THE DIET PROBLEM REVISITED

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The "diet problem" has long fascinated economists because it is a direct and obvious application of the constrained cost minimization procedure that economists teach to students at all levels. The relevance of the diet problem to economist and citizen alike has been intensified by the recent surge in food prices. The need of consumers to minimize the cost of satisfying basic nutritional requirements is hardly an inconsequential matter in a world where at least 80 percent of all individuals fall short of meeting these requirements in any given year.[1]

This paper updates and extends the original diet problem presented by George Stigler in the year 1945.[2] Since Stigler's seminal work, important changes have occurred which have a strong bearing not only upon the formulation of the problem, but also upon the methodology by which the problem itself is solved. First, food prices have risen much more rapidly than the Consumer Price Index since Stigler's work.[3] Second, nutritionists today have identified almost twice as many basic nutritional requirements as existed in 1945. Third, there are far more varieties of foods available today than existed in 1945. For example, frozen foods are commonplace in supermarkets today, but were rare in 1945. Fourth, it was not until 1947 that George Dantzig perfected the simplex algorithm for the solution of linear programming problems such as the diet problem.[4] Hence, there exists both academic and practical interest in the solution of a 1975-vintage diet problem.[5]

I. 1975 Foods, Prices, and Nutritional Content

Stigler utilized 77 different foods in an application of the diet problem to nutritional and price situation of August, 1939, and increased this number of foods to 80 in an August, 1944, application. The current application utilizes 117 different foods commonly available at a typical supermarket in July, 1975. The prices of the foods are those actually observed in supermarkets in the Bloomington-Normal, Illinois, Standard Metropolitan Statistical Area on July 15, 1975. It was decided that these prices were preferable to Bureau of Labor Statistics price indexes for individual foods because the Bureau's data do not include many of the 117 items included here.

Since a total of 16 quantifiable nutritional requirements now exist[6] (an increase of seven over the year 1945), there are 16 pieces of nutritional data associated with each particular food. All entries are stated in terms of nutrient received per ounce of the food. The nutritional requirements themselves are reported in Table 1 for years 1945 and 1975. The 1975 recommendations represent the "recommended daily dietary allowances" for a male, aged 23–50, who is 154 pounds in weight and 69 inches in height. The 1945 recommendations pertain only to an undifferentiated 154 pound man. It is apparent from Table 1 that not only are there seven additional nutritional requirements in 1975, but also six of the nine 1945 requirements have been altered in size since that time. A closer perusal of Table 1 reveals that in every case where an existing nutritional requirement was revised, the new requirement is less demanding. For example, the 1945 requirement stipulated 3,000 calories, whereas the 1975 requirement was only 2,700 calories. This means that, holding prices and dietary requirements constant at their 1945 levels, it is possible to satisfy all of the 1945 requirements less expensively in 1975.

II. The Model and the Solution: 1975

The task is to minimize an objective function representing the cost of food purchases subject to constraints reflecting the 16 nutritional requirements. That is,

Minimize:

$$\alpha = P_1X_1 + P_2X_2 + \cdots + P_NX_N \qquad (1)$$

Subject to:

$$\begin{aligned} a_{11}X_1 + a_{21}X_2 + \cdots + a_{N1}X_N &\geq K_1 \\ a_{12}X_1 + a_{22}X_2 + \cdots + a_{N2}X_N &\geq K_2 \\ \vdots \qquad\quad \vdots \qquad\qquad \vdots \qquad\quad &\vdots \\ a_{1M}X_1 + a_{2M}X_2 + \cdots + a_{NM}X_N &\geq K_M \end{aligned} \qquad (2)$$

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TABLE 1: Recommended Daily Dietary Allowances: 154 Pound Man: 1945 and 1975

Nutrient	1945 Requirements	1975 Requirements
1. Calories	3,000 calories	2,700 calories
2. Protein	70 grams	56 grams
3. Calcium	800 milligrams	800 milligrams
4. Iron	12 milligrams	10 milligrams
5. Vitamin A	5,000 Intl. Units	5,000 Int'l. Units
6. Thiamine (Vit. B_1)	1.8 milligrams	1.4 milligrams
7. Riboflavin (Vit. B_2)	2.7 milligrams	1.6 milligrams
8. Niacin	18 milligrams	18 milligrams
9. Ascorbic Acid (Vit. C)	75 milligrams	45 milligrams
10. Phosphorus	—	800 milligrams
11. Magnesium	—	350 milligrams
12. Vitamin B_6	—	2 milligrams
13. Vitamin B_{12}	—	3 μ-grams
14. Vitamin E	—	15 milligrams
15. Zinc	—	15 milligrams
16. Folacin	—	400 μ-grams

NOTE: The 1945 Recommended Daily Dietary Allowances were reported by George J. Stigler, "The Cost of Subsistence," *Journal of Farm Economics*, 27 (May, 1945), p. 305, and were taken from National Research Council, Recommended Dietary Allowances, Reprint and Circular Series No. 115, January, 1943. They represent recommendations for a 154 pound man. The 1975 Recommended Daily Dietary Allowances were taken from Recommended Daily Dietary Allowances, Food and Nutrition Board, National Academy of Sciences-National Research Council, *Nutrition Reviews*, 33 (May, 1975), p. 154–5. They represent recommendations for a 154 pound man, aged 23–50, whose height is 69 inches.

where:

X_i = foods
P_i = prices of the foods per ounce
a_{ij} = number of units of nutritional requirement "j" that are satisfied by one ounce of food "i"
K_j = nutritional requirement "j" in terms of
$N = 117$
$M = 16$

The optimal solution for July, 1975, is reported in Table 2. Despite the addition of seven nutritional requirements, only seven foods are needed to minimize the cost of meeting 16 basic nutritional requirements. By way of contrast, only five foods were needed to satisfy the nine nutritional requirements imposed by Stigler in each of his applications. Table 3 indicates the percent of the daily nutritional allowance that is satisfied by the seven foods. These range from a low of 100 percent for seven different requirements, to a high of 308 percent for the Vitamin B_{12} requirement.

Column (3) in Table 2 reveals that the daily cost of optimal 1975 solution is 43.72 cents, while

TABLE 2: Minimum Cost Diet Satisfying 16 Nutritional Requirements July, 1975

(1) Food item	(2) Daily amount in ounces	(3) Daily cost in cents	(4) Yearly amount in pounds	(5) Yearly cost in dollars
1. Kidneys, beef	.403 oz.	.73c	9.18 lbs.	$ 2.66
2. Milk, non-fat solid	1.068	6.32	24.36	23.07
3. Beans, red	4.734	8.38	107.99	30.59
4. Cabbage	2.299	3.01	52.45	10.99
5 Spinach	2.139	3.08	48.79	11.24
6. Flour, all-purpose	11.595	9.39	264.50	34.27
7. Flour, whole wheat	9.777	12.81	223.03	46.76
Totals	32.015 oz.	43.72c	730.30 lbs	$159.58

TABLE 3: Percent of Daily Allowances Fulfilled by Minimum Cost Diet

Nutrient Requirement	Recommended Daily allowance	Percent of allowance fulfilled by diet
1. Calories	2,700 calories	100
2. Protein	56 grams	206
3. Calcium	800 milligrams	100
4. Iron	10 milligrams	235
5. Vitamin A	5,000 Int'l. Units	100
6. Thiamine (Vit. B_1)	1.4 milligrams	182
7. Riboflavin (Vit. B_2)	1.6 milligrams	100
8. Niacin	18 milligrams	152
9. Ascorbic Acid (Vit. C)	45 milligrams	100
10. Phosphorus	800 milligrams	275
11. Magnesium	350 milligrams	197
12. Vitamin B_6	2 milligrams	106
13. Vitamin B_{12}	3 μ-grams	308
14. Vitamin E	15 milligrams	291
15. Zinc	15 milligrams	100
16. Folacin	400 μ-grams	100

TABLE 4: Comparative Diet Problem Solutions

(1) Stigler, August, 1939

Wheat Flour Evaporated Milk Cabbage Spinach Dried Navy Beans	Daily Cost = 10.94¢ Annual Cost = $39.93

(2) Stigler, August, 1944

Wheat Flour Cabbage Spinach Pancake Flour Pork Liver	Daily Cost = 16.41¢ Annual Cost = $59.88

(3) 1975: 1945 Constraints, 1945 Allowances

Kidneys, beef Milk nonfat solid Cabbage Soybeans, dried Spinach, canned Flour, all-purpose	Daily Cost = 44.83¢ Annual Cost = $163.63

(4) 1945 Constraints, 1975 Allowances

Kidneys, beef Milk, nonfat solid Cabbage Soybeans, dried Spinach, canned Flour, all-purpose	Daily Cost = 38.12¢ Annual Cost = $139.14

(5) 1975: 1975 Constraints, 1975 Allowances

Kidneys, beef Milk, nonfat solid Beans, red Cabbage Spinach, raw Flour, all-purpose Flour, whole wheat	Daily Cost = 43.72¢ Annual Cost = $159.58

the annual cost is $159.58. Again, by way of contrast, Stigler's August, 1939, solution cost 10.94 cents per day, and $39.93 annually. His August, 1944, solution cost 16.41 cents per day, and $59.88 annually.

A large proportion of the increased cost of the minimum cost diet is due to the price increases that have occurred for most foods since Stigler's work. However, some of the increased cost of the minimum cost diet is due to the additional seven nutrient requirements that have been imposed since that time. Table 4 reports the 1975 solution noted above, but also reports the 1975 solution for the case where only nine nutrient constraints are imposed as per Stigler's work. Since six of the nine daily allowances in the constraints have also been adjusted since that time, a result is reported both for the 1945 level allowances and the 1975 level allowances. When only nine nutritional constraints are imposed, and current allowances used,

the cost of the minimum cost diet falls from 43.72 cents per day ($159.58 annually) to 38.12 cents per day ($139.14 annually). On the other hand, because the 1945 daily nutrient allowances were higher in six of nine cases, the cost of the minimum diet in terms of those allowances (see Table 1) rises to 44.83 cents per day ($163.63 annually). As Table 4 reveals, the cost of satisfying the seven additional nutrient requirements is (in terms of 1975 prices and allowances) $159.58 − $139.14 = $20.44.

III. Final Considerations

The minimum cost diet found by Stigler in August, 1944, cost 16.41 cents per day; the comparable minimum cost diet in 1975 (for 1945 constraints and allowances) cost 44.83 cents per day. This is a 173.2 percent increase. During the same time period, however, the food price index of the Bureau of Labor Statistics increased over 237.7 percent.[7] Hence, the cost of more efficient food sources declined relative to the cost of other foods. An important reason for this may be the propensity of today's consumers to include in their diets increasingly large amounts of food purchased at "fast food" outlets. The purchaser of such an item may well "deserve a break today;" however, the item in question is demonstrably an inefficient purchase in terms of nutrients obtained per penny of expenditure.[8] It should also be pointed out here that the cost of the minimum cost diet has increased less than the Consumer Price Index during the 1945–1975 time period as well.[9] Hence, the constrained, cost-minimizing consumer finds the minimum cost diet relatively less of a burden upon his income in the year 1975 than he did in the year 1945.

It is interesting to note that despite the obvious changes that occurred since 1945 in prices, foods available, and nutrient requirements, that the food items included in the various diet problem solutions recorded in Table 4 are substantially similar. All five of the food items included in Stigler's August, 1939, solution are either included in the 1975 solution, or very close substitutes appear instead. Relatively "efficient" foods (in terms of nutrient obtained per penny of expenditure) have not changed much in identity since Stigler's original rendition of the problem. This is somewhat remarkable in view of the many changes that have occurred since that time.

A final set of caveats is necessary. The minimum cost diet is computed without reference to the taste, appearance, or esthetic value of the food involved. Similarly, no weight of any kind is given to the circumstances or atmosphere in which food is consumed. There can also be considerable variance in the nutrient value of a particular food due to differing conditions of storage, preparation, and aging. Further, it is not always possible to extract all nutrient values from a food and in many cases some or all of a particular food may be wasted in preparation. Hence, one cannot apply the solution dictated by the minimum cost diet blindly without reference to these factors. Nonetheless, the data contained in the Appendix and the solution to the 1975 diet problem can be utilized by consumers for efficient and intelligent diet planning.

Notes

1. Sohan L. Monocha, *Nutrition and Our Overpopulated Planet* (Springfield, Illinois: Charles C. Thomas, 1975), Chapter One. See also Alan D. Berg, *The Nutrition Factor* (Washington, D.C.: The Brookings Institution, 1973).
2. George J. Stigler, "The Cost of Subsistence," *Journal of Farm Economics*, 27 (May, 1945), pp. 303–14.
3. The price index for food (1967 = 100.0) was 50.7 in 1945 and 171.2 in April, 1975. The Consumer Price Index for all items was 53.9 in 1945 and was 158.6 in April, 1975. Therefore, the food price index increased 237.7 percent during that time period, while the Consumer Price Index increased only 194.2 percent in the same time period.
4. Stigler generated a solution to his original diet problem by logic and careful deduction, since the simplex algorithm was not available at the time. For example, he eliminated all foods from consideration whose nutritional values per dollar were exceeded in every category by some other food. Stigler commented that "Thereafter the procedure is experimental because there does not appear to be any direct method of finding the minimum of a linear function subject to linear conditions." George J. Stigler, p. 310.
5. The exposition of the diet problem has been commonplace in textbooks since 1945. However, there have been comparatively few general applications of it since then. Among the notable exceptions are Victor E. Smith, *Electronic Computation of Diets* (East Lansing, Michigan: Bureau of Business and Economic Research, Graduate School of Business Administration, Michigan State University, 1964); and, Jose D. Langier, *Economical and Nutritional Diets Using Scarce Resources* (East Lansing, Michigan: Institute for International Business and Economic Development Studies, Graduate School of Business Administration, Michigan State University, 1969).
6. The Food and Nutrition Board of the National Academy of Sciences, National Research Council, actually identifies 19 different recommended daily dietary allowances. However, the quantity of each of these nutrients in certain foods is not always known, even in common foods. Hence, it was necessary to eliminate three of these requirements from further consideration.
7. See footnote 3 for a derivation of the increase in the food price index.

8. "How Nutritious Are Fast-Food Meals?" *Consumer Reports,* 40 (May, 1975), pp. 278–81.
9. As footnote 1 indicates, the increase in the Consumer Price Index during the time period 1945–1975 was 194.2 percent. The increase in the cost of the minimum cost diet during the August, 1944–April, 1975, time period was only 171.2 percent.

References

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