Steer calves of Angus and Hereford breeding averaged considerably higher in price than Shorthorn, Hereford-Shorthorn and Hereford-Angus steers. Also, when the three breeds—Angus, Hereford, and Shorthorn—were combined, they averaged significantly higher in price than the crossbred steers. Heifer calves of Angus breeding showed an obvious price advantage over Hereford and Shorthorn heifers. The straightbred Angus, Hereford, and Shorthorn heifers averaged substantially more per hundredweight than comparable crossbred calves.

Buyers patronizing the special sales during the period covered by this study preferred calves weighing between 400 and 500 pounds. Calves weighing between 300 and 400 or between 500 and 600 pounds were penalized slightly in price per hundredweight. Calves weighing over 600 pounds sold for considerably less per hundredweight than lighter-weight calves.

The average price spread between "medium," "good," and "fancy and choice" steer calves was in excess of $2.00 per hundredweight. The spread in price between similar grades of heifer calves was slightly less than $1.50 per hundredweight.

THE DIET PROBLEM REVISITED: A LINEAR PROGRAMMING MODEL FOR CONVEX ECONOMISTS

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IT IS almost axiomatic that a text in operations research or linear programming includes an example of the classic diet problem, preferably in an introductory chapter, before moving on to more practical applications, e.g., nut and gasoline blending problems.[1] Typically, the objective function to be minimized is the monetary cost of the diet, subject to a number of constraints which pertain to minimum food value requirements. However, in today's affluent society it may be argued that the principle dietary problem of the modern sedentary American is one of opulence, or perhaps corpulence, rather than cost and that the proper objective of a diet should be to minimize caloric input rather than monetary output. Consequently, the purpose of this paper is to utilize the general linear-programming technique in order to derive such an optimal diet and still provide some leeway for indulgence so that the

[1] For example, see R. Dorfman, P. Samuelson, and R. Solow, *Linear Programming and Economic Analysis* (New York, 1958), pp. 9-31; D. Gale, *Theory of Linear Economic Models* (New York, 1960), pp. 1-4; W. Garvin, *Introduction to Linear Programming* (New York, 1960), pp. 58-61; and S. Gass, *Linear Programming* (New York, 1958), pp. 9-10.

finicky diner is not restricted to an unpalatable sequence of identical meals.

The original linear-programming formulation of the minimum-cost diet was subject to nine constraints and included 86 choice variables, i.e., foods and slack variables, hence restricting the diet to a maximum of nine foods.[2] Two solutions were derived, one via the simplex procedure and the other via a systematic trial-and-error method. The simplex-derived diet consisted of assorted quantities of wheat flour, evaporated milk, cabbage, spinach, dried navy beans, corn meal, peanut butter, lard, potatoes, and beef liver, whereas the other excluded the latter five foods. The annual cost of either diet was under $40 in 1939 prices.[3]

The Revised Problem

The revised diet problem is as follows: select $x_1, x_2, \ldots, x_n$ from the field of real numbers so as to minimize the linear function

$$z = c_1x_1 + c_2x_2 + \cdots + c_nx_n, \tag{1}$$

subject to

$$\begin{array}{l} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \geqq b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \geqq b_2 \\ \cdots\cdots\cdots\cdots\cdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \geqq b_n \end{array} \tag{2}$$

$$x_j \geqq 0, \qquad (j = 1, 2, \cdots, n),$$

where the x_j denote the quantities of foods; the c_j denote the caloric content of the foods; the b_j are assorted minimum daily requirements of designated vitamins and minerals; and the a_{ij} denote the vitamin or mineral content with respect to a unit of the jth food and the ith nutrient requirement.

A simple hypothetical model which can be solved graphically in a two-dimensional Cartesian coordinate system may help to illustrate the problem and its solution. Consider two foods and three nutrient requirements as shown in Table 1. Furthermore, the minimum daily requirements for vitamin B_1 and protein consumption are one milligram and 70 grams, respectively. Hence we wish to minimize the objective function

$$z = 30x_1 + 229x_2 \tag{1'}$$

[2] G. Stigler, "The Cost of Subsistence," *Journal of Farm Economics*, Vol. 27 (1945), pp. 303-314.

[3] Stigler later revised the cost of the abbreviated diet up to $8 per month in 1945 prices. See *The Theory of Price* (New York, 1946), p. 2. Of course, changes of relative prices over the period probably made the diet no longer optimal.

TABLE 1

Food	Vitamin B_1 content (milligrams)	Protein content (grams)	Iron (milligrams)	Calories
beef broth (one cup)	0	4	0	30
pecan nuts (10)	1/10	3	4/5	229

subject to

$$0x_1 + \frac{1}{10}x_2 \geq 1 \tag{2a'}$$

$$4x_1 + 3x_2 \geq 70 \tag{2b'}$$

$$0x_1 + \frac{4}{5}x_2 \geq 10, \tag{2c'}$$

where x_1 is the number of cups of beef broth in the diet and x_2 (times ten) is the daily intake of pecan nuts. The results are drawn in Figure 1. The inequalities associated with the constraints generate a convex set, S, in two-dimensional space[4] such that all points in S satisfy the inequalities. Constraint (2c′) is immediately noted to be redundant inasmuch as it is automatically satisfied by the more binding (2a′). The optimal solution is found where the objective function is tangent to an extreme point of the solution space S in a southwestern direction, and it is easily seen that the optimal diet consists of ten cups of beef broth and 100 pecan nuts, with a daily intake of 2,590 calories. Fortunately, a larger quantity of foods, even though accompanied by a greater number of constraints, makes it possible to affect substantial reductions in daily calorie consumption, albeit at the cost of a substantial increase in computational difficulties.

The Optimal Diet

It is extremely difficult to determine what constraints should be considered in the model, especially since minimum requirements for some nutrients have never been established. Moreover, available data are likely to be inadequate inasmuch as the vitamin and mineral contents of many kinds of food are apt to be dependent upon the nature of the soil in which they are grown, etc. Finally, these problems are compounded in that some vitamins, particularly the B complex, tend to sup-

[4] Briefly, a convex set S in two-space is defined as a space such that all of the points on a straight line drawn between any two points in the space lie within S. The implications of the paper's title should now be clear to the non-mathematical reader.

plement each other so that in the proper combinations the requirements are less restrictive than when each vitamin is considered as a separate entity.

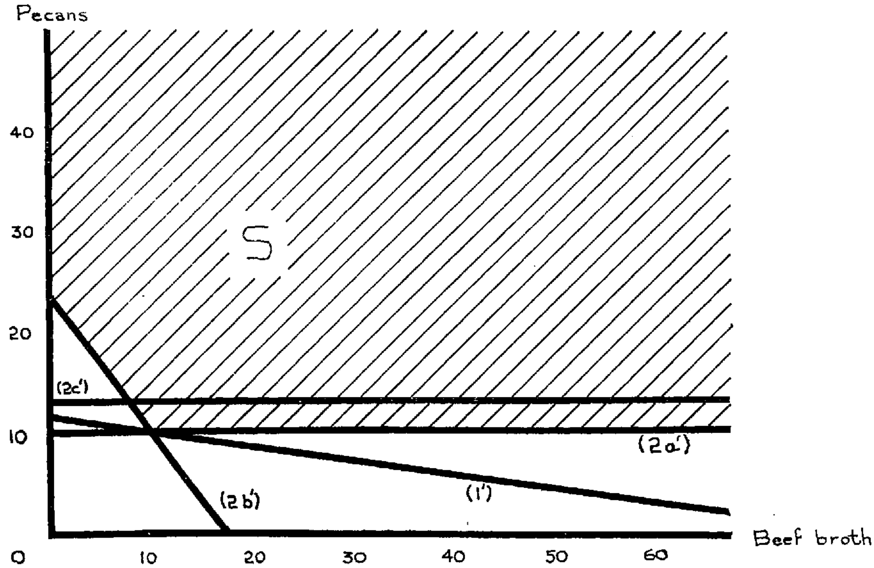

FIGURE 1

Table 2 shows the nutrient requirements included in the problem's constraints and the minimum daily needs with respect to each.[5] At most, therefore, eight foods can be included in the optimal diet, placing a severe strain on the palate of the dieter. A possible escape from this unappetizing dilemma would be to construct the initial model so as to take into consideration a larger variety of foods. This can be done by including within the constraints small non-negative quantities of other foods.[6]

TABLE 2. MINIMUM DAILY REQUIREMENTS

Nutrient	Minimum daily requirement
Vitamin A	5,000 U.S.P. units
Vitamin B_1	1 milligram
Vitamin B_2	2 milligrams
Vitamin C	30 milligrams
Vitamin E	30 milligrams
Calcium	750 milligrams
Iron	10 milligrams
Protein	70 grams

[5] The diet is set up on a daily basis because the human body is capable of storing vitamin C only for very short periods of time.
[6] Gass, *op. cit.*, pp. 171-172.

However, the approach to be taken here is that such variety can be taken into account, once the optimal diet is determined, in such a manner so as to set the desired calorie intake at any desired maximum subject to the constraint that the maximum must be at least as large as the minimum solution arrived at by the linear-programming solution.

Almost 400 foods were considered for inclusion into the model. Many were found to be dominated completely by other foods, i.e., to contain smaller quantities of nutrients while having more calories, and were hence dropped from the model. Others were eliminated on the basis of not being available during certain seasons of the year or falling into those categories of foods which are not readily found in neighborhood supermarkets, e.g., Japanese persimmons. Finally, some foods were discarded as being unpalatable.[7] The remaining foods are shown in Table 3. It should be noted that all of the foods contained in the optimal solution of the original diet problem were deliberately included.

Hence the simplex tableau contains eight constraints and 40 variables, including eight slack variables, which were inserted in order to convert the inequalities into equalities and provide an initial basic feasible solution.[8] The optimal daily diet was found to consist of specific quantities of only four foods, namely 14.08 ounces of haddock, 40.3 lettuce leaves, 1.4 cups of cooked turnip tops, and slightly less than $\frac{1}{32}$ cup of wheat germ.[9] The minimal value of the objective function was 349.06 calories per diem. Vitamin and other nutrient contents of the diet are given in Table 4. It is worth remarking that some of the results are substantially above the minimum daily requirements.

If the commercial 900 calorie diets, which have recently occupied prominent places on drug store and supermarket shelves, are accepted as criterion, the above diet has obvious advantages. On the one hand, it contains 550 fewer calories. Secondly, even though consisting of only four foods, it clearly offers more variety than any of the powdered or

[7] The writer confesses to making certain value judgments in this case. Readers who may be placed on a higher indifference curve by the addition to their diet of watercress and barley will undoubtedly wish to formulate their own models.

[8] Impecunious economists and graduate students may wish to add a ninth constraint relating to maximum food expenditures, inasmuch as the above diet would require a monthly outlay of approximately $25, assuming that one is able to acquire turnip tops at no charge from grocery stores. Otherwise, one would be in the position of requesting checkers to twist off the tops and throw the turnips away.

[9] A second-best diet consisted of 14.28 ounces of haddock, 66.2 lettuce leaves, ½ cup of cooked turnip tops, and 0.43 cups of cabbage, with a total calorie count of 349.28. Hence cabbage, which was also included in the optimal subsistence diet, clearly possesses the multiple virtues of high nutritive value, low cost, and slight calorie content.

TABLE 3. FOODS OF THE MODEL

beans, dried	dandelion greens, cooked
beans, Lima, green, cooked	lard
beans, string, green, cooked	lettuce, green
beef, lean	liver, beef
beet greens, cooked	milk, condensed
bread, whole-wheat	peanut butter
cabbage, green	peas, fresh, cooked
carrots	potatoes, white, baked
cauliflower	pork chops
cheese, Cheddar	prunes, dried
cheese, cottage	spinach, cooked
cornmeal, yellow	steak, beef
egg, whole	turnip tops, cooked
haddock	wheat germ
lamb chops	yeast, brewer's diet

TABLE 4

Nutrient	Daily consumption
Vitamin A	23,513.34 U.S.P. units
Vitamin B_1	1.00 milligrams
Vitamin B_2	2.60 milligrams
Vitamin C	210.21 milligrams
Vitamin E	47.26 milligrams
Calcium	750.00 milligrams
Iron	13.14 milligrams
Protein	70.05 grams

liquid commercial preparations. Moreover, hungry economists can add other foods to the minimal calorie diet and remain within a 900 calorie maximum, while those addicted to steak can consume almost ¾ pound of lean beef.

As previously noted, an alternative method of adding variety to the diet is to include minimal daily amounts of certain foods among the constraints. This can be done in such a manner as not to increase the number of variables and constraints over that of the original problem and has an advantage over the approach suggested in the previous paragraph in that the nutrient values of the added foods are included in the optimal solution, hence reducing the required intakes of haddock, lettuce, turnip tops, and wheat germ. At the same time, this technique has the disadvantage of increasing the caloric content of the optimal diet.

Finally, it should be remarked that the solution set is open to the northeast, suggesting that no finite solution exists if the objective function is to be maximized, rather than minimized. Hence those wishing to gain weight can do so without bound unless an additional constraint relating to some maximum capacity is added to the system.

The Dual

Using the previous notation, the dual of the revised diet problem may be written as follows: maximize

$$z' = b_1y_1 + b_2y_2 + \cdots + b_my_m \tag{3}$$

subject to

$$\begin{aligned} a_{11}y_1 + a_{21}y_2 + \cdots + a_{m1}y_m &\leqq c_1 \\ a_{12}y_1 + a_{22}y_2 + \cdots + a_{m2}y_m &\leqq c_2 \\ \cdots\cdots\cdots\cdots\cdots\cdots & \\ a_{1n}y_n + a_{2n}y_2 + \cdots + a_{mn}y_m &\leqq c_n \end{aligned} \tag{4}$$

$$y_i \geq 0, \qquad (i = 1, 2, \cdots, m).$$

Each of the numbers on the right-hand side of the constraints is a price in terms of calories, and the y_i hence may be defined as the caloric price of the nutrients. Therefore, the value of the objective function, z', is the imputed caloric value of the minimal nutrient requirements of the primal, and the dual problem amounts to assigning non-negative numbers to the vitamins and minerals so that the quantity of nutrition obtained in a diet is maximized and never exceeds its cost in calories.

CORRECTION: PH.D. DEGREES, 1960

The title of the thesis of Lowell Edwin Wilson was incorrectly reported in the listing of Ph.D. degrees conferred in Agricultural Economics in the May issue (43:495). The correct listing should be:

LOWELL EDWIN WILSON, B.S. Murray (Kentucky) State College 1953; M.S. University of Kentucky 1957; Ph.D. University of Illinois 1960, An Analysis of Distribution Costs and Margins for Fluid Milk.