Man's size in terms of fundamental constants

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(Received 26 September 1979; accepted 15 January 1980)

Why are we the size we are, instead of some very different size? Simple physical scaling laws and three "requirements" dictate that our size be of order $(\hbar^2/m_e e^2)(e^2/Gm_p{}^2)^{1/4}$. They also "predict" the mass and radius of the Earth. The three requirements are: (i) We are made of complicated molecules; (ii) we breathe an evolved planetary atmosphere; (iii) we are about as big as we can be without breaking.

Haldane's essay "On Being the Right Size," [1] discusses qualitatively but elegantly some concepts of elementary physics which cause various animal species to be the sizes that they are: weight increasing as the cube of size, while structural strength increases as only the square; force required to escape from the surface tension of liquid water; oxygen diffusion resistance in organisms such as insects, lacking lungs; heat loss; eye resolution; limits to brain weight. Very recently there has been a flurry of interest[2,3,4] in astrophysical and cosmological circles on a problem of similar character there, namely to understand as many as possible of the basic features of "galaxies, stars, planets, and the everyday world" [3] in terms of a few microscopic physical constants (c, G, $\hbar$, e, etc.) plus the constraint that these constants have values consistent with the possibility of intelligent life evolving so as to observe them. This additional constraint is usually called the "anthropic principle." [2]

A few years ago, in connection with an elementary physics course, I attempted to fuse Haldane's general approach with the more quantitative astrophysical arguments, and to derive an order of magnitude expression for L_H, the size of man (the term used generically to include male as well as female) in terms of fundamental constants. Since the result of my calculation has now found its way into the literature on the anthropic principle (e.g., Ref. 3) and since the line of argument is suitable in level for use in an undergraduate general physics course, I sketch the calculation here.

Let us assume only that man satisfies three properties: (i) he is made of complicated molecules; (ii) he requires an atmosphere which is not (primordial, cosmological) hydrogen and helium; and (iii) he is as large as possible, to carry his huge brain, but he is liable to stumble and fall; and in so doing he should not break. These three properties do not differentiate between a man and, say, an elephant of size L_E; however $L_E \approx L_H$ to the accuracy of our calculation, and we should not expect to distinguish elephants from men by dimensional arguments.

The characteristic atomic size of all matter is set by the Bohr radius

$$a_0 = \hbar^2/m_e e^2 = 0.53 \times 10^{-8}\ \text{cm}, \tag{1}$$

where $\hbar$ is Planck's constant, m_e the mass of the electron, and e its charge. The characteristic density of all matter is therefore set by the density of one (or so) proton in a cubic Bohr diameter, so we can define a scale density

$$\rho_0 = m_p/(2a_0)^3 = 1.4\ \text{g/cm}^3, \tag{2}$$

where m_p is the mass of a proton. The scale of energies for all molecular binding is set by the characteristic energy of the hydrogen atom,

$$e^2/2a_0 = 1\ \text{Ry} = 13.6\ \text{eV}. \tag{3}$$

According to (i) above, man's chemistry is complicated, so his binding energies are a small fraction, say ϵ, of a Rydberg. A reasonable value for ϵ is 0.003; but we will see below that in fact L_H depends on ϵ only very weakly. We next observe that man must live in an environment whose temperature is given in order of magnitude by

$$T \sim (\epsilon/k)\ \text{Ry} = 470(\epsilon/0.003)\ \text{K} \tag{4}$$

(where k is Boltzmann's constant). If the temperature were much larger, his chemistry would be disrupted; if it were much lower, his internal chemical processes would proceed at an exponentially smaller rate; thus he would be immobile and unlikely to stumble, violating assumption (iii) above. (Empirically, man does in fact live at close to the temperature of his chemical bond energy; this, incidentally, makes cooking practicable.)

We now use property (ii). Since man's atmosphere is not hydrogen, but is also not vacuum, the escape velocity from the surface of his planet (Earth) should be greater, but not too much greater, than the thermal velocity of hydrogen at his ambient temperature (ϵ/k) Ry. This condition gives

$$GM_\oplus/R_\oplus \sim \epsilon\ \text{Ry}/m_p, \tag{5}$$

where G is the gravitational constant, $M_\oplus$ and $R_\oplus$ are the mass and radius of the Earth. From Eq. (2) we have

$$M_\oplus/R_\oplus^3 \sim m_p/(2a_0)^3. \tag{6}$$

Equations (5) and (6) can be solved for $M_\oplus$ and $R_\oplus$ separately, giving

$$R_\oplus \sim \epsilon^{1/2}(2a_0)\left(\frac{e^2}{Gm_p{}^2}\right)^{1/2} = 6.5 \times 10^8 \left(\frac{\epsilon}{0.003}\right)^{1/2}\ \text{cm}, \tag{7}$$

$$M_\oplus \sim \epsilon^{3/2} m_p \left(\frac{e^2}{Gm_p{}^2}\right)^{3/2} = 3.8 \times 10^{26} \left(\frac{\epsilon}{0.003}\right)^{3/2}\ \text{g}, \tag{8}$$

which are to be compared with the actual values 6.4×10^8 cm and 5.9×10^{27} g.

If man is of a size L_H, his mass is roughly

$$M_H \sim \rho_0 L_H^3. \tag{9}$$

When, according to property (iii), he stumbles and falls, the

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energy of his fall is of order $M_H L_H (GM_\oplus / R_\oplus^2)$, the last factor being the acceleration of gravity at planetary surface. The number of atoms that man contains is about M_H/m_p. His breaking involves a disruption only on a two-dimensional surface which contains of order $(M_H/m_p)^{2/3}$ atoms, and each atom is bound with an energy ϵ Ry, so the scale of man's breaking energy is set by the combination $\epsilon\,\mathrm{Ry}(M_H/m_p)^{2/3}$. Property (iii) thus takes the form of an equation,

$$M_H L_H \, GM_\oplus / R_\oplus^2 \sim \epsilon\,\mathrm{Ry}(M_H/m_p)^{2/3}. \tag{10}$$

Using Eqs. (2), (5), (7), and (9), this can be solved for L_H in terms of known quantities,

$$\begin{aligned} L_H &\sim \epsilon^{1/4}(2a_0)(e^2/Gm_p^2)^{1/4} \\ &= 2\epsilon^{1/4}\hbar^2 e^{-3/2} m_e^{-1} m_p^{-1/2} G^{-1/4} \\ &= 2.6(\epsilon/0.003)^{1/4}\,\mathrm{cm}, \end{aligned} \tag{11}$$

which is very insensitive to changes in the assumed value of ϵ.

The observed value of L_H is, we note, about a factor 10^2 larger than that given by our dimensional calculation. This disagreement is not surprising, considering the crudeness of the estimate. Even so, it is not difficult (using, however, more information than the three properties originally assumed) to see where the disagreement arises: we have underestimated man's breaking energy [right-hand side of Eq. (10)] by a factor of about 10^4–10^5, and this factor enters the estimate for L_H as a square root. Equation (10) assumed implicitly that man was "brittle," i.e., that the energy of a fall would be concentrated as stress along his weakest fault plane. If this were true, a 100-kg man would break under an energy of order 10^6 erg; in actuality his breaking energy is of order 3×10^{10} erg (a 3-m fall). Probably the reason for this excess strength is that man's molecular structure is polymeric rather than amorphous, so that stresses are distributed over a rather larger volume than that of a single monatomic fault plane. As a very crude model, one might take the fault to be about as wide as the length of a polymer, and therefore equate the excess breaking energy (10^4–10^5) to the number of atoms in a protein; this gives fairly good agreement with observation.

We might turn now to a different problem, that of estimating the characteristic human *timescale* (or lifespan). In this regard, we first notice that if the ambient temperature (ϵ/k) Ry is to be maintained by solar radiation, then the scale of the solar constant is determined by the Stefan–Boltzmann law, namely,

$$f \sim \sigma(\epsilon/k\,\mathrm{Ry})^4 = 2.9\times10^6\,(\epsilon/0.003)^4\,\mathrm{erg\,cm^{-2}\,sec^{-1}} \tag{12}$$

(compared to a measured value $1.4\times10^6\,\mathrm{erg\,cm^{-2}\,sec^{-1}}$). Here σ is the Stefan–Boltzmann constant which is defined from fundamental constants by

$$\sigma = \pi^2 k^4/60\,c^2\hbar^3. \tag{13}$$

A natural characteristic time t_H is obtained by equating the total energy of chemical bonds in man to t_H multiplied by the solar flux incident on man's surface area L_H^2. If man were a plant, this time would be a characteristic growth time, hence set the scale for his lifespan, but in fact we are not plants; we are at the end of a complicated food chain. Another possible interpretation is that the time t_H is a "shelter-seeking" time, in which the solar flux is likely to be damaging to an unprotected man.

In any case, the value obtained is

$$\begin{aligned} t_H &\sim \frac{k^4}{\sigma}\frac{\rho_0}{m_p}\frac{L_H}{(\epsilon\,\mathrm{Ry})^3} \\ &= \frac{120}{\pi^2}\epsilon^{-2.75} c^2\hbar^5 e^{-7.5} m_c^{-1} m_p^{-1/2} G^{-1/4} \\ &= 5.0\times10^4\left(\frac{\epsilon}{0.003}\right)^{-2.75}\mathrm{sec} \approx 14\left(\frac{\epsilon}{0.003}\right)^{-2.75}\mathrm{h}. \end{aligned} \tag{14}$$

Unfortunately, the strong dependence on ϵ here allows us to put little credence in the numerical result. The universal constants seem to determine the scale of man's size quite narrowly, but they seem to determine the scale of his time only very poorly. This result, if true, may be relevant to the problem of interstellar communication, since we should probably not hope to establish communication with intelligences whose time scale is either very much shorter or very much longer than our own.

ACKNOWLEDGMENT

I am pleased to acknowledge interesting and helpful discussions on these matters with Douglas Eardley and V. F. Weisskopf.

[1] J. B. S. Haldane, in *Possible Worlds* (Harper, New York, 1928); reprinted in *The World of Mathematics*, edited by James R. Newman, (Simon and Schuster, New York, 1956), Vol. 2.

[2] B. Carter, in *Confrontation of Cosmological Theories with Observation*, edited by M. S. Longair (Reidel, Dordrecht, 1974), p. 291.

[3] B. J. Carr and M. J. Rees, Nature **278**, 605 (1979).

[4] J. Barrow and F. Tipler (unpublished monograph).