutrition problem, the prior results of Stiebeling, Stinger, Smith and Balintfy, and their respective associates are closely related to this project.

Hazel K. Stiebeling, 1933

Stiebeling and Ward[20] worked out four different plans to fit four different levels of income based on 1931–32 price levels. Plan 1, an "adequate diet at minimum cost," which may be comparable to our Model 1, cost 16.9¢ a day for a moderately active man. Plan 2, an "adequate diet at minimum cost" provides more "protective" foods such as vegetables, fruit and milk and a higher proportion of proteins, for 23.6¢ a day. Plan 3, an adequate diet at moderate cost" provides even greater protein content and more minerals and vitamins for 41.5¢. Plan 5, a "liberal diet" gives better than average nutrition with leeway to satisfy personal tastes for 50.8¢.

Rather than identify particular shelf items as most later studies do, Stiebeling and Ward categorized foods according to twelve major groups: flour, cereal or bread; milk, potatoes, sweet potatoes; dried beans, peas, nuts; tomatoes, citrus fruits; leafy green and yellow vegetables; dried fruits; other vegetables, fruits; fats; sugars; lean meat, poultry, fish; and eggs. For each of the 4 plans, some foods are to be selected from each of the twelve groups. The Stiebeling and Ward diet plans were popularized in a book by Hambidge[8]. Variations of them have appeared in various USDA publications authored or co-authored by Stiebeling[2, 17–19].

George J. Stinger, 1945

One of the most creative solutions to the minimum cost diet problem was obtained by Stinger in his classic paper "The Cost of Subsistence"[21]. Using *Recommended Dietary Allowances* (Ref. [9] is a recent edition) as a guide to nutritional needs and Bowes and Church *Food Values* (Ref. [3] is a recent edition) and other references as guides to the nutrient contents of foods, he obtained two five-food subsistence diets, one at August, 1939 prices and one at August, 1944 prices. These diets cost 10.9¢ per day for 1939 and 16.4¢ per day for 1944, for a moderately active adult male. Stinger's August, 1939 cost of 10.9¢ is approximately (1/2) of Stiebeling's most nearly comparable lowest cost diet at January–October, 1938 prices (a range of 17.8¢–22.8¢), given in [19, p. 334].

It is remarkable that Stinger's diets, which were obtained laboriously by hand computations before the simplex method was discovered by Dantzig, are practically as good as the LP solutions. For example, the LP solution for the 1939 data was obtained in 1947 by Laderman and Dantzig at the RAND Corporation. It costs only 25¢ per yr less than 1939 prices. See Dantzig[4, p. 551] and Smith[13, pp. 12–14, 18–19].

Victor E. Smith

In *Electronic Computation of Human Diets* [13] Smith introduced palatability considerations and presented various solutions to the minimum cost diet problem obtained by LP. Prepared with great attention to meticulous details, this book may be viewed as a continuation of Stinger's 1945 paper.

For nutrition only, comparable to the Stinger diet, Smith presented the "Smith Midget Model Diet" for a family of three (two 45-yr-old adults and their 18-yr-old daughter) at May, 1955

Table 1. Stinger's "subsistence" diets

commodity	August, 1939		August, 1944	
	quantity	cost	quantity	cost
wheat flour	370 lb.	$13.33	535 lb.	$34.53
evaporated milk	57 cans	3.84	----	----
cabbage	111 lb.	4.11	107 lb.	5.23
spinach	23 lb.	1.85	13 lb.	1.56
dried navy beans	285 lb.	16.80	----	----
pancake flour	----	----	134 lb.	13.08
pork liver	----	----	25 lb.	5.48
total cost		$39.93		$59.88

prices in Lansing, Michigan[13, p. 21]. The cost was $368.29 for the family, an average of $122.76 per yr or 33.6¢ per person per day[13, p. 21]. This diet has minimum levels for 11 nutrients, and both upper and lower limits for calories. Six commodities are served, including fresh milk, oleomargarine, fresh carrots, fresh potatoes, ham butts and enriched flour. It is noteworthy that Smith obtained an interesting distribution among food classes, with only six foods.

Besides the nutrition-only "midget model diet" Smith also developed three different diets, with respectively increasing palatability. The added constraints in each case were supported from survey and expenditure data. Unfortunately, these diets are not presented in the book in a single tabular form which would make it possible for us to compare our results to his. Also, with only one exception he does not explicitly give the cost.

The "midget model with cooking aids" is the original midget model with five cooking aids added: baking powder, baking soda, flavoring extracts, vinegar and prepared mustard. The "small model" adds complementarity restraints (foods served together, such as bread and butter), requirements for specific amounts of certain foods and maximum limits on the quantities of other foods, and includes a total of 10 cooking aids.

The "large model" sets minimum quantities of 41 foods that were consumed by at least 90% of the families that had participated in an expenditure survey. The "large model" diet included 62 different commodities, at a cost of $43.96 for the family for 4 weeks or 52.3¢ per day per person. Although we do not know exactly what is in the "large model" diet because the tables are not included, it seems to be comparable to Model 2.

Smith hypothesized that calories, vitamin A and vitamin C tend to be "scarce" [13, p. 22], that is, at the lowest levels permitted by the constraints, in minimum cost diets. Our findings differ. Calories are scarce, as Smith hypothesizes, but vitamin C and vitamin A are not. There can be a variety of explanations for this, including the fact that we set upper and lower limits on practically all nutrients, whereas Smith used lower limits only on all except calories. Also, the way in which we paid attention to serving sizes and portions might be a contributing factor to the differences between these results and Smith's.

Besides his contributions to the problem of minimum-cost nutrition in the United States, Smith has been using computers to develop diets for the developing nations Nigeria and Colombia under the sponsorship of various academic consortia, the Rockefeller Foundation, and the National Science Foundation. His Nigerian studies are very extensive and are reported upon in Refs. [14] and [16]. The objective is to define efficient agricultural production patterns for the country as a whole "for the attainment of specified nutrient goals and identify those foods, crops and methods which are nutritionally efficient" [16, p. 41].

Although the Nigeria study both takes account of individual dietary needs and makes extensive use of LP, it has a different scope than those dealing with least-cost nutrition in the United States. First, it deals with different sections of the country as a whole; in others words, a different optimum diet is obtained for each section of Nigeria. Second, it incorporates a well developed and fairly sophisticated protein model with a nonlinear constraint, that determines a least-cost diet in which the quality and the quantity of the protein are jointly and optimally determined, based upon Ref.[15]. Because of this nonlinearity, the separability feature of LP is employed in the solution.

Smith identified for each of six areas of Nigeria the foods which are most nutritionally efficient. Examples include maize, sorghum, groundnut, goat meat, mutton, millet, beer and kola nut, which do not appear in any of the American minimum-cost diets [16, p. 78]. While the results differ from one geographical section of the country to another, Smith finds calories to be an expensive nutritional need in all sections, riboflavin in some sections, and vitamin A, iron, or calcium in others [16, p. 172].

The Colombia nutrition study is documented in two reports by Florencio and Smith [5, 6] and in a published abstract[11]. This deals with problems of nutrition of undernourished families who spent little on food, and spent that inefficiently, and for whom least-cost diets had been calculated. Tables are provided which give for 8 cities the list of foods occurring in least-cost diets, the frequency of appearance of different foods in actual and least-cost diets and the percentage contribution of cost for energy and proteins by each of 9 groups of foods in the diet.

Joseph L. Balintfy

Balintfy, formerly at Tulane and now with the University of Massachusetts, who is the developer of the proprietary CAMP (Computer Assisted Menu Planning) system for institutional menu planning is quite famous for his work in applying mathematical programming to nutrition. He has been actively involved in experimental work in modelling food preferences. Wing [23, pp. 10–20] gives a summary of his work in the area of modelling nutrition, including a special bibliography of 25 papers published since 1963.

The CAMP system is used interactively by dietitians at hospitals and other institutions around a time-shared computer and a dedicated disc file. The dietary staff has on-line access to centrally shared data files, to various libraries and to the menu planning program. Meals are planned for regular diets, modified diets of cafeteria meals, to satisfy all nutritional and palatability considerations. Results have shown raw food cost savings of 9–34%, while satisfying all nutritional needs[7].

It is not possible to explicitly compare Models 1 and 2 to those generated by the CAMP system for a variety of reasons. CAMP plans menu items (e.g. omelets, roasts, casseroles) whereas we select food items (eggs, butter, carrots) as purchased in a supermarket at retail. CAMP is also designed for volume feeding, whereas our plans are for individuals.

Balintfy's LP techniques, however, apparently are closely related to those we used. For example, Ref.[1] describes four different types of constraints incorporated into a menu planning model: upper bounds, nutrient constraints, structural constraints and attribute constraints. An "upper bound" is the maximum number of times in a 31-day period that a given item is served; "nutrient constraints" specify how much of each nutrient to include; "structural constraints" relate to the composition of each of the 3 meals served in a day. In addition, the model counts servings. All of these concepts have counterparts in both Models 1 and 2.

DATA SOURCES

The food items considered in this study were selected from a recent edition of Bowes and Church[3]. This provides both food values (nutrient contents) and the size of the portion in grams. The list was reduced to 392 for which price data are readily available in Stillwater and which are not obviously too expensive (e.g. caviar). The cost of a serving was adjusted to the true cost of the edible portion, based on information obtained from "Handbook 8" of the USDA[22].

Eleven (11) different nutrients are considered in this study, listed in Table 2. Recommended allowances of each nutrient were obtained from[9]. These allowances were modified so as to provide both upper and lower limits for each nutrient, in consultation with nutritionists Leidigh and West. Palatability and availability of nutrient data served as guides for selecting the upper and lower limits, as well as the possibility of developing toxicity due to excessive consumption of a nutrient, or deficiency. Since the planning period is a 30-day month, all dietary allowances are given in 30-day units.

NUTRITIONAL ASPECTS

Subsistence

Table 2 presents monthly nutrition requirements for an average male age 23–50, giving both upper and lower limits.

Palatability

For Model 2, palatability constraints were added, including: "separations," the minimum

Table 2. Nutrition requirements

nutrient	units	lower limit	upper limit
calories	calories	75,000	85,500
calcium	mg.	15,000	36,000
phosphorus	mg.	21,000	42,000
thiamin	mg.	27	75
niacin	mg.	450	none
vitamin A	I.U.	111,000	165,000
protein	gm.	1,500	3,500
magnesium	mg.	3,000	12,000
iron	mg.	180	750
riboflavin	mg.	42	75
ascorbic acid	mg.	900	2,500

number of meals between two successive servings of the same food; lower limits, minimum servings of categories such as milk products, meat or fish, vegetables, and fruits; upper limits for candy, cereal, butter or margarine, non fruit desserts and fish; and "complementarity" requirements for foods served together such as milk with cereal, and tartar sauce with fish.

Separations

Dietitians use a "separation rating (SR)" to define the minimum number of meals between any two successive servings of a given food. A separation rating is easily translated into an upper bound on the number of servings in a month which is lower than the standard upper bound of 90 used in Model 1. The upper bound is 90/(SR + 1). For example, suppose a food has a separation rating of 8; then there are at least 8 meals between any two successive servings, or three full days. Thus the upper bound on the number of servings in a month is 10.

Separation ratings are provided for all of the 392 foods. The upper bounds calculated from these SR's are available in the MPSX-360 computer output which is reproduced in Appendix B of [10].

Palatability constraints

There are 12 conditions imposed upon the palatability model. Each of these is either a lower limit on the number of servings of a certain class of food, an upper limit, or a complementarity requirement. Each of these conditions becomes a linear constraint to the LP model, in addition to those of Table 2.

(a) lower limits
  1. At least 60 servings of milk, whole, nonfat or skim, buttermilk or chocolate milk
  2. At least 60 servings of meat, poultry and game or fish
  3. At least 90 servings of vegetables
  4. At least 120 servings of bread and cereal products†
  5. At least 30 servings of fresh fruit or fruit juice

(b) upper limits
  1. At most 30 servings of breakfast cereal
  2. No more than four candy bars
  3. No more than 90 servings of either butter or oleomargarine
  4. No more than 10 servings of nonfruit desserts
  5. No more than four servings of fish

(c) complementarity constraints
  1. One serving of milk, whole, nonfat, or skim with each serving of breakfast cereal
  2. One serving of tartar sauce with every serving of fish

RESULTS AND ANALYSIS

MIP models

Let x_j, $j = 1, \ldots, 392$, x_j an integer, denote the number of servings in a month of a food, with serving cost c_j and upper bound u_j, respectively. For each of the 11 nutritional constraints, there are both upper and lower limiting values, because there are both upper and lower limits for each nutrient. Let b_{il}, $i = 1, \ldots, 11$, denote the lower limit and b_{iu} the upper limit and a_{ij} the "technological coefficients" of the model.

Model 1 for "subsistence only" is

$$\text{minimize} \sum_{j=1}^{392} c_j x_j, \text{ cost in dollars}$$

$$\text{subject to: } \sum_{j=1}^{392} a_{ij} x_j \begin{cases} \geq b_{il}, & i = 1, \ldots, 11, \text{ limits in nutrients} \\ \leq b_{iu} \end{cases}$$

$$0 \leq x_j \leq u_j,\ j = 1, \ldots, 392, \text{ number of servings} \tag{1}$$

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† It has been pointed out that this constraint, combined with the other constraints of the model, force exactly 120 servings. This could be unreasonable.

Table 3. Model 1 subsistence diet for an adult male age 25–30

food	servings	unit weight (grams)	unit cost	total cost
unsalted almonds	90	10	$.013	1.17
roasted unsalted peanuts	1	20	.036	.04
bread, cracked wheat	29	23	.022	.64
hominy grits, enriched	89	36	.027	2.40
macaroni, dry enriched	1	110	.11	.11
macaroni and cheese, baked	2	225	.173	.35
unsalted crackers	1	7.2	.01	.01
chocolate snaps	89	3.8	.005	.44
coconut cookies, snack size	1	3.5	.005	.00
lemon snap cookies	1	3.1	.006	.00
vanilla wafers	88	3.2	.004	.35
oleomargarine	90	28	.018	1.62
beef kidney, stewed	2	93	.15	.30
calf liver, cooked	3	72	.156	.47
mayonnaise	20	14	.029	.58
brown sugar	90	14	.01	.90
white sugar	90	8	.008	.72
beans, dry pinto	65	100	.073	4.74
raw cabbage	18	100	.037	.67
raw onions	1	100	.033	.03
total				$15.55

In Model 1, $u_j = 90$ for all foods x_j, $j = 1, \dots, 392$, since all foods are counted in serving units, and the maximum number of servings in a month is 90.

Model 2 for "nutrition plus palatability" includes all 11 constraints and the 392 variables in (1). Instead of the upper bounds u_j all being 90 as in Model 1, each bound is an upper limit to the number of monthly servings calculated from the separation index. In addition, Model 2 has 12 additional palatability constraints, defined respectively by the lower limits, upper limits and complementarity requirements.

Model 1

The optimum diet for subsistence only given in Table 3 includes 20 foods and costs $15.55 a month, or 51.8¢ a day.

This is clearly a better diet than its 1939 and 1945 Stinger counterparts. There is a greater variety of foods. Also it has a bigger variety than Smith's midget model diet which has only six items, although the latter is probably a better diet, since it includes "protective" foods such as fresh milk, green and yellow vegetables and potatoes. An undesirable feature is too much of an inclination to sugar and sugar-rich cookies: *both* brown and white sugar are served at every meal, as well as *both* chocolate snaps and vanilla wafers. This is a clear reflection of the calorie shortage in minimum-cost diets, as hypothesized by Smith.

This selection costs less than in 1976 prices than Stinger's both in absolute and relative terms. A price check at local supermarkets on 15 July, 1976, showed that the Stinger August, 1939 diet in Table 1 with only 5 foods costs 71.2¢ a day and the August, 1944 diet costs 62.9¢. A comparison in relative terms was made by the BLS food price index, which was 359.4% in January, 1976 of the August, 1944 level. Thus, in index terms, food costing 16.4¢ a day in 1944 would come to 59.0¢, or approximately 7¢ more than our subsistence diet, which has a greater variety of foods.

A comparison of this type, however, covering a span of 32 yr, should be interpreted with caution because of changes in the products and marketing practices, and in the composition of the indices. For example, frozen spinach was substituted for spinach in the price check. Comparisons in index terms based upon a very small subset of the foods available in grocery stores and supermarkets may not be valid. Nevertheless, these results show that given a sufficiently large group of foods to choose from, LP is a powerful tool in minimizing the cost of subsistence.

Model 2

The Model-2 optimum diet for subsistence plus palatability in Table 4 has 68 foods and costs

$34.54 per month, or $1.15 a day. This diet in Table 4 appears to be a relatively good one in the opinion of nutritional consultants Leidigh and West. Not only does it have variety, but every major category of food is represented (appetizers, beverages, breads, etc.). There are exactly 60 servings of meat or chicken, two for each day. There is an interesting variety of vegetables, and salad dressings. There are no fresh fruits, however, since canned fruit juices appear to be a more economical way to obtain the same food values.

As in the case with Model 1, the solution contains sugar and sweets and sugar-rich cookies. This is a reflection of the calorie scarcity in minimum-cost diets hypothesized by Smith. While these results tend to support the calorie-shortage theory, Smith's hypotheses about vitamin A and vitamin C deficiencies are not confirmed in Model 2. There is more discussion on this point in the sequel.

The separation ratings and corresponding upper bounds on servings and the 12 palatability constraints were very effective in causing variety, since 80% of the foods in this diet (56 out of 68) are at their respective upper bounds. The computer had to seek out other foods to substitute for those at their upper limits, but did so in the most economical way possible so as to minimize the cost of consumption.

Table 4. Model 2 diet, nutrition plus palatability

food	servings	unit weight (grams)	unit cost	total cost
appetizers and snack foods				
almonds	2	10	.013	.03
bananas	8	75	.046	.37
carrots	10	30	.021	.21
stalk celery	7	10	.049	.34
peanut butter	14	20	.04	.56
roasted peanuts	2	20	.036	.07
sour pickles	4	30	.03	.12
sweet gherkin	1	10	.023	.02
potato chips	8	10	.027	.22
beverages				
buttermilk	3	244	.098	.29
chocolate milk	12	244	.132	1.58
whole milk	45	244	.10	4.50
breads				
bran raisin bread	9	48	.06	.54
cracked wheat bread	64	23	.022	1.41
rye bread	1	23	.022	.02
white muffins	10	40	.046	.46
cornmeal muffins	10	45	.066	.66
cereal products				
quick cream of wheat	11	38	.043	.47
farina	12	38	.043	.52
hominy grits	7	36	.027	.19
enriched dry macaroni	3	110	.11	.33
spaghetti, cooked	2	150	.087	.17
macaroni and cheese	3	225	.173	.52
macaroon cookies	6	14.6	.044	.26
oatmeal cookies	6	18.8	.033	.20
sandwich cookies	6	20	.039	.23
dairy products or substitutes				
oleomargarine	90	28	.018	1.62
desserts				
peanut cookies	2	12	.023	.05
frozen apple pie	5	160	.215	1.08
blueberry pie	3	160	.186	.56
fruit juices				
grapefruit juice	1	100	.042	.04
grape juice drink	12	100	.044	.53
pineapple juice	9	30	.016	.14
meats				
beef, chuck	8	113	.32	2.56
corned beef hash	2	115	.195	.39
hamburgers	16	85	.175	2.80
chili con carne w/beans	2	142	.18	.36
deviled meat	1	20	.105	.10
beef kidney	2	93	.15	.30
tripe beef	1	85	.11	.11
luncheon meat	6	30	.13	.78
pork sausage	4	100	.30	1.20
vienna sausage	4	18	.04	.16

Table 4. (*Contd*).

nut products				
shelled mixed nuts	1	15	.044	.04
peas	4	100	.064	.26
sunflower seed kernels	1	100	.235	.24
poultry and game				
chicken fryer breast	12	96	.20	2.40
stewing chicken hen	2	100	.201	.40
salad dressing				
French dressing	1	14	.033	.03
Italian dressing	6	14	.033	.20
mayonnaise	6	14	.029	.17
thousand island	6	14	.033	.20
sauce				
butterscotch sauce	1	44	.071	.07
syrup and sugar				
brown dark sugar	1	14	.01	.01
white sugar	20	8	.008	.16
sweets				
plain caramels	2	28	.063	.13
grape jam	3	20	.028	.08
orange marmalade	3	20	.026	.08
vegetables				
canned beans w/ pork and tomato	4	125	.094	.38
dry pinto beans	4	100	.073	.29
red beets	4	83	.035	.14
cabbage	9	100	.037	.33
mature onion	13	100	.033	.43
white potato	12	100	.064	.77
tomato catsup	10	17	.013	.13
white root turnip	4	100	.044	.18
yams	4	100	.075	.30
prepared brown mustard	4	5	.005	.02
total				$34.51

A qualifying statement

The discussion in this paper is as if the Model-1 and Model-2 solutions in Tables 3 and 4 are the optimum solutions of their respective MIP problems, defined by the sets of constraints and bounds. For both cases, however, it is not possible at this time to prove unconditionally that these are the all-integer (i.e. in the number of servings) optimum. The reason for this is that in both cases the computer branch-and-bound iterations, each of which is the optimum solution of a completely specified LP problem, did not converge in the sense of completing the search of the entire decision tree of possibilities.

Although optimality cannot be guaranteed, these solutions are close enough that the additional cost in computer time is not worth further search. Approximately 8 hr of computer time (360 – 65) were consumed altogether spread out over several experiments in arranging sets of integer variables to try to induce convergence. There were about 75,000 branch-and-bound iterations altogether. Suffice it to say that the all-integer subsistence only solution in Table 3 is only 10¢ per month ($15.55 rather than $15.45) higher than the noninteger optimum, and the all-integer Model 2 solution in Table 4 is only 1¢ per month ($34.51 rather than $34.50) higher than the noninteger optimum.

The fact that the solution did not converge can be explained by two factors: first, the size of the problem, and second, the fact that there is a very large number of nearly optimal LP solutions which are almost as good. The size of the problem is a factor, because there are 392 integer variables, each one representing a food. This means that the size of the decision tree to be searched can be up to approximately 2^{392} (approximately 10^{100}) nodes.

Near optimality of many solutions is also an explanation of lack of convergence, because even in a very large MIP problem solved by branch-and-bound, the speed of convergence to the integer optimum depends on whether or not some sets of basic solutions clearly dominate others, or whether or not some branches lead to infeasibilities or unbounded solutions. The fact that the computer continued iterating for a long time after finding an integer solution, and the run had to be prematurely terminated, shows that there are still many feasible unbounded solutions with nearly all-integer optima to be examined before converging.

Measuring the Stinger Gap

In his 1963 book, Smith[13, p. 122] coined the term "Stinger gap", and used it in several different ways, one applying to commodities, the difference between the price of a pound of a commodity and the dollar value of the nutrients that a pound of it contains; the other meaning is the difference between the cost of subsistence and the cost of a diet which satisfies cultural or taste preferences. The latter meaning is the appropriate one for this study. The Stinger gap estimate at 1976 prices is the difference between the total cost of the Model-2 and Model-1 diets, $1.15 - $0.52 = $0.63.

Since this measurement depends on current prices in a particular locality, average nutrient contents and the kind of constraints used to obtain both diets, it is difficult to generalize from this result as to what the gap is or should be. Obviously it varies from country to country, locality to locality, and time to time. Further search by nutritionists may help to ascertain whether this is a useful concept, and if it is, to measure it under a greater diversity of circumstances.

Satisfying nutritional needs

At best, specifying the nutritional needs for an adult male age 23–50 or for any age group can be a conjecture. It seems more natural to give a range of values, as in Table 2, rather than a specific quantity of each nutrient as in[9]. These ranges, however, are not standardized. Further research by nutritional scientists to determine better upper and lower limits for each nutrient would probably lead to improved LP solutions of the diet problem in the future.

LP has the remarkable feature that a solution is not feasible unless all constraints are satisfied. Once a model is specified and the data are given, there is no possibility of reaching an unacceptable conclusion unless there is an error in the model, the data or both. Since the nutritional needs of Table 2 are constraints for both Model 1 and Model 2, it is interesting to determine how these needs are met. Table 5, which gives both the upper and lower monthly limits for each of the 11 nutrients, and the quantity of each nutrient in both models, facilitates this comparison.

Calories and magnesium are at their respective lower limits in both models; calcium, niacin, riboflavin and vitamin C (ascorbic acid) are at or near lower limits in Model 1; phosphorous is at the upper limit in both models; and vitamin A is at the upper limit in Model 2. It should also be noted that vitamin A is at the upper limit in the noninteger optimum solution in Model 1, which preceded the branch-and-bound iterations to derive the integer optimum. Vitamin C is at the lower limit in Model 1. Thus, from our point of view, Smith's hypotheses about "shortages" in minimum cost diets are disprovable.

Table 5. Comparison of Model 1 and Model 2

nutrient	units	Model 1	Model 2	lower limit	upper limit
calories	calories	75,005	75,004	75,000	85,500
calcium	mg.	15,080	27,990	15,000	36,000
phosphorus	mg.	40,949	41,165	21,000	42,000
thiamin	mg.	75	37	27	75
niacin	mg.	450	808	450	
vitamin A	I.U.	149,535	164,992	111,000	165,000
protein	gm.	2,236	2,543	1,500	3,500
magnesium	mg.	3,016	3,000	3,000	12,000
iron	mg.	647	428	180	750
riboflavin	mg.	42	52	42	75
ascorbic acid	mg.	901	1,448	900	2,500

Sensitivity to price changes and alterations of the model

Considering the fact that the constraints, limits, serving sizes and separation indices are all somewhat arbitrary, the food values are averages, and food prices fluctuate, no attempt should be made to interpret these results as the optimum diet. Indeed, extensive experience with branch-and-bound iterations shows that there are possibly many near optimum diets.

This diversity of optimum solutions should be a source of encouragement to nutritionists and dietitians who may be interested in computerized institutional menu planning or the construction of economical grocery shopping lists for low-income families. It shows that there is a possibility of a wide variety of choices to satisfy local or regional preferences or tastes, prices and constraints.

SUMMARY

The primary objective of this paper was to measure the "Stinger gap", the difference between the cost of subsistence, and the cost of an optimum diet with palatability, in current prices. Based on January, 1976 prices, this is the difference in cost between Model 2 and Model 1 or $1.15 − $0.52 = $0.63.

Smith's hypothesis about a calorie scarcity in minimum cost diet is confirmed. His hypotheses about vitamin C and vitamin A shortages are contradicted. Vitamin C is in ample supply in both diets, but at the lower limit in Model 1. Vitamin A is at the upper limit. This may be explained by the fact that both upper and lower limits were set on nutrients, whereas Smith used only lower limits, except for calories, and also that we used better control of portions and serving sizes.

This paper is a further demonstration in the applicability of LP to the solution of problems in human nutrition. It shows that LP is well suited to this task, and that many solutions can be obtained, which can be suited to a wide variety of dietetic theories and local preferences. Parenthetically, it also shows that MIP is uneconomical because of convergence problems, and that satisfactory solutions are obtained with ordinary LP.

Further research may be helpful in establishing whether the concept of a "Stinger gap" is a useful one, and, if so, how it can be measured with a more or less standardized model which makes proper allowances for regional tastes and preferences. It would be helpful for this purpose if the recommended dietary allowances for nutrients in[9] were given as ranges rather than as specified values.

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