The slingshot effect: explanation and analogies

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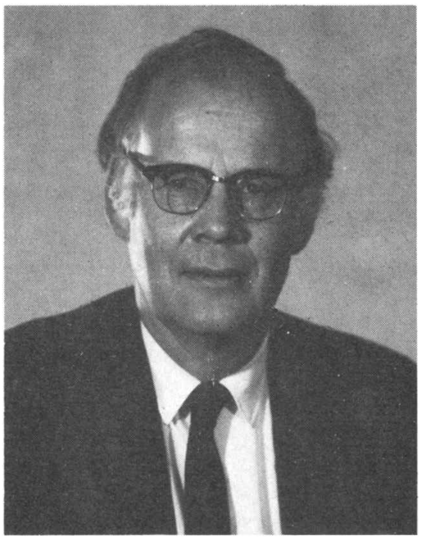

I. Introduction

As scientists send spacecraft on silent circuits through distant domains of the solar system (and beyond), they often use the "slingshot effect" of the gravitational field of a planet to increase or decrease the velocity of the spacecraft. This is frequently referred to as a "gravity assist." We wish to have a simple understanding of this effect.

II. Examples

The Voyager missions provide dramatic examples of the use of the slingshot effect. In a NASA report we read:

> The spacecraft will be launched in the late summer of 1977, and will fly past Jupiter in 1979. Using the big planet's immense gravity to boost them on their way, the (two) Voyagers will be accelerated toward Saturn, reaching the ringed planet in 1980 and 1981.[1]

Later in the same report we read.

> Controllers and scientists might pick a new path, using Saturn's gravity to boost Voyager 2 toward distant Uranus.[1]

In the abstract of the article "Voyager Mission Description," we read:

> Gravity-assist swingbys of Jupiter are utilized in order to reduce the launch energy demands needed to reach Saturn. In addition, a gravity-assist targeting option at Saturn will be maintained on the second-arriving Voyager for a possible continuation on to Uranus with its arrival in January of 1986.[2]

In the same article it is reported that:

> Two Voyager spacecrafts will be launched during a one-month launch opportunity which opens on August 20, 1977. This opportunity...was identified with a special alignment of the outer planets allowing the spacecraft to use multiple gravity assists to go from Jupiter to Saturn, then to Uranus, and finally to Neptune...[2]

The gravity assist was used recently in the mission of another spacecraft whose trajectory remains closer to the earth. Under the dateline "Greenbelt MD" the newspaper story reports that:

> Radio controllers guided an American spacecraft narrowly around the backside of the moon on Thursday, flinging it into a course that should carry the tiny observatory to a rendezvous with a comet in 1985.[3]

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The story continues:

> The International Sun-Earth Explorer 3, a [450 kg] observatory launched five years ago to do another job, got a slingshot-like gravity boost from the moon to put it on a course between the earth and the sun that should allow it to pass through the tail of the Giacobini-Zinner Comet on September 11, 1985.[3]

The story also reported that:

> The lunar maneuver not only snapped the spacecraft to a new trajectory but boosted its speed by about [9.8×10^2m/s to 2.3×10^3m/s]. The comet and the spacecraft will be [7.1×10^{10}m] from the earth at the moment of rendezvous.[3]

The complex maneuvering of this spacecraft, to bring it out to the comet, started in June of 1982 and involved a total of five close encounters with the moon.[4]

III. The Puzzle

These reports of the slingshot effect may well raise questions in the minds of our students. "The gravitational field is a conservative field. How can a space probe have a greater kinetic energy at a given distance from a planet, when the probe is leaving the planet, than it had at the same distance when it was approaching the planet?"

IV. The Explanation: Qualitative.

The answer is straightforward.[5] It can be illustrated by a simple example as shown in Fig. 1. A person (representing the earth) stands beside a busy street (which represents the orbit of an approaching planet). The person tosses a soccer ball (which represents the spacecraft) with a small initial velocity in front of a truck (which represents the planet) that is approaching with a high velocity V. The ball, nearly stationary, is struck by the truck. Let us assume that the collision is elastic. Measurements made by the person on the curb will show that the ball gained an amount of kinetic energy equal to the kinetic energy lost by the truck. The fractional increase in the kinetic energy of the ball is very large while the fractional loss of kinetic energy of the massive truck is very small. Now let's look at the collision in the frame of reference of the truck. Because of the large mass ratio, the frame of reference of the truck is almost coincident with the center of mass frame. The truck driver will see the ball approaching with a velocity V before the collision and will see it receding with a velocity B after the collision. The driver will report that the kinetic energy of the ball after the collision was the same as before the collision. The driver will also report that the interaction between the truck and the ball could be represented by a conservative force field. To the curbside observer, the interaction would not appear to be described by a conservative force field. Thus a given elastic interaction of two particles may or may not appear to be governed by a conservative field of force, depending on the frame of reference in which the interaction is observed.

This serves to remind us that all of our textbook discussions of conservative fields deal with frames of reference in which the source of the field is at rest. Our textbook discussions are all in the frame of reference of the truck driver.

It is very easy to translate this example of the truck and the ball to the interaction of a spacecraft and Jupiter. The observer on Jupiter will see the spacecraft having the same kinetic energy, at a given distance from the center of Jupiter, when the spacecraft is approaching as it has when it is leaving. The observer on Jupiter will describe the interaction in terms of a conservative force-field. An observer on the Sun will see the massive planet Jupiter moving with a high speed. A tiny spacecraft with very little kinetic energy is "hit" by Jupiter in an elastic collision and as a result, the spacecraft leaves the site of the "impact" with a velocity greater than Jupiter's. The observer on the Sun would be tempted to say that the elastic interaction is not governed by a conservative field of force.

There is one important difference between the ball-truck interaction and the spacecraft-Jupiter interaction. The ball-truck interaction is a repulsive force while the spacecraft-Jupiter interaction is the attractive interaction of Newton's Law of gravitation. In order that the small object gain energy, we have to throw the ball in front of the approaching truck; we have to throw the spacecraft behind the approaching Jupiter.

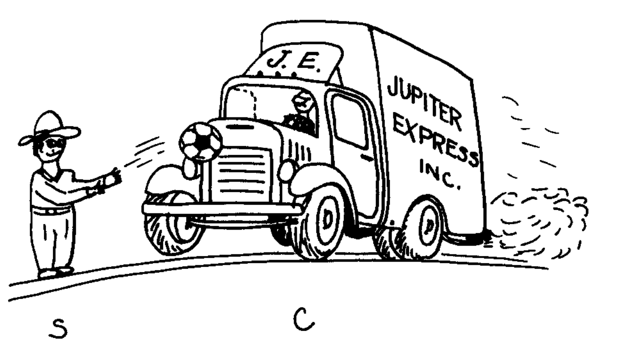

Fig. 1. The person S on the curb throws a soccer ball in front of a heavy truck that is approaching at high speed. The truck collides elastically with the ball. Person S sees the ball gain a great deal of kinetic energy in the collision, while the truck drive C sees the ball's kinetic energy unchanged by the collision.

V. Calculations: Outline

The details of orbit and trajectory calculations are analytically complex. Let us do a simple approximate calculation of a hypothetical interaction of a Voyager spacecraft with Jupiter. This will give us values for the angular deviation of the path of Voyager and for the changes in velocity, momentum, and energy that take place. For simplicity let us treat the interaction of Voyager and Jupiter as an elastic collision in two dimensions of two point particles. Thus we will examine the momenta of the two colliding particles (Jupiter and Voyager) only when they are separated by large distances before and after the "collision," which permits us to ignore the analytical complexities of the orbit of Voyager in the vicinity of Jupiter.

We will view the collision in two different frames of reference. The C frame is the one in which the center of mass of the two-particle system (Jupiter and Voyager) is at rest. The S frame is the one in which the Sun is at rest.

In a two-dimensional elastic collision of point particles of known constant mass with known initial velocities, we have four unknown quantitities after the collision. These are the magnitudes and directions of the velocities of the two particles. Only three equations are available to solve the problem. These three represent conservation of the x component of momentum, conservation of the y component of momentum, and conservation of energy. Thus it is necessary to assume a value of one of the four unknown quantities after the collision. Values of the other three quantities after the collision can then be determined from the simultaneous solution of the three equations. In order to complete the characterization of the collision we need to specify (in the S frame) the magnitude and direction of the velocity with which Voyager approaches Jupiter, the magnitude and direction of the velocity of Jupiter, and in the C frame we will choose to specify the direction of the path along which Voyager departs from Jupiter.

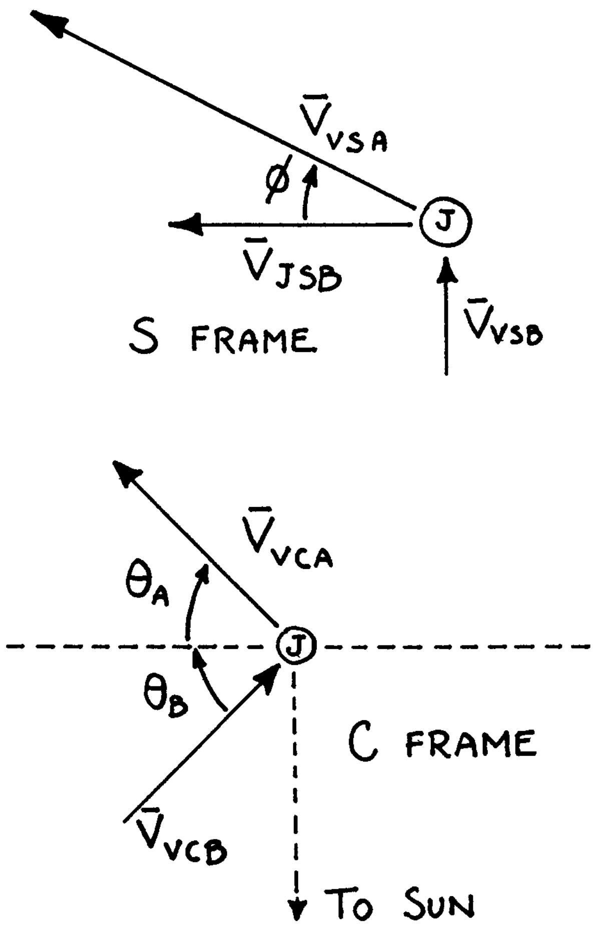

Fig. 2. The upper figure shows the "collision" of Voyager and Jupiter in the S frame and the lower figure shows the collision in the C frame. In the C frame we have chosen to examine the case where $\theta A = \theta B$.

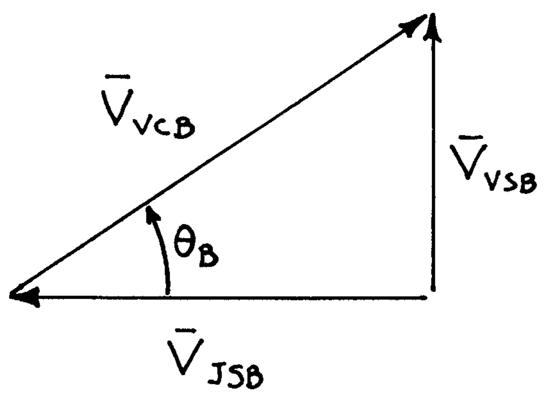

Fig. 3. In the S frame Jupiter is moving to the left with the velocity $\vec{V}_{JSB}$, which is also the velocity of the C frame with respect to the S frame. The velocity of the C frame plus the velocity of Voyager as measured in the C frame, gives the velocity $\vec{V}_{VSB}$ of Voyager in the S frame.

VI. Data

Mass of Voyager	= 825 kg (2)
Mass of Jupiter	= 314.5 × Mass of the Earth
	≅ $10^2\pi$ × Mass of the Earth
	= 1.88×10^{27} kg
Distance, Sun-Jupiter	= 7.78×10^{11} m
Period of Jupiter	= 4332.6 Sidereal days
	= 3.73×10^{8} s

If we assume that Jupiter is in a circular orbit around the Sun, then the speed of Jupiter is:

$V_{JSB} = 1.31 \times 10^4$ m/s

We are here introducing a triple subscript. The first character identifies the object (J for Jupiter or V for Voyager). The second identifies the frame of reference (S for Sun or C for Center of Mass) with respect to which the velocity is measured. The third identifies whether we are speaking of the value of the quantity before the collision (B) or after (A).

Because the interaction time of Jupiter and Voyager is short compared to the orbital period of Jupiter around the Sun, we will give no further consideration to the circular nature of this orbit. We will treat the motion of Jupiter as straight line motion with an initial velocity V_{JSB}.

VII. Calculation: Details

Let us assume that

$\vec{V}_{VSB} = 1.00 \times 10^4$ m/s

in the positive y direction, and use the value above

$\vec{V}_{JSB} = 1.31 \times 10^4$ m/s

and assume it is in the negative x direction. We will specify the direction of the velocity of Voyager after the collision, V_{VCA}, by assuming that V_{VCB} and V_{VCA} make equal angles with a line through Jupiter that is perpendicular to the line from Jupiter to the Sun. We have now specified sufficient detail to permit us to calculate the velocity V_{VSA} with which Voyager leaves Jupiter. Figure 2 shows the velocity vectors of Voyager before and after the collisions in the S and C frames.

1. Calculate the ratio of the velocities in the C frame

$$\frac{\vec{V}_{JCB}}{V_{VCB}} = \frac{M_V}{M_J} = \frac{825}{1.88 \times 10^{27}} = 4.39 \times 10^{-25} \quad (1)$$

This ratio is so small that we can say that the errors are negligible if we assume:

a. The velocity of Jupiter in the S frame is the velocity in the S frame of the center of mass of the Jupiter-Voyager system, and
b. The velocity of Jupiter is unchanged by the collision, $\vec{V}_{JSB} = \vec{V}_{JSA}$.

2. Calculate the initial velocity of Voyager in the C frame. From Fig. 3 we see that

$$\vec{V}_{VSB} = \vec{V}_{JSB} + \vec{V}_{VCB} \quad (2)$$

The velocity of the C frame relative to the S frame ($\vec{V}_{JSB}$) plus the velocity of Voyager in the C frame ($\vec{V}_{VCB}$) equals the velocity of Voyager in the S frame ($\vec{V}_{VSB}$).

$|\vec{V}_{VSB}| = 1.00 \times 10^4$ m/s

$|\vec{V}_{JSB}| = 1.31 \times 10^4$ m/s

It is easy to calculate that

$$\vec{V}_{VCB} = 1.65 \times 10^4 \text{ m/s} \quad (3)$$
$$\theta B = 37.4°$$

3. Calculate the final velocity of Voyager in the C frame. The basis for our analysis of the collision is to note that in the C frame the elastic collision requires that

$$\left|\vec{V}_{VCB}\right| = \left|\vec{V}_{VCA}\right| \quad (4)$$

These velocity vectors are shown in Fig. 2 which is drawn in accord with our assumption that $\theta B = \theta A$. In the C frame

$$\left|\vec{V}_{VCB}\right| = \left|\vec{V}_{VCA}\right| = 1.65 \times 10^4 \text{ m/s} \quad (5)$$

4. Calculate the final velocity of Voyager in the S frame. The vector diagram is shown in Fig. 4

$$\vec{V}_{VSA} = \vec{V}_{JSA} + \vec{V}_{VCA}$$

We have the values

$$\vec{V}_{JSA} = 1.31 \times 10^4 \text{ m/s}$$
$$\vec{V}_{VCA} = 1.65 \times 10^4 \text{ m/s}$$
$$\theta A = 37.4°$$
$$\phi = 20.9°$$

From these,

$$V_{VSA} = 2.80 \times 10^4 \text{ m/s} \quad (6)$$

5. Energies and momenta

In the C frame the kinetic energy of Voyager after the collision is the same as before. The kinetic energy of Jupiter is the same after as before. The four momenta (Voyager and Jupiter before and after) all have the same magnitude in the C frame.

6. Kinetic Energy Gain of Voyager

The ratio of Voyager velocities in the S frame before and after the collision is

$$\left|\frac{\vec{V}_{VSA}}{\vec{V}_{VSB}}\right| = \frac{2.80 \times 10^4}{1.00 \times 10^4} = 2.8$$

The ratio of kinetic energies in the S frame is then

$$\frac{KE_{VSA}}{KE_{VSB}} = 2.8^2 = 7.9 \quad (7)$$

In this example, Voyager leaves Jupiter with 7.9 times as much kinetic energy as it had on its approach to Jupiter.

The initial KE of Voyager in the S frame is

$$\tfrac{1}{2} \times 825 \times (1.0 \times 10^4)^2 = 4.13 \times 10^{10} \text{ J}$$

The final KE of Voyager is

$$\tfrac{1}{2} \times 825 \times (2.8 \times 10^4)^2 = 3.23 \times 10^{11} \text{ J}$$

The kinetic energy gain of Voyager in the S frame is 2.82×10^{11} J.

7. Reduction of Velocity of Jupiter

In one of the descriptions of the Voyager missions we read:

> Nearly a billion kilometers from home, the travels of the Voyagers have just begun. Voyager 1, having gained energy by imperceptibly slowing Jupiter, now races toward Saturn. From Saturn it will depart from the solar system at a steady rate of about three astronomical units (AU) per year.[5]

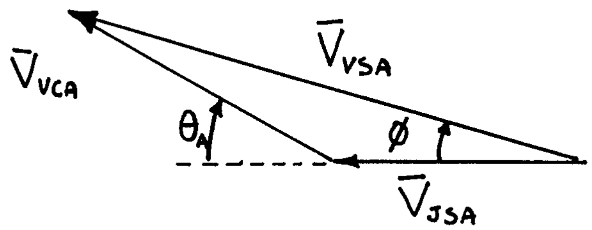

Fig. 4. The velocity of the C frame $\vec{V}_{JSA}$, plus the velocity of Voyager measured with respect to the C frame, is the velocity of Voyager in the S frame.

Although we assumed that the velocity of Jupiter in the S frame was unchanged by the collision, we know that Voyager's gain of KE is equal to the loss of KE of Jupiter. The velocity reduction of Jupiter due to this "collision" is easy to estimate. The KE of Jupiter in the S frame is

$$KE_J = \tfrac{1}{2} M_J V_{JSB}^2 \quad (8)$$

The change of Jupiter's velocity dV_{JS} is related to its change of KE by differentiating both sides of Eq. 8.

$$dV_{JS} = \frac{d(KE_J)}{M_J V_{JSB}} \quad (9)$$

Because the "collision" is elastic, the KE gain of Voyager is equal to the KE loss of Jupiter. Both are measured in the S frame.

$$dV_{JS} = \frac{2.82 \times 10^{11}}{1.88 \times 10^{27} \times 1.31 \times 10^4}$$
$$= 1.15 \times 10^{-20} \text{ m/s} \quad (10)$$

This change in velocity is truly negligible. In 10^9 y this would lead to a displacement change of 3.6×10^{-4}m!

VIII. The Natural Order of Things

Questions have been raised by environmentalists about the effect on Jupiter of these gravity-assisted accelerations of spacecraft. The implication is that these disturb the orbit of Jupiter and thus disrupt "the natural order of things." It is true that the orbit of Jupiter is disturbed by giving gravity-assists to spacecraft, but we must ask, "compared to what?" The key is the mass ratio. The mass of Voyager is roughly the same as that of a very small car. The mass of Jupiter is about 300 times the mass of the Earth. If a car accelerates with respect to the Earth at the same rate as Voyager accelerates with respect to Jupiter, then the recoil acceleration of the Earth will be around 300 times that of Jupiter. The Earth's motion is disturbed in a calculable amount each time a person accelerates by walking or driving. The only way we can avoid disturbing the "natural order of things" is to lie quietly—permanently.

The surfaces of planets and satellites are continuously bombarded by particles whose masses range from micrograms to megatons. The effect of a megaton impact will far exceed the effect of a gravity-assist to a spacecraft, yet these impacts are a part of "the natural order of things."

Thus we can see that the accelerations of Jupiter that result from these gravity assists to spacecraft, are comparable to what each of us does everyday to the earth and are negligible compared to the effects of asteroid impacts.

IX. Slingshot Effect Can Reduce Velocity

The slingshot effect may be used either to increase or to decrease the velocity of a spacecraft. This may come as a surprise because one usually thinks of space missions as requiring high velocities in order to cover great distances in the shortest possible time, with the expenditure of the smallest possible quantity of fuel. Thus the use of the slingshot effect to increase the KE of a spacecraft has obvious advantages. One may sometimes use the slingshot effect to slow a spacecraft in order to put it into orbit around a distant planet.

X. Gravitational Potential Energy

The PE of masses M_J and M_V when they are separated by a distance r is

$$PE = -G \frac{M_J M_V}{r} \tag{11}$$

The total energy (TE = KE + PE) is positive if M_V has an appreciable KE when it is infinitely far removed from M_J. If M_V is in a bound orbit of radius r around M_J then

$$\frac{M_V v_V^2}{r} = G \frac{M_V M_J}{r^2} \tag{12}$$

As is shown in elementary physics texts, the TE of the system then is negative. A spacecraft with a positive total energy cannot exist in a stable circular orbit around a planet. A spacecraft approaching a planet from a great distance must lose a large quantity of kinetic energy if it is to enter a stable orbit around a planet. The spacecraft engines can be used to generate a reverse thrust to reduce the velocity and total energy of the spacecraft or, if one wants to save fuel, the slingshot effect can be used in reverse.

In order to do this the planet must have satellite moons. The spacecraft can approach a moon along such a trajectory that the reverse slingshot effect between the spacecraft and the moon reduces the spacecraft velocity sufficiently, so that the total energy of the system (spacecraft and planet) is negative. The spacecraft can then be said to be "captured" in a stable orbit around the planet. The spacecraft cannot later escape from this stable orbit unless it increases its kinetic energy and its total energy. This can be done through the use of fuel or it might be done if a way could be found to use the slingshot effect to gain energy at the expense of the KE of a passing moon of the planet.

XI. Numerical Example

The reverse slingshot effect can be seen by making a motion picture of the numerical example in Section VII and then viewing the motion picture with the projector running backward. We now see Voyager traveling with a speed of 2.80 x 10^4 m/s overtaking Jupiter. Voyager passes in front of Jupiter and as a result, Voyager's velocity is reduced to 1.0 x 10^4 m/s and its kinetic energy is reduced to about 13% of its initial value.

XII. Contrast the Two Slingshot Interactions

We can contrast the two types of "slingshot" effects. If Voyager crosses Jupiter's path ahead of Jupiter, then Jupiter will gain energy and Voyager must lose energy. If Voyager crosses Jupiter's path behind Jupiter, then Jupiter will lose energy and Voyager must gain energy.

A similar effect can be imagined with the soccer ball and the truck which interact via the repulsive force. Let the truck velocity be V as seen in the S frame. This is very nearly the velocity of the C frame as seen from the S frame. After the truck has passed, the person at the curb hurls the ball with a velocity 2V (in the S frame) at the back of the truck. In the C frame the ball approaches the truck with a velocity 2V - V = V and after impact, it rebounds from the back of the truck with the velocity V. Thus the observer at the curb (at rest in the S frame) sees the ball at rest in the S frame after the impact. All of the KE of the ball was given to the truck.[7] We can see that if the ball-truck interaction takes place behind the truck, the ball will always lose energy and the truck will gain energy. If the interaction takes place ahead of the truck, the ball will always gain energy and the truck will lose energy. Table I summarizes the interactions, giving, for attractive and repulsive forces, the location (ahead or behind) of the region where the interactio takes place in order to have the small object gain or lose KE.

Table I. This table shows the region ahead or behind the massive moving object, where the interaction (attractive or repulsive) must take place in order that the small object (Voyager or the ball) gains or loses kinetic energy.

	Interaction Attractive (Jupiter-Voyager)	Interaction Repulsive (truck-ball)
small object gains KE	behind Jupiter	ahead of truck
small object loses KE	ahead of Jupiter	behind truck

XIII. Other Examples

Other interactions of moving objects and radiation with moving "reflectors" can produce effects that are analogous to the slingshot effect. This can be seen by examining the event in the C frame in which the reflector is at rest and in the S frame in which the reflector is moving. As in the case of the ball and truck, a ball bouncing elastically from a stationary wall will experience no change in KE. When the event is viewed from a moving frame, the ball will gain or lose KE depending on whether the collision has a "head on" component or whether it is an "overtaking" collision. A similar thing is seen in acoustics. The echo from an approaching wall has a higher frequency and a higher energy density than the echo from a stationary wall. Light reflected from an advancing mirror has a higher frequency than light reflected from a stationary mirror. An electrically charged object will "reflect" oppositely charged particles. If the charged object is approaching the particles, the KE of the particles after "reflection" will be greater than it was before reflection. Persons riding surfboards slowly into the advancing face of a high wave, return toward the shore with greater KE than they had when leaving the shore.

Magnetic fields can "reflect" particles. The "collision" of Fig. 2 as seen in the C frame, could easily be imagined to represent the interaction of a charged particle with a small region of magnetic field perpendicular to the diagram. Since, in the C frame, the direction of the magnetic field is always perpendicular to the particle velocity, the magnetic field cannot change the KE of the particle, yet in the S frame the particle gained or lost energy. In the more general case, when a charged particle travels in a magnetic field it will spiral around the magnetic field lines. If the spiraling particle moves into a region of a strong field, the force vector has a component perpendicular to the direction of advance of the particle along the field lines and the particle is "reflected" without change in KE by a "magnetic mirror." If either of these reflections is viewed from a moving frame, the charged particles will be seen to gain and lose energy just as in

Table II. (from Ref. 7)

NAVIGATION
TOUR HISTORY/ASSESSMENT M/S (meters/second)

TOUR	ΔV DET	ΔV 90 STAT	ΔV TOTAL
78-3	72	188	260
79-1	104	127	231
80-23	27	142	169
ΔVEGA	10	140	150
84-02	77	119	196
85-01*	57 (52)	132 (120)	189 (172)
85-02*	67 (61)	102 (93)	169 (154)
85-05*	70 (64)	128 (116)	198 (180)
85-06*	68 (62)	118 (107)	186 (169)

Note: All tours normalized to 11 encounters.
() - values for the 10 encounter tours.
* - preliminary assessment
Detailed analysis for [the tour] 85-01 [is] in progress
Current allocation is (187) m/s total ΔV for 10 encounters.

the case of the truck and the soccer ball. Fermi proposed this as a possible mechanism for the acceleration of cosmic rays.

> The main process of acceleration is due to the interaction of cosmic (charged) particles with wandering magnetic fields....Such fields have a remarkably great stability because of their large dimensions (of the order of magnitude of light years) and the relatively high electrical conductivity of interstellar space.[8]

Fermi notes that

> On a collision both a gain or a loss of energy may take place....gain and loss, however do not average out completely, because....a head-on collision is slightly more probable than an overtaking collision due to the greater relative velocity.[8]

The same thing happens with a truck moving through the air. The truck gives more energy to molecules that collide with its front, than it gains by collisions from molecules that are overtaking it and striking it on the rear. Thus the truck loses energy in its passage through the air.

XIV. A Thermodynamic Analogy

A sample of an ideal gas is confin:ed adiabatically by a cylinder and piston. If the cylinder moves forward to compress the gas, the molecules will rebound elastically from the surface of the advancing piston with a greater velocity than they had before they struck the advancing piston. The average velocity of the molecules is increased and their temperature is raised. The energy required to accomplish this increase of the internal energy of the gas, must have been supplied by the external agent that maintained the velocity of the piston. When the piston is retreating, the effects are reversed. The gas molecules lose energy, the temperature of the gas decreases, and work is done by the gas to increase the velocity of the piston or to do work on the agent that is retarding the free motion of the piston. When the slingshot effect is used to increase the KE of Voyager, we are adiabatically "heating" Voyager. If the effect is used to reduce the KE of Voyager, we are "cooling" Voyager adiabatically.

XV. Example

The Galileo mission to Jupiter is planned for launch in mid-1986. Many detailed plans for the mission have been studied intensively as planners seek an optimum mission. Two criteria of optimization can be thought of at once: (1) maximize the opportunities for close observation of Jupiter and its moons, and (2) minimize the integrated total change in velocity ΔV which must be achieved through the burning of fuel.

Table II is an assessment of some possible tours around Jupiter and through its moons.[9] The numbers given are the integrated total changes in velocity in meters per second that must be supplied by fuel on board the spacecraft after the craft arrives in the vicinity of Jupiter, in order to complete 11 encounters with Jupiter and its satellites. Since the fuel requirements increase with increasing ΔV, a low ΔV is desirable. The left column assumes that all the ephemeris data on Jupiter and its satellites are accurate. The "STAT" column is what must be allowed to account for statistical uncertainties in the ephimeris data and these add to give the total ΔV. The variations in the ΔV's for the different paths are used partly by the differing extents to which the path is able to take advantage of gravity assists (speedup or slowdown) from Jupiter and from the moons. Figures 5, 6, 7, and 8 show one possible configuration of the mission. One can see that the detailed calculation of the trajectory of a particular mission is an enormously complex many-body problem. Each encounter will involve a slingshot effect although path changes in encounters with Jovian moons are often very small.

'86 INTERPLANETARY TRAJECTORY

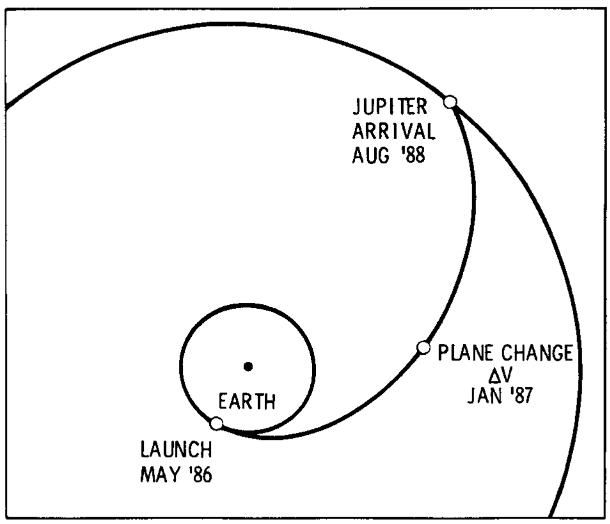

Fig. 5. Spacecraft path from the Earth to Jupiter. The Sun is the black dot in the center of the circle that represents the earth's orbit around the Sun. The earth's orbital velocity counterclockwise around the sun adds to the velocity given by the burning fuel of the rocket. The spacecraft overtakes Jupiter in August 1988 (from Ref. 9).

ARRIVAL GEOMETRY

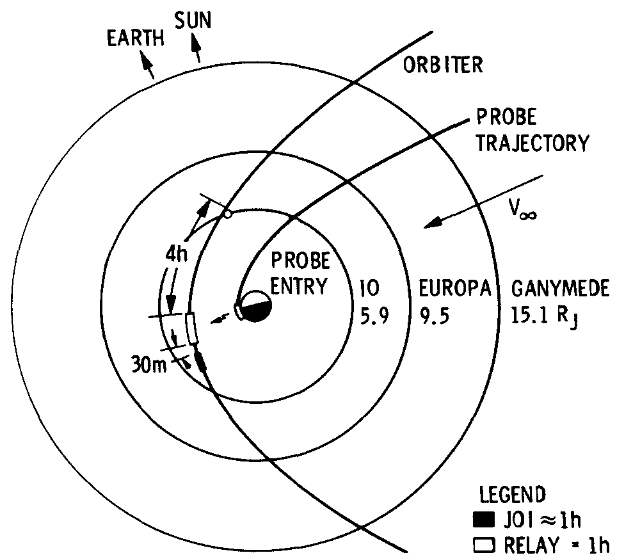

Fig. 6. As the spacecraft approaches Jupiter it separates into both a probe that lands on the surface of Jupiter, and an orbiter which will then move about among Jupiter's moons. Jupiter's velocity is to the right. In this part of the trajectory as seen in the C frame, the spacecraft is gaining energy as it approaches Jupiter. The open rectangle represents the portion of the trajectory in which the probe near or on the surface of Jupiter relays data to the spacecraft for relay to earth. The solid rectangle represents the Jovian Orbit Insertion (JOI), which is the fuel burn needed to reduce the velocity of the spacecraft so that it is "captured" in an orbit around Jupiter (from Ref. 9).

XVI. Other Sources

A comprehensive review of the history and mechanics of the gravity-assist procedures has been given by Nock and Uphoff[10] who point out that

> Voyager uses the gravity assist of Jupiter to bend and speed up the trajectory to reach Saturn. This assist is considerable, equivalent to 11,500 m/s, and in fact results in a solar system escape trajectory.

In regards to orbital capture missions they report that

> At Jupiter the ΔV required to alter the hyperbolic trajectory to an elliptical orbit is of the order of 1000 m/s or greater for favorable approach conditions....Although the planet itself is not sufficient to achieve orbit capture, the gravity assist of a massive satellite (or of several satellites) may be applied in order to reduce the capture ΔV.

Nock and Uphoff give a detailed historical review, with an extensive bibliography. They point out that

> The first known reference to the use of a satellite to reduce energy requirements for celestial navigation was in a manuscript by Yu. V. Kondratvuk (of the U.S.S.R.) about 1918-1919 (unpublished until 1964).

XVII. Conclusion

The slingshot effect appears puzzling because it seems to violate our understanding of the behavior of a particle in a conservative force field. Because it is puzzling, one might conclude that the "slingshot" effect is remote, complex, and beyond simple understanding. We have shown that the tools of elementary physics are sufficient to permit beginning students to gain a good understanding of the slingshot effect and its applications. By examining the asymptotic behavior of Jupiter and Voyager when they are far from one another before and

SATELLITE TOUR - DETAIL

ENCOUNTER	DATE
1	18 MAR 89
2	21 MAY 89
3	22 JUL 89
4	1 SEP 89
5	10 OCT 89
6	9 NOV 89
7*	2 DEC 89
7	3 DEC 89
8	21 DEC 89
9	17 JAN 90
10*	22 FEB 90
10	23 FEB 90
11	13 APR 90
11*	14 APR 90

*NON-TARGETED ENCOUNTERS

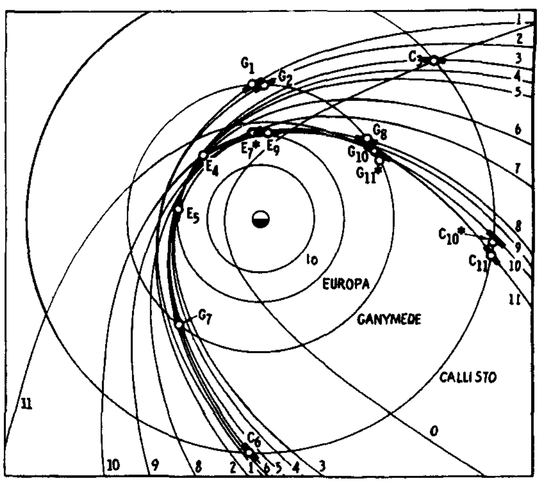

Fig. 7. The spacecraft approaches Jupiter on the path that enters at the top right near the circle marked "150 R_J" (150 times the radius of Jupiter). It passes close to Jupiter (the small dot) where the JOI burn takes place to reduce its velocity. It then moves out on the large loop at the right labeled "APOJOVE 1." Following APOJOVE 1 the spacecraft orbits Jupiter repeatedly, using small amounts of fuel from time to time to give optimum encounters with Jovian satellites (from Ref. 9).

'86 SATELLITE TOUR-EXAMPLE

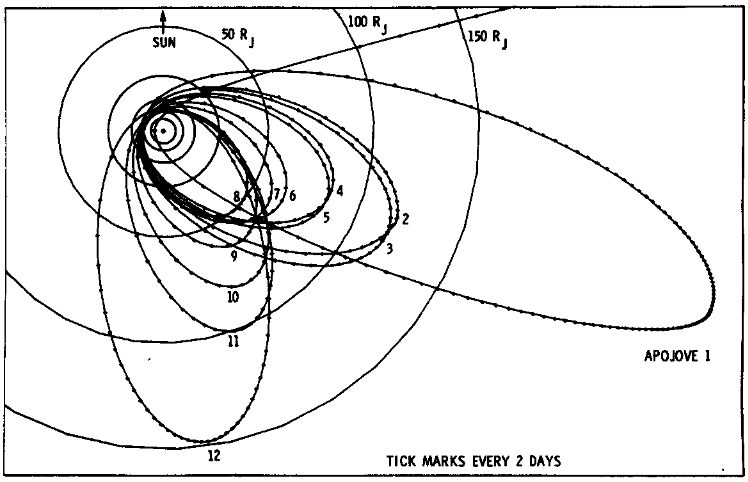

Fig. 8. Closeup details of the orbits of the spacecraft in the vicinity of Jupiter. Eleven encounters are marked with Europa (E), Gannymede (G) and Callisto (C). The numbers along the bottom and right margins identify the sequence of passes near Jupiter, starting with 0 near the lower right (from Ref. 9).

after the collision, we can see that the details of the collision are not of significance; thus any interaction that produces an elastic collision can be substituted in place of the gravitational interaction. The slingshot effect is then an effect that is observed when an elastic collision is viewed in a frame of reference other than the C frame.

The NASA report closes by noting that

> Years after launch, perhaps 30 times farther from the Sun than the Earth is, their attitude control gas spent, the two Voyagers will be unable to respond to attitude correction commands from their Earth masters, and communications will fade and disappear as they drift out of range. Their mission of discovery and exploration complete, the two crafts will sail on forever (see footnote 1).

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