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ARITHMETICAL PRODIGIES.

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I.

A great deal has been said and written about these phenomenal persons in a very uncritical manner; on the one hand they are regarded as almost supernatural beings, while on the other hand no notice has been taken of them scientifically. Nevertheless, we can perhaps gain light on the normal processes of the human mind by a consideration of such exceptional cases. The first object of the present article is to give a short account of these persons themselves, and to furnish for the first time an approximately complete bibliography of the subject. Thereupon the attempt will be made to make such a psychological analysis of their powers as will help in the comprehension of them, and will perhaps furnish more than one hint to the practical instructor in arithmetic.

NIKOMACHOS.—Lucian said that he did not know how better to praise a reckoner than by saying that he reckoned like Nikomachos, of Gerasa.[1] Whether this refers to the reckoning powers of Nikomachos (about 100 A. D.), or to the famous Introduction to Arithmetic written by him, we are left in doubt. De Morgan inclines to the former opinion,[2] Cantor holds the latter.[3] The literal translation of the pas-

[1] Lucianus, Philopatris, "ἀριθμέεις ὡς Νικόμαχος."
[2] Smith's Dictionary of Greek and Roman Biography v. Nikomachos.
[3] Cantor, *Vorlesungen über Geschichte der Mathematik*, Leipzig, 1880, I, 363.

sage places Nikomachos undoubtedly among the skillful calculators.

AFRICAN SLAVE DEALERS.—Perhaps brought to the front or produced by the necessity of competing with English traders armed with pencil and paper, many of the old-time slave-dealers of Africa seemed to have been ready reckoners, and that, too, for a practical purpose,—a point overlooked by more than one of the later calculators. "It is astonishing with what facility the African brokers reckon up the exchange of European goods for slaves. One of these brokers has perhaps ten slaves to sell, and for each of these he demands ten different articles. He reduces them immediately by the head into bars, coppers, ounces, according to the medium of exchange that prevails in the part of the country in which he resides, and immediately strikes the balance."[1] The ship-captains are said to have complained that it became more and more difficult to make good bargains with such sharp arithmeticians. It was also an African who was the first to appear in this rôle in America.

TOM FULLER.—The first hand evidence in regard to Fuller consists of the following: A letter read before the Pennsylvania Society for the Abolition of Slavery by Dr. Rush of Philadelphia, which is published, more or less completely, in three places;[2] and the obituary which appeared in the Columbian Centinel.[3] On the foundation of these documents several later accounts have been given.[4]

[1] [T. Clarkson.] An Essay on the Slavery and Commerce of the Human Species, particularly the African. 2d Ed., London, 1788. (The passage quoted does not appear in the American editions, Phila., 1786, 1787, 1804).

[2] American Museum, Vol. V, 62, Phila., 1799.

Steadman. Narrative of a five years expedition against the Revolted Negroes of Surinam, South America, 2v. 4°, London, 1796, Vol. II, 260. In the French translation, Vol. III, 61.

Needles, Historical Memoir of the Penn. Society for the Abolition of Slavery; Phila., 1848, p. 32.

[3] Columbian Centinel of Boston, Dec. 29, 1790, No. 31 of Vol. XIV.

[4] For example, Grégoire; An Enquiry concerning the Intellectual and Moral Faculties, and Literature of Negroes, followed with an Account of the Life and Works of Fifteen Negroes and Mulattoes; Translated by D. B. Warden; Brooklyn, 1810. (The translation is from Grégoire's original manuscript.) Brissot de Warville; New Travels in the United States of America. performed in 1788; London, 1792, p. 287; 2d Ed., London, 1794, vol. I, 243; Boston, 1797 (reprint of 1st ed.), p. 158; in the original French edition, vol. II, p. 2. Williams; History of the Negro Race in America; New York, 1883, vol. I, 399. Didot's Nouvelle biographie générale v. Fuller.

Thomas Fuller, known as the Virginia Calculator, was stolen from his native Africa at the age of fourteen and sold to a planter. When he was about seventy years old, "two gentlemen, natives of Pennsylvania, viz., William Hartshorne and Samuel Coates, men of probity and respectable characters, having heard, in travelling through the neighborhood in which the slave lived, of his extraordinary powers in arithmetic, sent for him and had their curiosity sufficiently gratified by the answers which he gave to the following questions: First, Upon being asked how many seconds there were in a year and a half, he answered in about two minutes, 47,304,000. Second: On being asked how many seconds a man has lived who is 70 years, 17 days and 12 hours old, he answered in a minute and a half 2,210,500,800. One of the gentlemen who employed himself with his pen in making these calculations told him he was wrong, and that the sum was not so great as he had said—upon which the old man hastily replied: 'top, massa, you forget de leap year. On adding the amount of the seconds of the leap year the amount of the whole in both their sums agreed exactly."[1] Another question was asked and satisfactorily answered. Before two other gentlemen he gave the amount of nine figures multiplied by nine. He began his application to figures by counting ten and proceeded up to one hundred. He then proceeded to count the number of hairs in a cow's tail and the number of grains in a bushel of wheat. Warville says in 1788, "he has had no instruction of any kind, but he calculates with surprising facility."[2] In 1790 he died at the age of 80 years, having never learned to read or write, in spite of his extraordinary power of calculation.[3]

JEDEDIAH BUXTON.—Jedediah Buxton[4] was born in 1702, at Elmton, in Derbyshire, England, where he died in 1772.[5]

[1] American Museum, V, 62.

[2] Warville, New Travels, p. 158.

[3] Columbian Centinel, loc. cit.

[4] Gentleman's Magazine, 1751, Vol. XXI, p. 61, 347; 1753, vol. XXIII, p. 557; 1754, vol. XXIV, p. 251, which are the original authorities. Chalmer's General Biogr. Dictionary, London, 1812, v. Buxton; Rose, New General Biogr. Dictionary, London, 1848, v. Buxton; Didot's Nouvelle biographie générale, v. Buxton; Michaud's Biographie universelle v. Buxton.

[5] The dates are given on the authority of Lyson's Magna Britannia, London, 1817, vol. V, Derbyshire, p. 157.

Although his father was schoolmaster of the parish and his grandfather had been the vicar, his education was by some chance so neglected that he was not able to scrawl his own name.[1] All his attainments were the result of his own pure industry; the only help he had was the learning of the multiplication table in his youth; "his mind was only stored with a few constants which facilitated his calculations; such as the number of minutes in a year, and of hair's-breadths in a mile."[2] He labored hard with his spade to support a family,[3] but seems to have shown not even usual intelligence in regard to ordinary matters of life. The testimony as to his arithmetical powers is given by two witnesses. George Saxe says: "I proposed to him the following random question: In a body whose three sides are 23,145,789 yards, 5,642,732 yards, and 54,965 yards, how many cubical ⅛ths of an inch? After once naming the several figures distinctly, one after another, in order to assure himself of the several dimensions and fix them in his mind, without more ado he fell to work amidst more than 100 of his fellow-laborers, and after leaving him about five hours, on some necessary concerns (in which time I calculated it with my pen) at my return, he told me he was ready: Upon which, taking out my pocket-book and pencil, to note down his answer, he asked which end I would begin at, for he would direct me either way. . . . I chose the regular method, . . . and in a line of twenty-eight figures, he made no hesitation nor the least mistake."[4] "He will stride over a piece of land or a field, and tell you the contents of it, almost as exact as if you measured it by the chain. . . . He measured in this manner the whole lordship of Elmton, of some thousand acres, . . . and brought the contents, not only in acres, roods and perches, but even in square inches; . . . for his own amusement he reduced them to square hairs-breadths,

[1] "His total want of education has been attributed to his excessive stupidity when a child, and an invincible unwillingness to learn anything." Lyson's Magna Britannia, V, 157, note.

[2] Journey Book of Engl., Derbyshire, p. 79.

[3] "A day-labourer," Lyson's Magna Britannia, loc. cit. "Either a small land-owner or a day-labourer; but probably the former," The Journey-Book of England, Derbyshire; London, 1841, p. 79.

[4] Gentleman's Magazine, XXI, 61.

computing (I think) 48 to each side of the inch."[1] Various other problems were solved by him with like facility on later occasions, before a different witness.[2]

From May 17 to June 16, 1725, he was (to use his own expression) drunk with reckoning, by which a kind of stupefaction was probably meant. The cause was the effort to answer the following question: In 202,680,000,360 cubic miles how many barley-corns, vetches, peas, wheat, oats, rye, beans, lintels, and how many hairs, each an inch long, would fill that space, reckoning 48 hairs in breadth to an inch on the flat? His table of measures, which he founded on experiment, used in answering this was:

200 Barley Corns,	are contained in one solid inch.[3]
300 Wheat Corns,	
512 Rye Corns,	
180 Oats,	
40 Peas,	
25 Beans,	
80 Vetches,	
100 Lintels,	
2304 Hairs 1 inch long,	

Quite curious is Buxton's notation for higher numbers. His system is: Units, thousands, millions, thousands of millions, millions of millions, thousand millions of millions, tribes, thousands of tribes, etc., to thousand millions of millions of tribes; cramps, thousands of cramps, etc., to thousand million of million of cramps; tribes of cramps, etc. to tribes of tribes of cramps.

In regard to subjects outside of arithmetic, his mind seemed to have retained fewer ideas than that of a boy ten years old. On his return from a sermon he never brought away one sentence, having been busied in dividing some time or some space into the smallest known parts. He visited London in 1754, and was tested by the Royal Society. On this visit he was taken to see King Richard III performed at Drury Lane playhouse, but his mind was employed as at church. During the dance he fixed his attention upon the number of steps; he attended to Mr. Garrick only to count the words that he

[1] Gentleman's Magazine, XXI. 61.
[2] Gentleman's Magazine, XXIII, 557, XXIV, 251.
Gent. Mag., XXI, 348.

uttered.[1] At the conclusion of the play they asked him how he liked it. He replied "such an actor went in and out so many times and spoke so many words; another so many, etc."[2] He returned to his village and died poor and ignored.

AMPÈRE.—The first talent shown by André Marie Ampère,[3] *1775, at Lyon, †1836, at Marseilles, was for arithmetic. While still a child, knowing nothing of figures, he was seen to carry on long calculations by means of pebbles. To illustrate to what an extraordinary degree the love of calculation had seized upon the child, it is related that being deprived of his pebbles during a serious illness, he supplied their places with pieces of a biscuit which had been allowed him after three days strict diet.

As soon as he could read he devoured every book that fell into his hands. His father allowed him to follow his own inclination and contented himself with furnishing him the necessary books. History, travels, poetry, romances and philosophy interested him almost equally. His principal study was the encyclopedia in alphabetical order, in twenty volumes folio, each volume separately in its proper order. This colossal work was completely and deeply engraved on his mind. "His mysterious and wonderful memory, however, astonishes me a thousand times less than that force united to flexibility which enables the mind to assimilate without confusion, after reading in alphabetical order matter so astonishingly varied."[4] Half a century afterwards he would repeat with perfect accuracy long passages from the encyclopedia relating to blazonry, falconry, etc.

At the age of eleven years the child had conquered elementary mathematics and had studied the application of algebra to geometry. The parental library was not sufficient to

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[1] Gentleman's Magazine, XXIV, 251.

[2] Memoir of Zerah Colburn, p. 174.

[3] Bibliography (for his life), Saint-Beuve, M. Ampère, sa jeunesse, ses études diverses. etc., in the Revue des deux Mondes, 1837, 4 ième série, t. IX, p. 389. M. F. Arago, Eloge d' Ampère (given in a somewhat condensed form by E. Arago, in Michaud's Biogrgraphie universelle, nouv. éd., v. Ampère), translation in Smithsonian Reports, 1872, p. 111. Didot's Nouvelle biographie générale, v. Ampère. Valson, Vie d' Ampère, Lyon, 1886.

[4] Arago, Eulogy on Ampère, Smithsonian Reports, 1872, p. 113; Michaud's Biogr. universelle, I, p. 597.

supply him with further books, so his father took him to Lyon, where he was introduced to higher analysis. He learned of himself according to his fancy, and his thought gained in vigor and originality. Mathematics interested him above everything. At eighteen he studied the Mécanique analytique of Lagrange, nearly all of whose calculations he repeated; he said often that he knew at that time as much mathematics as he ever did.

In 1793 his father was butchered by the revolutionaries, and young Ampère was completely paralyzed by the blow. Rousseau's botanical letters and a chance glance at Horace roused him after more than a year from an almost complete idiocy; and he gave himself up with unrestrained zeal to the study of plants and the Augustan poets. At the age of twenty-one his heart suddenly opened to a new passion and then began the romantic story of his love, which is preserved in his *Amorum* and his letters.[1] Ampère became professor of mathematics, chemistry, writer on probabilities, poet, psychologist, metaphysician, member of the Academy of Sciences of Paris, discoverer of fundamental truths of electrodynamics, and a defender of the unity of structure in organized beings.[2]

Just as he began by learning completely the encyclopedia of the 18th century, he remained encyclopedic all his life, and his last labors were on a plan for a new encyclopedia.

GAUSS.—The arithmetical prodigies might be divided into two classes, the one-sided and the many-sided. The former would include those who like Buxton, Colburn and Dase were mere "reckoning-machines," the other would consist of men in whom the calculating power was only a part of gifts of mathematical talent like Safford, or even of the highest mathematical genius like Gauss.

Carl Friedrich Gauss was born in 1777, in Braunschweig. He was the offspring of a poor family that had in nowise distinguished themselves, although his mother seemed to have been of finer mental build than the paternal stock. Moreover his maternal uncle was a man of unusual talent: com-

[1] André Ampère, Correspondence et souvenirs, Paris, 1873.
[2] List of Works in Michaud's Biogr. universelle, nouv. éd., I, p. 611.

pletely uninstructed he learned to produce the finest damask; in Gauss's opinion "a natural genius had been lost in him."[1] At an early age the genius of Gauss began to show itself. With the assistance of friends and of persons of the nobility he was enabled to get a school-education. At the age of eleven he entered the gymnasium where he mastered the classical languages with incredible rapidity. In mathematics also he distinguished himself. It is said that a new professor of mathematics handed back thirteen-year-old Gauss's first mathematical exercise with the remark that it was unnecessary for such a mathematician to attend his lessons in the future.[2] The Grand Duke, hearing of his talent, sent for him. The court was entertained by the calculations of the fourteen-year-old boy, but the duke recognized the genius and gave him his support. It is to be regretted that we have not fuller accounts of his early calulations, but his later achievements have so completely occupied the world of science that less attention has been paid to his calculating powers. It is curious to think that if he had had the misfortune to have been gifted with nothing else, he would probably have distinguished himself as Dase or Mondeux did; he might even have proclaimed himself in the Colburn fashion, as a miraculous exception from the rest of mankind; as it is, he was only the greatest mathematician of the century.

After leaving the gymnasium in 1795, he entered the University of Göttingen. As early as 1795, he discovered the method of the least squares, and in 1796 he invented the theory of the division of the circle.

In 1798 he promoted *in absentia* as Dr. phil. at the university of Helmstedt.[3]

In 1801, at the age of twenty-four, his *Disquisitiones arithmeticæ* were published; the work was quickly recognized as one of the milestones in the history of the theory of numbers. From this point on his life was a series of most brilliant discoveries till his death at Göttingen, 1855.

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[1] Hänselmann, K. F. Gauss, Leipzig, 1878, p. 15.
[2] Hänselmann, K. F. Gauss, p. 25.
[3] His dissertation was entitled: *Demonstratio nova theorematis, omnem functionem algebraicum rationabilem integram unius variabilis in factores reales primi vel secundi gradus resolvi posse, Helmstedt, 1798.*

It is much to be regretted that no adequate life of Gauss has yet been written; nevertheless, the story of his discoveries is too well known[1] to need mention. We are here interested in his talent for calculation, for Gauss was not only a mathematical genius,—he was also an arithmetical prodigy, and that, too, at an age much earlier than any of the others.

An anecdote of his early life, told by himself, is as follows: His father was accustomed to pay his workmen at the end of the week, and to add on the pay for overtime, which was reckoned by the hour at a price in proportion to the daily wages. After the master had finished his calculations and was about to pay out the money, the boy, scarce three years old, who had followed unnoticed the acts of his father, raised himself and called out in his childish voice: "Father, the reckoning is wrong, it makes so much," naming a certain number. The calculation was repeated with great attention, and to the astonishment of all it was found to be exactly as the little fellow had said.[2]

At the age of nine Gauss entered the reckoning class of the town school. The teacher gave out an arithmetical series to be added. The words were scarcely spoken when Gauss threw his slate on the table, as was the custom, exclaiming, "There it lies!" The other scholars continue their figuring while the master throws a pitying look on the youngest of the scholars. At the end of the hour the slates were examined; Gauss's had only one number on it, the correct result alone.[3] At the age of ten he was ready to enter upon higher analysis. At fourteen he had become acquainted with the works of Euler and Lagrange, and had grasped the spirit and methods of Newton's Principia.

He was always distinguished for his power of reckoning, and was able to carry on difficult investigations and extensive numerical calculations with incredible ease. His unsurpassed memory for figures set those who met him in astonishment; if he could not answer a problem at once, he stored it up for

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[1] Except to Mr. Sully, who in an article "Genius and Precocity," in the *Nineteenth Century*, never even mentions Pascal, Ampère and Gauss.

[2] Waltershausen; Gauss zum Gedächtniss, Leipzig, 1856, p. 11.

[3] Hänselmann; Karl Friedrich Gauss, Zwölf Kapitel aus seinem Leben, Leipzig, 1878.

future solution. At once, or after a very short pause, he was able to give the properties of each of the first couple thousand numbers. In mental calculation he was unsurpassed. He had always in his mind the first decimals of all the logarithms, and used them for approximate estimates while calculating mentally. He would often pursue a calculation for days and weeks, and—what distinguishes him from all other calculators,—during such a calculation he continually invented new methods and new artifices.

Perhaps the best picture of his genius is given by Waltershausen: "Gauss showed a remarkable, perhaps unprecedented, combination of peculiar talents. To his eminent ability to work out in himself abstract investigations on all sides and from all standpoints, there were joined a marvellous power of numerical calculation, a peculiar sense for the quick apprehension of the most complicated relations of numbers, and an especial love for all exact observation of nature."[1]

From Gauss's opinion of Pfaff we get a hint of what he regarded as the essential of genius, "never to leave a matter till he had investigated wherever possible."

WHATELY.—Richard Whately, *1787, Archbishop of Dublin from 1831 to 1863, author of "Historic Doubts relative to Napoleon Bonaparte," "Elements of Logic," "Elements of Rhetoric," and numerous other works, mostly religious, displayed a singular precocity in regard to calculation. At six years old he astonished his family by telling Parkhurst, a man of past sixty, how many minutes he was old.

"There certainly was something peculiar in my calculating faculty," wrote Whately in his Commonplace Book. "It began to show itself between five and six, and lasted about three years. One of the earliest things I can remember is the discovery of the difference between even and odd numbers; . . . I soon got to do the most difficult sums, always in my head, for I knew nothing of figures beyond numeration, nor had I any names for the different processes I employed. But I believe my sums were chiefly in multiplication, division and the rule of three. In this last point I believe I surpassed the famous American boy, though I did

[1] Waltershausen, Gauss, p. 83.

not, like him, understand the extraction of roots. I did these sums much quicker than any one could upon paper, and I never remember committing the smallest error." . . .

"When I went to school, at which time the passion was worn off, I was a perfect dunce at cyphering, and so have continued ever since."[1]

ZERAH COLBURN.[2]—Autobiographies do not always furnish the most trustworthy evidence in regard to the man himself; when, moreover, the author is convinced that he is nothing less than a modern miracle; and, finally, when having had no scientific and little literary education, he at a later date writes the memoirs of his youth, we are obliged to supply the lacking critical treatment of the narrative. The main source of information in regard to Colburn's youthful powers consists of his memoirs published by him in 1833.[3] Only one contemporary account of his earliest exhibitions in America isto be found, we must rely mostly on his own statements, probably derived from recollections of his friends, and on a "Prospectus," a sort of advertisement, published in London in 1813.

Zerah Colburn,* 1804,† 1840, of Cabot, Vt., was considered a very backward child. In the year 1810, a short time after

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[1] Jane Whately, Life and Correspondence of Richard Whately, D. D., London, 1866, I, 4.

[2] Bibliography; A Memoir of Zerah Colburn, written by himself, Springfield, Mass.,1833; Medical and Philosophical Journal and Review, New York, 1811, Vol. III, p. 19; Philosophical Magazine, XL., London, 1812, p. 119; Philosophical Magazine, XLII, London, 1813, p. 481 [report of the proceedings of the Royal Society, in which a letter concerning the extra digits on members of the Colburn family was read]; Analectic Magazine, Vol. I, year 1813, p. 124 ff. [contains a reprint of an article by the calculator, Mr. Francis Baily, in the Literary Panorama, Oct. 1812, which article is almost identical with the one in the Philosophical Magazine, 1812, p. 119.] Graves; Life of Sir William R. Hamilton, I, 78 ff.; American Almanac, 1840, p. 307. In Faraday's commonplace book there is an unpublished account of Zerah Colburn, who visited Faraday in 1816 and explained to him his method of calculation; Jones, Life of Faraday, London, 1870, I, 221. Gall, Functions of the Brain, Organology, XVIII; R. A. Proctor, On Some Strange Mental Feats, Cornhill Mag., Aug. 1875, Vol. XXXII, p. 157, reprinted in Science Byways, London, 1875, p. 337. [This is an attempt at explaining Colburn's powers; the objections to it will be found below.] R. A. Proctor, Calculating Boys, in Belgravia, Vol. XXXVIII, p. 450 [contains a further explanation.] Carpenter, Mental Physiology, Chap. VI, § 205 [quoting from Baily's account.]

[3] There is no statement regarding the time at which they were written, or even a date to the preface; the last year mentioned in the book is 1827.

a six weeks attendance at the district school, in which he had learned no arithmetic [unless from the recitations of other boys in the class-room], his father heard him saying "5 times 7 are 35," "6 times 8 are 48," etc., and upon examining him and finding him perfect in the multiplication table, he asked the product of 13 $\times$ 97, to which 1261 was instantly given in answer. The account given by Zerah himself, when stated in plain terms, amounts to this; nevertheless, one is tempted to ask for the authority on which the statements were made. If Zerah remembered the exact figures himself till the time of writing his memoirs, then his power of memory for long periods must have been extraordinary, yet he never mentions such powers. On the other hand, if these statements are made from the stories current about him, the general untrustworthiness of such evidence does not allow us to put too much faith in the figures.

Before long Zerah's father took him to Montpelier, Vt., where he was exhibited. Of his performances here Colburn gives only three specimens. "Which is the most, twice twenty-five, or twice five and twenty (2×25 or $2 \times 5 + 20$?) Ans.—Twice twenty-five. Which is the most, six dozen dozen, or half a dozen dozen ($6 \times 12 \times 12$ or 6×12?) Ans.—6 dozen dozen. It is a fact, too, that somebody asked how many black beans would make five white ones? Ans.—5, if you skin them."[1] It is at once apparent that these questions do not demand any extraordinary calculating powers, but on the other hand, a sharpness of wit and an analytical quickness of comprehending puzzles that would be phenomenal in a joker and riddle-maker of ripe years. If it is really true that the child answered the last of these questions, then the real miracle is that he should on not a single other occasion of his life have shown a sign of the Yankee quickness and shrewdness here implied.

On the journey to Boston, Zerah's wonderful gifts convinced A. B., Esq., that "something had happened contrary to the course of nature and far above it;" he was compelled by this "to renounce his Infidel foundation, and ever since has been established in the doctrines of Christianity." At

[1] Memoirs, p. 12.

Boston he gave public exhibitions. "Questions in multiplication of two or three places of figures, were answered with much greater rapidity than they could be solved on paper. Questions involving an application of this rule, as in Reduction, Rule of Three, and Practice, seemed to be perfectly adapted to his mind." The extraction of the roots of exact squares and cubes was done with very little effort; and what has been considered by the Mathematicians of Europe an operation for which no rule existed, viz., finding the factors of numbers, was performed by him, and in course of time he was able to point out his method of obtaining them. "Questions in Addition, Subtraction and Division were done with less facility, on account of the more complicated and continued effort of the memory [sic.] In regard to the higher branches of Arithmetic, he would observe that he had no rules peculiar to himself; but if the common process was pointed out as laid down in the books, he would carry on the process very readily in his head."[1]

Among the questions answered at Boston were the following:[2] "The number of seconds in 2000 years was required?"

730,000 days,
17,520,000 hours,
1,051,200,000 minutes,
63,072,000,000 seconds, } Answer.

"Supposing I have a corn-field, in which are 7 acres, having 17 rows to each acre; 64 hills to each row; 8 ears on a hill, and 150 kernels on an ear; how many kernels on the corn-field? Answer, 9,139,200."

At this time he was a child only six years old, unable to read and ignorant of the name or properties of one figure traced on paper. The exercise of his faculty under such circumstances causes him later to exclaim: "for it ever has been, and still is, as much a matter of astonishment to him as it can be to any other one; God was its author, its object and aim are perhaps still unknown."[3]

Shortly afterward, on a steamboat journey up to Albany, a gentleman taught Zerah the names and the powers of the

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[1] Memoirs, p. 15.

[2] P. 171, of the Memoirs, perhaps on the authority of the London Prospectus mentioned above, although Colburn does not say so.

[3] Memoirs, p. 15.

nine units, of which he had been previously ignorant. In June, 1811, he visited Portsmouth and answered the following: "Admitting the distance between Concord and Boston to be 65 miles, how many steps must I take in going this distance, allowing that I go three feet at a step? The answer, 114,400, was given in ten seconds. "How many seconds in eleven years? Answer, in four seconds, 346,896,000. What sum multiplied by itself will produce 998,001? In less than four seconds, 999."[1]

Next summer Zerah's father took him to England and made efforts to secure the patronage of the nobility. At a meeting of his friends "he undertook and succeeded in raising the number 8 to the sixteenth power, 281,474,976,710,656. He was then tried as to other numbers, consisting of one figure, all of which he raised as high as the tenth power, with so much facility that the person appointed to take down the results was obliged to enjoin him not to be too rapid. With respect to numbers of two figures, he would raise some of them to the sixth, seventh and eighth power, but not always with equal facility; for the larger the products became the more difficult he found it to proceed. He was asked the square root of 106,929, and before the number could be written down he immediately answered 327. He was then requested to name the cube root of 268,336,125, and with equal facility and promptness he replied 645 [Extracted from a Prospectus printed in London, 1813]."[2]

"It had been asserted . . . . that 4,294,967,297 ($=2^{32}+1$) was a prime number. . . . Euler detected the error by discovering that it was equal to $641 \times 6,700,417$. The same number was proposed to this child, who found out the factors by the mere operation of his mind. [Ibid.]"[3]

Colburn is undoubtedly the one referred to as the Russian boy in the Gentleman's Magazine of 1812. He showed him-

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[1] Memoirs, p. 171.

[2] Memoirs, p. 37.

[3] Memoirs, p. 38. It requires considerable faith to accept this statement, although S. B. Morse met him in London, and a friend of Morse writes that "There was some great arithmetical question, I do not exactly know what, which he solved almost as soon as it was put to him, though it for several years baffled the skill of some of the first professors." Prime, Life of S. B. Morse, New York, 1875, p. 68.

self to the merchants of the London Stock Exchange; one of them gave the boy a guinea of William III, and demanded to know how many years, months and days had elapsed since its coinage; all of which he answered promptly.[1] This is confirmed by a passage in a letter from a friend of S. B. Morse: "Zerah Colburn . . . has called on us. . . . He has excited much astonishment here, and, as they are very unwilling just at this time to allow any cleverness to the Americans, it was said in some of the papers that he was a Russian."[2]

The father and son, after a visit to Ireland and Scotland, returned to London. In 1814 they proceeded to Paris, where the people manifested very little interest in his calculations. This neglect he can only explain by a national defect of character or a crushing historical event. "Whether it were principally owing to the native frivolity and lightness of the French people, or to the painful effect produced by the defeat of their armies and the restoration of the exiled Louis XVIII, cannot be correctly stated; probably it was owing to the former, etc."[3]

He was introduced to and examined by the members of the French "Institute," among whom was La Place. "Three months had now elapsed that he had not been exhibited, but had given his attention to study; even in this short space it was observable that he had lost in the quickness of his computations."[4] Before long his calculating power left him entirely.

By the exertions of Washington Irving, at that time in Paris, the boy obtained admission to the Lyceum Napoleon (or Royal College of Henri IV.) Zerah gives an interesting account of this institution, which was under strict military discipline, and also of Westminster School, in which he was placed on his return to England.

Being in financial straits the father suggests the stage, and so Zerah makes an unsuccessful attempt at acting. There-

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[1] Gentleman's Magazine, 1812, Vol. LXXXII, Pt. II, p. 584.
[2] Prime, Life of S. B. Morse, New York, 1875, p. 68.
[3] Memoirs, p. 74.
[4] Memoirs, p. 76.

after, in 1821, he starts a private school, which was given up after somewhat more than a year. After his return to America he joined the Congregational church, but soon went over to the Methodists and began to hold religious meetings. He was ordained deacon, and labored thenceforth as an itinerant preacher, till, in 1835, he was appointed "Professor of the Latin, Greek, French and Spanish Languages, and English Classical Literature in the seminary styled the Norwich University."[1] Here he died at the age of 35, leaving a wife and three children.

It is to be remarked that Colburn's calculating powers, such as they were, seemed to have absorbed all his mental energy; he was unable to learn much of anything, and incapable of the exercise of even ordinary intelligence or of any practical application. The only quality for which he was especially distinguished was self-appreciation. He speaks, for example, of Bidder as "the person who approached the nearest to an equality with himself in mental arithmetic."[2] Again, "he thinks it no vanity to consider himself first in the list in the order of time, and probably first in the extent of intellectual power."[3]

Colburn possessed bodily as well as mental peculiarities. His father and great-grandmother had a supernumerary digit on each hand and each foot; Zerah and three (or two?[4]) brothers possessed these extra members, while they were wanting in two brothers and two sisters. These digits are attached to the little fingers and little toes of the hands and feet, each having complete metacarpal and metatarsal bones.[5] Zerah leaves it a matter of doubt "whether this be a proof of direct lineal descent from Philistine blood or not (see 1 Chronicles xx. 6)."[6] A portrait of Colburn was made in Philadelphia in 1810, and placed in the museum,[7] and another

[1] American Almanac, 1840, p. 307, where he is spoken of as Rev. Zerah Colburn. The University of Norwich (Vt.), after a fire in 1866, was removed to Northfield, Vt.

[2] Memoirs, p. 175.

[3] Memoirs, p. 176.

[4] Memoirs, p. 72.

[5] Philos. Mag. XLII, 481.

[6] Memoirs, p. 72.

[7] Memoirs, p. 20.

was engraved in London in 1812. The origin of the portrait prefixed to his memoirs is not given; it shows a large head, with unusual development of the upper parts; the forehead is rather small and angular, the occiput is small;[1] the eyes are quite large with projecting orbital arch. Gall, who examined the boy without any previous intimation of his character, "readily discovered on the sides of the eyebrows certain protuberances and peculiarities which indicated the presence of a faculty for computation."[2]

MANGIAMELE.—In the year 1837 Vito Mangiamele, who gave his age as 10 years and 4 mos., presented himself before Arago in Paris. He was the son of a shepherd of Sicily, who was not able to give his son any instruction. By chance it was discovered that by methods peculiar to himself, he resolved problems that seemed at the first view to require extended mathematical knowledge. In the presence of the Academy Arago proposed the following questions: "What is the cubic root of 3,796,416? In the space of about half a minute the child responded 156, which is correct. What satisfies the condition that its cube plus five times its square is equal to 42 times itself increased by 40? Everybody understands that this is a demand for the root of the equation: $x^3 + 5x^2 - 42x - 40 = 0$. In less than a minute Vito responded that 5 satisfied the condition; which is correct. The third question related to the solution of the equation: $x^5 - 4x - 16779 = 0$. This time the child remained four to five minutes without answering; finally he demanded with some hesitation if 3 would not be the solution desired. The secretary having informed him that he was wrong, Vito, a few moments afterwards, gave the number 7 as the true solution. Having finally been requested to extract the 10th root of 282,475,249, Vito found in a short time that the root is 7."[3] At a later date a committee, composed of Arago, Cauchy and others, complains that "the masters of Mangiamele have always kept secret the methods of calculation which he made use of."[4]

[1] Medical and Philos. Journal and Rev., N. Y., 1811, Vol. III, p. 21

[2] Memoirs, p. 77.

[3] Comptes rendus des séances de l'Academie des Sciences, 1837, IV, 978.

[4] Comptes rendus, etc., 1840, XI, 952; reprinted in Oeuvres complétes de A. Cauchy, Paris, 1885, Ire serie, tome V, p. 493.

ZACHARIAS DASE.[1]—Zacharias Dase (also, Dahse) *1824, †1861, was born with a natural talent for reckoning; in his own opinion his early instruction had very little influence on him; but his powers were later developed by practice and industry.[2] He spent most of his life in Hamburg, but made many journeys through Germany, Denmark and England, giving exhibitions in ready reckoning in the most important towns. He became acquainted with many learned men, among whom were Gauss, Schumacher, Petersen, Encke, et al. On one occasion Petersen tried in vain for six weeks to get the first elements of mathematics into his head. Schumacher credits him with extreme stupidity.

In 1840, Dase exhibited in Vienna. He attended the lectures of Prof. Strasznicky on the elements of mathematics, who seems to have brought him to such a point that under the guidance of a good mathematician he could do scientific work. He was induced to reckon out the value of π, which he did in two months with the formula $\frac{1}{4}\pi = \text{arc tang } \frac{1}{2} + \text{arc tang } \frac{1}{8} + \text{arc tang } \frac{1}{5}$. The result, which is published in Crelle's Journal (loc. cit.), agreed with that of Thibaut. In 1844, he had a position in the Railroad Department at Vienna; in 1845 he appears in Mannheim; in 1846 he seems to have had a position in Berlin.

Dase was ambitious to make some use of his powers in the service of science. In 1847 he had reckoned out the natural logarithms (7 places) of the numbers from 1 to 1,005,000, and was seeking a publisher.[3] In reckoning on paper he possessed all the accuracy of mental calculation, and added to

[1] Bibliography: Briefwechsel zwischen Gauss und Schumacher, herausg. von Peters, Altona, 1861. III, 382; V, 30, 32, 277, 278, 295, 296, 297, 298, 300, 301, 302, 303, 304; VI, 27, 28, 78, 112. Crelle; Journal für Mathematik, 1844, vol. XXVII, p. 198. Dase; Factoren-Tafel für alle Zahlen der siebenten Million, Hamburg, 1862; (the introduction contains remarks on Dase and a letter from Gauss.) Schröder's Lexikon der hamburgischen Schriftsteller, Hamburg, 1851, v. Dase. Auszug aus dem Album des Zacharias Dase, Wien, 1850. Anhang dazu, Hamburg, 1850. Preyer, Counting Unconsciously, in Pop. Science Monthly, XXIX, p. 221; reprinted from Die Gartenlaube, 1886, Bd. I. Littell's Living Age, 1857, LIV, p. 61, "On Mental Calculation"; reprinted from the Atheneum. Accounts of Dase are given in two periodicals at present not accessible to me: Allgemeine Literatur-Zeitung, 1861; Zeitschrift für österreichische Gymnasien, vol. XII.

[2] Schröder's Lexikon gives the account of Dase "nach Selbstbericht."

[3] Briefwechsel zw. Gauss und Schumacher, V, 277.

this an incredible rapidity in doing long problems. In the same year he had completed the calculations for the compensation of the Prussian triangulations. In 1850 the largest hyperbolic table, as regards range, was published by him at Vienna, under the title, "*Tafel der natürlichen Logarithmen der Zahlen*"; the same was reprinted in the annals of the Vienna observatory.[1]

In 1850 Dase went to England to earn money by exhibitions of his talents. Much the same is related of his great powers as in Germany; his general obtuseness also occasioned remark. He could not be made to have the least idea of a proposition in Euclid. Of any language but his own he could never master a word.

In 1849 Dase had wished to make tables of factors and prime numbers from the 7th to the 10th million. The Academy of Sciences at Hamburg was ready to grant him support, provided Gauss considered the work useful. Gauss writes him: "With small numbers, everybody that possesses any readiness in reckoning, sees the answer to such a question [the divisibility of a number] at once directly, for greater numbers with more or less trouble; this trouble grows in an increasing relation as the numbers grow, till even a practiced reckoner requires hours, yes days, for a single number; for still greater numbers, the solution by special calculation is entirely impracticable. . . . You possess many of the requisite qualities [for establishing tables of factors] in a special degree, a remarkable agility and quickness in handling arithmetical operations, . . . and an invulnerable persistence and perseverance."[2] The assistance was granted and Dase gave himself up to the execution of the task. Up to his death, in 1861, he had completed the 7th million and also the 8th, with the exception of a small portion. Thus he was able to turn his only mental ability to the service of science, forming a contrast to Colburn and Mondeux, who enjoyed even greater advantages yet failed to yield any results.

[1] Tafel der natürlichen Logarithmen, in Annalen der K. K. Sternwarte in Wien, Theil 34, neuer Folge Bd. XIV, Wien, 1851.

[1] Gauss's letter is given in the preface to Dase's Factoren-Tafeln, 7te Million, Hamburg, 1862.

He multiplied and divided large numbers in his head, but when the numbers were very large he required considerable time. Schumacher once gave him the numbers 79,532,853 and 93,758,479 to be multiplied. From the moment in which they were given to the moment when he had written down the answer, which he had reckoned out in his head, there elapsed 54 seconds.[1] He multiplied mentally two numbers each of 20 figures in 6 minutes; 40 figures in 40 minutes; and 100 figures in 8¾ hours, which last calculation must have made his exhibitions somewhat tiresome to the onlookers. He extracted mentally the square root of a number of 100 figures in 52 minutes.

It is curious that although Dase generally reckoned with astonishing accuracy, yet on at least two occasions his powers failed him. While he was in Hamburg, in 1840, he gave striking proofs of his talents, but at times made great mistakes, which luckily for him happened seldomer than his correct answers.[2] In 1845, Schumacher writes, "at a test which he was to undergo before me, he reckoned wrongly every time." This was explained as coming from a headache.

He had one ability not present to such a great degree in the other ready reckoners. He could distinguish some thirty objects of a similar nature in a single moment as easily as other people can recognize three or four. The rapidity with which he would name the number of sheep in a herd, of books in a book-case, of window-panes in a large house, was even more remarkable than the accuracy of his mental calculations.

PROLONGEAU.—A committee of the Academy of Sciences of Paris, including Arago and Cauchy, undertook in 1845 to investigate the powers of a child of 6½ years, who possessed an extraordinary aptitude for calculation. "He solves mentally with great facility problems relating to the ordinary operations of arithmetic and to the solution of equations of the first degree."[3]

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[1] Briefwechsel zw. Gauss und Schumacher, V, 302.
[2] Briefwechsel zw. Gauss und Schumacher, III, 382.
[3] Comptes rendus des séances de l'Academie des Sciences, 1845, t. XX, p. 1629.

GRANDMANGE.—In 1852 the attention of the Academy of Sciences of Paris was called to a young man of 16 years, C. Grandmange, born without legs or arms, who performed mentally very complicated calculations and solved difficult problems.[1] The committee appointed to investigate the case seems never to have reported.

MONDEUX.—Henri Mondeux,[2] *1826, †1862, was the son of a poor wood-cutter in the neighborhood of Tours. Sent at the age of seven to keep sheep, and deprived of all instruction, he amused himself in counting and arranging pebbles. At this period of life pebbles seem to have been his signs for numbers, for he was ignorant of figures. He learned to execute arithmetical operations mentally and to create for his own use ingenious methods of simplification. After long exercise at this calculation, he used to offer to persons he met to solve certain problems such as to tell how many hours or minutes were contained in the number of years which expressed their ages. This awakened the interest of M. Jacoby, a schoolmaster at Tours, who sought him out. Jacoby proposed several problems and received immediate answers and, finding that the boy could neither read, write nor cipher, and that he had no acquaintance with fractions or any of the ordinary rules of arithmetic, he offered to instruct him. Unfortunately the mind that could carry so many figures could not remember a name or an address, so the boy spent a month searching the city before he found his benefactor. He received instruction in calculation and was often shown in neighboring colleges and schools.

Although in other matters he showed only mediocre intelligence, yet he was something more than a mere calculating machine, as is shown for example in his way of solving the

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[1] Comptes rendus, etc., 1852, t. XXXIV, p. 371.

[2] Bibliography: Comptes rendus des séances de l'Academie des Sciences, XI, p. 820, 952. Oeuvres complètes de Cauchy, Ire Série, Paris, 1885, t. V, p. 493, (this is a reprint of Cauchy's report in the Comptes rendus.) Barbier, Vie d'Henri Mondeux, 1841; Jacoby, Biographie d'Henri Mondeux, 1846; Jacoby, la Clef de l'Arithmetique, 1860; Larousse, Dictionaire universelle, v. Mondeux; Didot's Nouvelle biographie générale, v. Mondeux; The Story of a Wonderful Boy Mathematician, in Every Saturday p. 118, vol. XI, June to Dec. 1871; Biographie universelle, Michaud, v. Mondeux.

following problem: "In a public square there is a fountain containing an unknown quantity of water; around it stands a group of people with vessels capable of containing a certain unknown quantity. They draw at the following rate: The first takes 100 quarts and $\frac{1}{13}$ of the remainder; the second 200 quarts and $\frac{1}{13}$ of the remainder; the third 300 quarts and $\frac{1}{13}$, and so on until the fountain was emptied. How many quarts were there? In a few seconds he gave the answer, and this is the simple process by which he obtained it: Take the denominator of the fraction, substract one; that gives the number of persons. Multiply that by the number of quarts taken by the first person—that is, by 100—and you get the equal quantities taken by each; square this number and multiply by the number of quarts, and you get the quantity in the fountain."[1]

In 1840, M. Jacoby presented the boy to the Academy of Sciences of Paris. Jacoby had taken note of the processes employed, and the boy was willing to unfold them himself before a commission. On this occasion two questions were given him, one of which was this: "How many minutes in 52 years? The child, who found the problem very simple, responded in a few moments: 52 years of 365 days each, are composed of 27,331,200 minutes, and of 1,639,872,000 seconds."[2] A committee, including Arago and Cauchy, made an exhaustive examination of his powers and reported on the processes used by him. "At present he easily executes in his head not only diverse operations of arithmetic, but also in many cases the numerical resolution of equations: he invents processes, sometimes remarkable, to solve various questions which are ordinarily treated with the aid of algebra."[3] In spite, however, of his marvellous power of inventing and applying arithmetical methods, he did not answer the expectations of his friends, but sank into obscurity and died almost unknown.

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[1] Every Saturday, XI, 118.
[2] Comptes rendus, etc., 1840, t. XI, p. 820.
[3] Comptes rendus, etc., 1840, XI, 953.

GEORGE BIDDER.—Geo. Bidder,[1] *1806, †1878, was the son of an English stonemason. His first and only instruction in numbers was received at about 6 years of age, from his elder brother, from whom he learned to count up to 10 and then to 100.

"I amused myself," he says, "by repeating the process [of counting up to 100], and found that by stopping at 10, and repeating that every time, I counted up to 100 much quicker than by going straight through the series. I counted up to 10, then to 10 again=20, 3 times 10=30, 4 times 10=40, and so on. This may appear to you a simple process, but I attach the utmost importance to it, because it made me perfectly familiar with numbers up to 100; . . . at this time I did not know one written or printed figure from another, and my knowledge of language was so restricted, that I did not know there was such a word as 'multiply'; but having acquired the power of counting up to 100 by 10 and by 5, I set about, in my own way, to acquire the multiplication table. This I arrived at by getting peas, or marbles, and at last I obtained a treasure in a small bag of shot: I used to arrange them in squares, of 8 on each side, and then on counting them throughout I found that the whole number amounted to 64: by that process I satisfied my mind, not only as a matter of memory, but as a matter of conviction, that 8 times 8 were 64; and that fact once established has remained there undisturbed until this day, . . . in this way I acquired the whole multiplication table up to 10 times 10; beyond which I never went; it was all that I required."[2]

Most of the child's time was spent with an old blacksmith.

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[1] Bibliography: Proceedings of the Institution of Civil Engineers, vol. XV., session 1855-56, London, 1856, p. 251 ff., "On Mental Calculation"; Vol. LVII, session 1878-79, part III, London, 1879, p. 294, "Memoirs of Deceased Members." A Memoir of Zerah Colburn, written by himself, Springfield, 1833, p. 175. Philosophical Magazine, XLVII, London, 1816, p. 314. Reviews of Bidder's speech are given in Littell's Living Age, 1856, XLIX, p. 254; 1857, LIV, p. 61. A correspondent in the Spectator, 1879, LII, p. 111, quotes from a pamphlet in his possession, the title-page of which is missing. The printers of this pamphlet were M. Bryan & Co., Bristol, and the date is estimated to be 1820. It contains a large number of questions proposed to Bidder at various places in the years 1816-19; the answers given are appended, often with the time it took him to perform the operation.

[2] Proceedings Civ. Eng., XV, p. 257.

On one occasion somebody by chance mentioned a sum and the boy astonished the bystanders by giving the answer correctly. "They went on to ask me up to two places of figures, 13 times 17 for instance; that was rather beyond me at the time, but I had been accustomed to reason on figures, and I said 13 times 17 means 10 times 10 plus 10 times 7, plus 10 times 3 and 3 times 7. . . . . "[1]

While remaining at the forge he received no instruction in arithmetic beyond desultory scraps of information derived from persons who came to test his powers, and who often in doing so gave him new ideas and encouraged the further development of his peculiar faculty, until he obtained a mastery of figures that appeared almost incredible. "By degrees I got on until the multiple arrived at thousands. Then . . . . it was explained to me that 10 hundreds meant 1000. Numeration beyond that point is very simple in its features; 1000 rapidly gets up to 10,000 and 20,000, as it is simply 10 or 20 repeated over again, with thousands at the end, instead of nothing. So by degrees I became familiar with the numeration table, up to a million. From two places of figures I got to three places; then to four places of figures, which took me up of course to tens of millions; then I ventured to five and six places of figures, which I could eventually treat with great facility, and as already mentioned, on one occasion I went through the task of multiplying 12 places of figures by 12 figures, but it was a great and distressing effort."[2]

Before long he was taken about the country by his father for the purpose of exhibition. This was so profitable for the father that the boy's education was entirely neglected. Even at the age of ten he was just learning to write; figures he could not make. Some of the questions he had answered were the following: "Suppose a cistern capable of containing 170 gallons, to receive from one cock 54 gallons, and at the same time to lose by leakage 30 gallons in one minute; in what time will the said cistern be full?" "How many drops are there in a pipe of wine, supposing each cubic inch to contain 4685 drops, each gallon 231 inches and 126 gallons

[1] Proceedings Civ. Eng., XV, p. 258.
[2] Proceedings Civ. Eng., XV, p. 259.

in a pipe?" "In the cube of 36, how many times 15228?"[1] Among others the famous Herschel came in 1817 to see the "Calculating Boy."

Shortly afterward he was sent to school for a while. Later he was privately instructed, and then attended the University of Edinburgh, obtaining the mathematical prize in 1822. Later he entered the Ordnance Survey, and then was employed by the Institution of Civil Engineers. He was engaged in several engineering works of importance; he is also to be regarded as the founder of the London telegraphic system. His greatest work was the construction of the Victoria (London) Docks. Bidder was engaged in most of the great railway contests in Parliament, and was accounted "the best witness that ever entered a committee room." He was a prominent member, Vice President, then President of the Institution of Civil Engineers. In his later years there was no appreciable diminution in Bidder's powers of retaining statistics in his memory and of rapidly dealing with figures. Two days before his death the query was suggested that taking the velocity of light at 190,000 miles per second, and the wave length of the red rays at 36,918 to an inch, how many of its waves must strike the eye in one second. His friend, producing a pencil, was about to calculate the result, when Mr. Bidder said, "You need not work it; the number of vibrations will be 444,433,651,200,000."[2]

The fact that Bidder became a highly educated man, and one of the leading engineers of his time; that his powers increased rather than diminished with age; and above all, that he has given a clear and trustworthy account of how he obtained and exercised his talent, renders his testimony of the highest worth, and provides the solution of many of the dark problems met with in the cases of Dase, Colburn, and others. Indeed, he seems to fill out just what is lacking in each case; Dase never gave a good account of the way in which he worked; Colburn could not till later explain his methods, and then only in the clumsy way to be expected from a young man of little education; finally, just the part we cannot understand in Buxton is here explained in full.

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[1] Philos. Mag. XLVII, p. 315.
[2] Proceedings Civ. Eng., LVII, 309.

In 1814 a witness to his powers states that he displayed great facility in the mental handling of numbers, multiplying readily and correctly two figures by two, but failing in attempting numbers of three figures. This same witness was present at an examination of the boy in 1816 by several Cambridge men. The first question was a sum in simple addition, two rows with twelve figures in each row; the boy gave the correct answer immediately. After more than an hour the question was asked, "Do you remember the sum in addition I gave you?" He repeated the twenty-four figures with only one or two mistakes. At this time he could not explain the processes by which he worked out long and intricate sums. "It is evident that in the course of two years his powers of memory and calculation must have been gradually developed."[1]

This development seems to have been steady. The following series shows the increasing rapidity with which the answers came:

1816 (10 years of age). What is the interest of £4,444 for 4,444 days, at 4½% per annum? Ans. in 2 min., £2,434, 16*s.* 5¼*d.*

1817 (10 years of age). How long would a cistern 1 mile cube be filling, if receiving from a river 120 gallons per minute without intermission? Ans. in 2 minutes—years 14,300, days 285, hours 12, minutes 46.

1818 (11 years of age). Divide 468,592,413,563 by 9,076. Ans. within 1 min., 51,629,838.

1818 (12 years of age). If the pendulum of a clock vibrates the distance of 9¾ inches in a second of time, how many inches will it vibrate in 7 years, 14 days, 2 hours, 1 minute, 56 seconds, each year being 365 days, 5 hours, 48 minutes, 55 seconds? Ans. in less than a minute, 2,165,-625,744¾ inches.

1819 (13 years of age). To find a number whose cube less 19 multiplied by its cube shall be equal to the cube of 6. Ans. instantly, 3.[2]

Sir Wm. Herschel put the following question to the boy:

[1] Spectator, 1879, LII, p. 47.
[2] Spectator, 1879, LII, p. 111.

Light travels from the sun to the earth in 8 minutes, and the sun being 98,000,000 miles off, if light would take 6 years and 4 months traveling at the same rate from the nearest fixed star, how far is that star from the earth, reckoning 365 days and 6 hours to each year, and 28 days to each month? Ans., 40,633,740,000,000 miles.[1]

Curious enough is the fact that Bidder and Colburn met in Derbyshire, and underwent a comparative examination, the result of which is said to have been to the total defeat of Colburn.[2]

Prof. Elliot, of Liverpool, who knew Bidder from the time they were fellow-students in Edinburgh, says he was a man of first-rate business ability and of rapid and clear insight into what would pay, especially in railway matters. As a proof of this statement we can accept the fact that Bidder became a wealthy man.

The Bidder family seem to have been distinguished for mental traits resembling George Bidder's in some part or another. Bidder was noted for his great mathematical ability and his great memory. One of his brothers was an excellent mathematician and an actuary of the Royal Exchange Life Assurance Office.[3] Rev. Thomas Threlkeld, an elder brother, was a Unitarian minister. He was not remarkable as an arithmetician, but he possessed the Bidder memory and showed the Bidder inclination for figures, but lacked the power of rapid calculation. He could quote almost any text in the Bible, and give chapter and verse.[4] He had long collected all the dates he could, not only of historical persons, but of everybody; to know when a person was born or married was a source of gratification to him.[5]

One of George Bidder's nephews at an early age possessed remarkable mechanical ingenuity.

Most interesting of all is the partial transmission of his peculiar faculties to his son, George Bidder, Q. C., and through him to two grandchildren. The second son was a

[1] Spectator, 1879, LII, p. 112.
[2] Spectator, 1879, LII, p. 112.
[3] Spectator, 1878, LI, p. 1635.
[4] Spectator, 1878, LI, p. 1635.
[5] Brierly's Journal, Jan. 25, 1879, quoted in the Spectator, 1879, LII p. 143.

first-class man in classics at Oxford, and Fellow of his college. The elder Bidder, however, possessed the peculiar faculties of the family is such proportions that he far exceeded the others in calculating powers.

GEORGE BIDDER, Q. C.—Bidder's calculating faculty was transmitted to his eldest son. It has caused some confusion that he bore the same name as his father. Some writers have lately referred to the father as G. P. Bidder, but since he was always known as Geo. Bidder, the only way out of the difficulty is to distinguish the son by adding his title.

George Bidder, Q. C., distinguished himself at Cambridge in mathematics, being seventh wrangler of his year. He is now a thriving barrister and Queen's Counsel.

He possesses a remarkable visual memory. He always sees mental pictures of figures and geometrical diagrams. "If I perform a sum mentally it always proceeds in a visible form in my mind; indeed, I can conceive no other way possible of doing mental arithmetic."[1]

He considers the special aids to mental calculation to be a powerful memory of a peculiar cast, in which figures seem to stereotype themselves without an effort, and an almost inconceivable rapidity of operation. The former he possessed in a high degree; the latter was no doubt congenital, but was developed by incessant practice and by the confidence thereby acquired.

Bidder says: "I myself can perform pretty extensive arithmetical operations mentally, but I cannot pretend to approach even distantly to the rapidity and accuracy with which my father worked. I have occasionally multiplied 15 figures by 15 [figures] in my head, but it takes me a long time and I am liable to occasional errors." Just before writing this he tried the following to see if he could still do it:

378,201,969,513,825
199,631,057,265,413

"I got in my head the answer 75,576,299,427,512,145,197,-597,834,725, in which I think you will find four figures out of the 29 are wrong."[2]

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1 Spectator, 1878, LI, p. 1634.
2 Spectator, 1878, LI, p. 1635.

We have no account from George Bidder, Q. C., to show whether he performs the operations rapidly or not.

The daughters of Mr. Bidder, Q. C., show more than the average but not extraordinary powers of doing mental arithmetic. To test their calculating powers Prof. Elliot in 1877 asked them, "At what point in the scale do Fahrenheit's thermometer and the centigrade show the same number at the same temperature?" The nature of the two scales had to be explained, but after that they were left to their own resources. The next morning one of the younger ones (about ten years old) said it was at 40 degrees below zero. This is the correct answer; she had worked it out in bed.[1]

Another granddaughter shows great visual memory. On one occasion she remarked, "When I hear anything remarkable read or said to me, I think I see it in print."

SAFFORD.[2]—Truman Henry Safford was born at Royalton, Vt.,[3] in 1836. Even in his earlier years his parents had amused themselves with his power of calculating. When six years of age he told his mother that if he knew how many rods it was round his father's large meadow he could tell the measure in barley-corns; on hearing that it was 1040 rods, he gave, after a few minutes the answer, 617,760, which he had reckoned out in his head. Before his eighth year he had gone to the extent of Colburn's powers. His abilities were won by means of study, and it was observed that he improved rapidly by practice and lost by neglect.

In 1845, Dr. Dewey wrote of him, "He is a regular reasoner on correct and established princibles, taking the easiest and most direct course. As he had Hutton's Mathematics, and wanted some logarithms, his father told me he computed the logarithms from 1 to 60 by the formula given

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[1] Spectator, 1878, LI, p. 1634.

[2] Bibliography: Appleton's Cyclopedia of American Biography, v. Safford; Chamber's Edinburgh Journal, July-Dec. 1847, Vol. VIII, p. 265, article "Truman Henry Safford," (this is founded on an article in the "Christian Alliance and Family Visitor," of Boston); Littell's Living Age, XVI, p. 82, has copied the article from Chamber's Journal; Leisure Hour, I, p. 540, contains an abstract of the same article.

[3] It is a curious fact that Safford was born within 40 miles of Colburn's birthplace, and 15 of Norwich.

by Hutton, which were afterwards found to be the same in a table of logarithms for the same number of decimals."[1]

In his return from a little tour, in which he had been introduced to various scientific men, he set about constructing an almanac which was put to press when the author was just 9½ years old. In the following year he calculated four different almanac calendars. While getting up the Cincinnati one he originated a new rule for getting moon-risings and settings, accompanied by a table which saves full one-fourth of the work in casting moon-risings. This rule and the manuscript almanacs are preserved in the Harvard library, as are also his new rules for calculating eclipses. At ten years of age he was carefully examined by Rev. H. W. Adams, with questions prepared beforehand. Adams says: "I had only to read the sum to him once. . . . Let this fact be remembered in connection with some of the long and blind sums I shall hereafter name, and see if it does not show his amazing power of conception and comprehension."[2] The questions given him became continually harder. "What number is that which, being divided by the product of its digits, the quotient is 3; and if 18 be added the digits will be inverted? He flew out of his chair, whirled around, rolled up his eyes and said in about a minute, 24." "What is the entire surface of a regular pyramid whose slant height is 17 feet and the base a pentagon, of which each side is 33.5 feet? In about two minutes after amplifying round the room, as his custom is, he replied 3354.5558. 'How did you do it,' said I. He answered: Multiply 33.5 by 5 and that product by 8.5 and add this product to the product obtained by squaring 33.5, and multiplying the square by the tabular area taken from the table corresponding to a pentagon."

"Multiply in your head 365,365,365,365,365 by 365,365,-365,365,365,365. He flew around the room like a top, pulled his pantaloons over the top of his boots, bit his hand, rolled his eyes in their sockets, sometimes smiling and talking, and then seeming to be in agony, until, in not more than one minute, said he, 133,491,850,208,566,925,016,658,299,941,-

[1] Chamber's Journal, VIII, p. 265.
[2] Chamber's Journal, VIII, p. 266.

583,225! . . . he began to multiply at the left hand and to bring out the answer from left to right."[1]

In the number of figures this exceeds Bidder's longest multiplication, but the repetition of the same figures renders it easier.

Safford had not a one-sided mind; "chemistry, botany, philosophy, geography and history are his sport." "His memory too is very retentive. He has pored over Gregory's Dictionary of the Arts and Sciences so much that I seriously doubt whether there can be a question asked him drawn from either of those immense volumes that he will not answer instantly." This reminds one of the story of Ampère and the encyclopedia.[2]

On an invitation of the Harvard University his father removed to Cambridge and Safford was placed under the charge of Principal Everett and Professor Peirce. At the age of 14 he calculated the elliptic elements of the first comet of 1849. After graduating from Harvard in 1854, he spent several years there in the observatory. Since this time he has made many important astronomical calculations and discoveries, and numerous contributions to the astronomical journals. He is at present Professor of Astronomy in Williams College.

In regard to the divisors of large numbers, Safford seemed to possess the power of recognizing in a few moments what numbers were likely to divide any given large number, and then of testing the matter by actual division with great rapidity.[3]

MISCELLANEOUS.—A boy from St. Poelten was exhibited by Gall in Vienna. He was the son of a blacksmith and had received no more instruction at school than his companions. At nine years of age, when they gave him three numbers each expressed by ten or twelve figures, asking him to add them, then to subtract them two by two, to multiply and then

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[1] Chamber's Journal, VIII, p. 266.

[2] See Arago's Eulogy on Ampère, translated in the Smithsonian Report, 1872.

[3] Belgravia, XXXVIII, p. 456.

divide them by numbers containing three figures, he would give one look at the numbers and announce the result before it could be obtained by others on paper.[1]

Gall says that an advocate came to him to complain that his son, aged five years, was occupied exclusively with numbers and calculations, and that it was impossible to fix his attention on anything else.[2]

Devaux, a boy of seven years, had a passion for going to all the fairs, and waiting for the traders at the moment when they had closed their accounts; when they made mistakes in their calculations, it was his greatest pleasure to discover the error.[3]

Mr. Van R. of Utica, U. S. A., at the age of six years distinguished himself by a singular facility for calculating in his head; at eight he entirely lost this faculty, and after that time he could calculate neither better nor faster than any other person. He did not retain the slightest idea of the manner in which he performed his calculations in childhood.[4]

The daughter of Lord Mansfield, seen by Spurzheim at London, when she was 13 years old, almost equaled Colburn; she extracted with great facility the square and cube root of numbers of nine places.[5]

Prof. Elliot tells of a half idiot who was remarkable in his own county district for his powers of calculation. He got him to put down his operations in a few cases on paper; his modes of abbreviation were ingenious.[6]

Huber tells of a blind Swiss who solved the most difficult arithmetical problems, and who was able to repeat in either way a line of 150 figures after hearing them only once.[7]

II.

The duty of a psychological analysis of the powers of arithmetical prodigies would be to determine the processes of

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[1] Huber; Das Gedächtniss, München, 1878, p. 43.
[2] Gall, Functions of the Brain, Organology, XVIII.
[3] Gall, loc. cit.
[4] Gall, loc. cit.
[5] Medical and Philosophical Journal and Review, New York, 1811, p. 22.
[6] Gall. loc. cit.
[7] Spectator, 1878, LI, p. 1634.

which such powers consist and to establish a series of gradations from the normal to the abnormal. It lies, however, outside of our present task to investigate the fundamental arithmetical processes, though just these cases seem to offer a means of clearing up some of the obscurity; we shall not go beyond facts such as, accuracy of memory, arithmetical association, etc., which for our purposes can be regarded as not requiring further analysis.

Speaking of the ability to reckon rapidly, Gauss remarks: "Two things must be distinguished here, a powerful memory for figures and a real ability for calculation. These are really two qualities entirely independent of each other, which can be united but are not always so."[1] Bidder's opinion was "that mental calculation depends on two faculties of the mind in simultaneous operation—computing and registering the result."[2] Nevertheless, there are some other important facts in the psychology of the ready reckoners; we shall accordingly consider them in respect to memory, arithmetical association, inclination to mathematics, precocity and imagination.

MEMORY. Perhaps aside from prococity the most remarkable fact in regard to ready reckoners is their power to do long calculations wholly in the mind without making a mistake; next to this would be placed the wonderful rapidity which some of them have shown.

Accuracy of Memory.—The performance of long calculations in the mind depends above all on the accuracy of the memory for a sufficient length of time. For longer periods of time there seems considerable variation among the several calculators, and indeed this power is not an absolute necessity.

Buxton had perhaps the most accurate memory of all. For example, he gave from memory an account of all the ale or strong beer that he had on free cost since he was 12 years of age; this list included 57 different persons and 2130

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1 Briefwechsel zwischen Gauss und Schumacher, V, 300.
2 Proceedings, Civ. Eng., XV, 262.

glasses. "He will leave a long question half wrought and at the end of several months resume it, beginning where he left off, and proceeding regularly till it is completed."[2] Buxton was very slow and clumsy, but extremely accurate in his calculations, a fact which shows that his powers depended on an accurate memory.

Much the same is related of Fuller. "Though interrupted in the progress of his calculation and engaged in discourse upon any other subject, his operations were not thereby in the least deranged so as to make it necessary for him to begin again, but he would go on from where he had left off, and could give any or all of the stages through which the calculation had passed."[3]

Of Dase it is related that, "after spending half an hour on fresh questions, if asked to repeat the figures he began with, and what he had done with them, he would go over the whole correctly."[4] Half an hour after using the two numbers mentioned on p. 45, it was asked if he remembered them. "He instantly repeated the two numbers together (as a number containing 25 figures) forwards and backwards; 9 quadrillion, 351 thousand, 738 billions, etc."[5]

Of Colburn we have no account that represents him as having a good memory for a long time, yet he, as well as all the others, must have possessed extensive multiplication tables stored up indelibly in their minds. This is not to be confused with what we ordinarily call accuracy of memory, by which we mean that a thing or a number once seen is always retained. We may, however, extend the term and speak of acquired accuracy, where the retention results from a proper impression on the mind by means of association and repetition. Bidder, and probably several of the others, possessed wonderful memories, especially for figures; the acquisition of such a memory was due to their peculiar training,

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[1] Gent. Mag. XXIII, 557.
[2] Gent. Mag. XXIV, 251.
[3] Columbian Centinel, Dec. 29, 1790, No. 31 of Vol. XIV.
[4] Littell's Living Age, LIV, 1857, p. 62.
[5] Briefwechsel zw. Gauss und Schumacher, V. 302. The notation follows the continental system; in English it would be 9 octillions, 351 septillions, 738 sextillions, etc.

and, we suspect, to a lack of the ordinary mind-killing processes found in our schools. Bidder says: "As regards memory I had in boyhood, at school and at college many opportunities of comparing my powers of memory with those of others, and I am convinced that I do not possess that faculty in a remarkable degree.[1] If, however, I have not any extraordinary amount of memory I admit that my mind has received a degree of cultivation in dealing with figures in a particular manner which has induced in it a peculiar power; I repeat, however, that this power is, I believe, capable of being attained by any one disposed to devote to it the necessary time and attention."[2]

Although an accurate memory for a long time may not be possessed by every rapid calculator, he must be able to retain before the mind with absolute accuracy the results of the various processes performed till he has finished the problem. This we can pre-suppose in the case of every one of the arithmetical prodigies, and indeed it seems to have been the one thing in which Buxton was superior to ordinary mortals.

One secret of such an accurate memory while performing a calculation, lies in relieving it of unnecessary burdens. It will be noticed that the ready-reckoners often divided a multiplier into two factors and multiplied first by one and then the other; *e. g.*, 432×56 would be $432 \times 8 = 3456$; 432 and 8 can be now forgotten and $3456 \times 7 = 24192$; whereas in the ordinary way $432 \times 6 = 2592$, must be held in memory, while $432 \times 50 = 21600$ is performed, in order that the partial products may be added together.

There are other means used to lighten the work of the memory. Every one of those about whom we know anything in this respect gave his answers and probably did his work from left to right.[3] Colburn's explanation shows how he began with the highest denominations: "the large numbers found first are easily retained because consisting of so many ciphers."[4]

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[1] Proceedings, Civ. Eng. XV, 253.
[2] Proceedings, Civ. Eng. XV, 253.
[3] Memoirs, p. 191.
[4] See Memoirs, p. 189, 190.

Bidder explains why beginning at the left is easier and necessary. "I could neither remember the figures [in the ordinary way of multiplying], nor could I, unless by a great effort, on a particular occasion, recollect a series of lines of figures; but in mental arithmetic you begin at the left hand extremity, and you conclude at the unit, allowing only one fact to be impressed on the mind at a time. You modify that fact every instant as the process goes on; but still the object is to have one fact and one fact only, stored away at one time."[1] In doing the example 373 × 279, "I multiply 200 in 300=60,000; then multiplying 200 into 70, gives 14,000. I then add them together, and obliterating the previous figures from my mind, carry forward 74,600," etc.

"For instance, multiplying 173 × 397, the following process is performed mentally:

100 × 397=39,700
70 × 300=21,000=60,700
70 × 90 . = 6,300=67,000
70 × 7 . . = 490=67,490
3 × 300 . . . = 900=68,390
3 × 90 . . . . = 270=68,660
3 × 7 . . . . . = 21=68,681.

The last result in each operation being alone registered by the memory, all the previous results being consecutively obliterated until a total product is obtained."[2]

In trying to follow the method used by these men we are hampered by our inability to keep the hundreds, thousands, etc., in their proper places. When a person asks you suddenly how many figures in a million, can you answer him instantly? In his instruction for a ready computer De Morgan gives the following rule: "In numeration learn to connect each primary decimal number, 10; 100; 1000, etc., not with the place in which the unit falls, but with the number of ciphers following. Call ten a *one-cipher* number; a hundred a *two-cipher* number; a million a *six-cipher*, and so on."[3]

Various other little helps were used. Bidder reveals some of them: *e. g.*, "in questions involving division of time, distances, weight, money, etc., it is convenient to bear in mind

[1] Proceedings, Civ. Eng. XV, 260.
[2] Proceedings, Civ. Eng. XV, 260.
[3] De Morgan, Elements of Arithmetic, London, 1857, p. 161.

the number of seconds in a year, inches or barley-corns in a mile, ounces and pounds in a cwt. and ton, pence and farthings in a pound sterling, etc. . . . These were always ready for use when they could be applied with advantage. . . . Suppose it is required to find the number of barley-corns in 587 miles, the ordinary process, viz.: $1{,}760 \times 587 \times 3 \times 12 \times 3 = 111{,}576{,}960$, when worked out, requires 56 figures; while, mentally, I should multiply 190,080, the number of barley-corns in a mile, by 587."[1] When we consider that certain stock questions continually recur among those answered by the prodigies, the assistance of such facts is apparent. Safford always remembered the divisors of any number he had examined.[2]

Extraordinary as their powers were these men are not the only ones distinguished for remembering numbers. After a whole day's public sale, Hortensius could tell from memory all the things sold and their prices. Niebuhr could dictate a whole column of statistics from memory.[3] It is related that Alex. Gwin at 8 years of age knew the logarithms of all numbers from 1 to 1000. He could repeat them in regular order or otherwise.[4]

Of Dirichlet it is said that he possessed an "extraordinary power of memory, by means of which he had at every moment completely before him what he had previously thought and worked out."[5]

Euler had a prodigious memory for everything; this gave him the power of performing long mathematical operations in his head. While instructing his children, the extraction of roots obliged him to give them numbers which were squares; these he reckoned out in his head. Troubled by insomnia, one night he calculated the first six powers of all

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[1] Proc. Civ. Eng., XV, 266.

[2] Belgravia, XXXVIII, 456.

[3] Lieber, Reminiscences of Niebuhr, Phila. 1835, p. 46; also, Lieber's Miscellaneous Writings, Phila. 1881, I, 74.

[4] Belgravia, XXXVIII, 462.

[5] Kummer, Gedächtnissrede auf Gustav Peter Lejeune-Dirichlet, Separatabdr. aus dem Abhandl. d. Kgl. Akad. d. Wiss. zu Berlin, 1860, p. 34.

the numbers under 20, and recited them several days afterwards.[1]

There is on record the case of Daniel McCartney, born 1817, near Mt. Pleasant, Westmoreland Co., Penn., as late as 1871 living in Salem, Columbiana Co., Ohio, who was examined in 1870 by W. D. Henkle, State Commissioner of Public Schools in Ohio. The man showed a remarkable memory. Among other questions put to him were the following which indicate a power not so great as Buxton's but yet remarkable: "Ques.—What is 123 times 456? Ans. (35 seconds), 56,088. Multiply 456 by 100; then 23 by 400; then add; multiply 23 by 56 and add. Ques.—What is 3756 times 182? Ans. ($4\frac{1}{2}$ minutes. He became confused), 683,-592. Ques.—What is the sum of 26, 67, 43, 38, 54, 62, 87, 65, 53, 44, 77, 33, 84, 56 and 14? (One minute occupied in calling the numbers.) Ans. (Instantly) 803."[2]

Still more remarkable is the case of Wallis, the mathematician. In a letter to Thomas Smith of Madalene College, Wallis tells his own story:

"December 22d, 1669.—In a dark night, in bed, without pen, ink or paper or anything equivalent, I did by memory extract the square root of 30000,00000,00000,00000,00000,-00000,00000,00000, which I found to be 1,77205,08075,68077,-29353, *ferè*, and did the next day commit it to writing."

"February 18th, 1670.—Joannes Georgius Pelshower (Regimontanus Borussus) giving me a visit, and desiring an example of the like, I did that night propose to myself in the dark without help to my memory a number in 53 places: 24681357910111214111315161820171921222426283023252729 31 of which I extracted the square root in 27 places: 15710301-6871482805817152171 *proximè;* which numbers I did not commit to paper till he gave me another visit, March following, when I did from memory dictate them to him.

Yours, etc., JOHN WALLIS."[3]

[1] Euleri commentationes arithmeticae collectae, Petropoli, 1849; tomus I, Éloge de L. Euler par N. Fuss, p. XLIX; see also Condorcet's eulogy of Euler.

[2] Remarkable Cases of Memory, in the Journal of Speculative Philosophy, 1871, Vol. V, p. 16.

[3] A copy of this letter is to be found in the Spectator, 1879, Vol. LII, p. 11.

We have here selected a series beginning with Hortensius and Niebuhr, who simply remembered numbers, and proceeding to men who used their memories in calculating with as much success as Buxton. None of these men could well be placed among the arithmetical prodigies, yet Buxton seems to have differed from McCartney only in his interest for figures, whereas in Euler and Wallis the calculating power was lost sight of. Like these men, Buxton showed none of the rapidity seen in all the other calculators.

Performing long calculations in the head has been compared to blindfold chess-playing. When rapidity is left out of consideration, as in Buxton's case, the same power of memory may perhaps account for both. Indeed, Geo. Bidder, Q. C., who possessed a strong power of visual imagery, is able to play two simultaneous games of chess without seeing the board.

Rapidity of Memory.—The rapidity of a memory will depend on the nature of the various processes of the mind which make up the phenomenon called by that name and also on the rapidity with which these processes work. Our power to rapidly commit a group of objects or a line of a dozen figures to memory and to call it up again instantly, depends on the ease and rapidity with which we can impress it on the mind, on the accuracy with which it is retained and the ease and rapidity with which it can be reproduced. The accuracy of retention, being of course only a manifestation of the accuracy of memory, has already been considered.

The ease and rapidity with which a number of objects can be impressed on the memory seem limited in ordinary persons to about five at a glance. Before the days of experimental psychology this was quite a matter of dispute,[1] but it has been in late years definitely settled. The first experiments seem to have been made by Stanley Jevons, who decides that his power does not reach to five with complete accuracy, and that the error in estimating numbers under such conditions, $= \frac{n}{9} - \frac{1}{2}$, where n is the number of objects.[2]

[1] Some of the opinions are given in Hamilton's Lectures on Metaphysics, Vol. I, 253.
[2] The Power of Numerical Discrimination, in Nature, Vol. III, 281.

Preyer has made some popular experiments, from which he concludes that after practice a person can estimate in general correctly up to nine objects seen for an instant, when these objects are irregularly grouped, and that acquaintance with a symmetric arrangement, as in cards or dominoes, raises the limit to about 40.[1] Cattell also made experiments on the extent of the focus of consciousness, which show that 4 to 5 unconnected impressions (lines, letters, figures) can be simultaneously apperceived. When these elements were placed in well known groups the number rose to 12 and 15.[2]

There are, however, two processes to be distinguished, the perception of the objects and the counting of them. Cattell's experiments show how many can be distinctly apperceived; but the power of counting them may depend on the maintenance of the apperceived and even the perceived objects in the memory for a sufficient time. Wishing to know how it is possible to count a number of objects seen for so brief a time, I exposed a few objects to the view for an instant; the person observing had then to tell how many objects were seen. One of the observers gave the first number thought of without being able to tell why; the other always counted or attempted to count the objects from a picture of them which he held in his memory.

All the arithmetical prodigies possessed a remarkable impressibility; they were able to grasp large numbers of figures on only once seeing or hearing them. Dase, morever, has given special proofs of his power by his experiments in rapid counting. "When you throw a handful of peas on the table, a casual glance is sufficient to enable him to tell you their number. He did the same . . . with the points of dominoes at which he gave only a momentary glance in order to tell their sum (117)."[3] "He counted the letters in a line on an octavo and a quarto page (47 and 63) after a hasty glance."[4] Dase's memory also possessed great impressibility

[1] Counting Unconsciously, in Pop. Science Monthly, XXIX, p. 221.
[2] Cattell, Psychometrische Untersuchungen, in Philos. Studien, III, 121; Wundt, Phys. Psychol. 3 Aufl. II, 247.
[3] Briefwechsel zw. Gauss und Schumacher, V, 277.
[4] Briefwechsel zw. Gauss und Schumacher, V, 302.

for figures. "Twelve figures being written down . . . he would just dip his eye upon them, not resting on them more than half a second. He would then repeat them backwards and forwards, and name any one at command, as the ninth or the fourth."[1] Dase can be contrasted with ordinary individuals in this respect. The experiments referred to in the AMERICAN JOURNAL OF PSYCHOLOGY, Vol. II, 607, 608, show that the largest number of numerals that could be learned by once hearing them at the rate of 120 to the minute, was 8.6 for boys of 19 years. Even Mondeux required 5 minutes to learn and retain a number of 24 figures divided into 4 periods, in such a way that he could give at will the six figures in each period.

Such quick apprehension of a number as Dase's can be explained by great impressibility; in which case the visual image would be in such a short time so firmly and vividly impressed on the memory that he could turn away his eyes and count the peas or domino points from a still persistent image, just as the person mentioned above did. The case would then be exactly like that of Robert Houdin and his son. They would pass rapidly before a toy shop and cast an attentive glance upon it; a few steps further on they tried which could describe the greatest number of objects seen. The son often reached 40, the father 30. An instance is also given in which the son saw at a glance and remembered the titles of many books of a library. This power of memory was not a natural gift. Houdin taught his son by laying dominoes before him; instead of letting him count the points the boy had to tell the total at once. In three days he could count six dominoes (from 15 to 61 points) and in a short time he could give instantaneously the sum of a dozen (up to 106 points.) In like manner it is possible to learn to commit a row of figures to memory in an instant. "A useful faculty, easily developed by practice, is that of retaining a retinal picture. A scene is flashed upon the eye: the memory of it persists, and details, . . . may be studied . . . in a subsequent vision."[2]

[1] Littell's Liv. Age, 1857, LIV, p. 62.
[2] Galton, Inquiries into Human Faculty, London, 1883; p. 107.

Arithmetical Association. The psychology of calculation is still an unexplored field; yet for our purpose we can regard the association of numbers as elementary, leaving further analysis for future investigation. The process is that which was taught us in school; we learned to say 1 and 1 make 2, 1 and 2 make 3, etc., 1 less 1 leaves 0, 2 less 1 leaves 1, etc., $1 \times 1 = 1$, $1 \times 2 = 2$, etc.; 1 divided by 1=1, 2 divided by 1=2, and so on through the rest of the tables. By this means firm associations are gradually established between any two numbers up to 10 (in older boys often to 12 and 15) in all of the four relations. After thoroughly learning these associations we are able to "do sums." Suppose we had this example to solve: What is the sum of 2571 and 4249? The process we go through is—when we write in the order we do it—as follows:

9 and 1 = 10, put down 0, carry 1.
4 and 7 = 11, and 1 = 12, put down 2, carry 1.
2 and 5 = 7, and 1 = 8.
4 and 2 = 6.
total, 6 thousand, 8 hundred and 20.

Or take an example in multiplication, *e. g.*, 136 by 43. What do you say to yourself while working it?

3 times 6 are 18, put down 8, carry 1;
3 times 3 are 9, and 1 are 10, put down 0, carry 1;
3 times 1 are 3, and 1 are 4;
total, 408.
4 times 6 are 24, put down 4, carry 2;
4 times 3 are 12, and 2 are 14, put down 4, carry 1;
4 times 1 are 4, and 1 are 5;
total 544;
8;
4 and 0 are 4;
4 and 4 are 8;
5;
result, 5848.

This is exactly the way in which children in school generally reckon, even when they have no distinct intention of reckoning aloud. I must also confess that although I long since left school, whenever my mind is tired or distracted I have to go through the same process and cannot put into practice the various methods of "cutting off" that have been since learned.

These "cut-offs" are found in all our activities, and consist in part of a train of thoughts or volitions becoming less and less conscious. Movements which regularly follow certain sense-perceptions have the tendency to become automatic, and *to occupy less time*.[1] In like manner it has been shown that a series of ideas can be gone through, although one of them can sink below consciousness without destroying the sequence.[2] In this way our arithmetical associations can be enormously shortened. In the above example we shall totally disregard the time used in recording results mentally, and try only to shorten the associations. From my own experience I can say that in the first place for most of the associations I can reduce the connecting links between the numbers to an extremely small degree of consciousness, with a greater or less saving of time. Instead of saying "plus," "less," "by," etc., I simply repeat the numbers and the results, and although I know perfectly what I am doing, and make no confusion among addition, multiplication, etc., nevertheless these relations do not rise above a very low degree of consciousness: "9, 5, 14," "9, 5, 4," "9, 5, 45," are perfectly distinct and clear, yet I do not think consciously of any of the operations performed. In like manner various connecting links can be cut out; so that for instance, the example given above would for me be reduced to

3, 6, 18, 1, 8;[3]
3, 3, 9, 1, 10, 1, 0;
3, 1, 3, 1, 4;
408;
4, 6, 24, 2, 4;
4, 3, 12, 2, 14, 1, 4;
4, 1, 4, 1, 5;
544
5848[4]

"The act of addition must be made in the mind without assistance; you must not permit yourself to say 4 and 7 are 11, 11 and 7 are 18, etc."[5] "Learn the multiplication table

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[1] See Wundt, Phys. Psych. 3 Aufl. II, 319.

[2] Scripture, Ueber den associativen Verlauf der Vorstellungen; Inaug. Diss., Leipzig, 1891.

[3] In such cases I always imagine the number, *e. g.*, 18, and then take away the 1, leaving the 8, so that the "carrying" occurs before the "putting down."

[4] In mental examples I also add from top to bottom, and in easy cases from left to right.

[5] De Morgan, Elements of Arith., p. 162.

so well as to name the product the instant the factors are seen; that is, until 8 and 7, or 7 and 8 suggest 56 at once, without the necessity of saying, 7 times 8 are 56."[1] Of course the saving of time is very great; and yet an educated person can work with just as much, perhaps more accuracy, than in the unabbreviated style.

Still another shortening can be made; by making an effort I can do "the carrying unconsciously so that in the above case I would say,

3, 6, 18, 8,

and not think of the 1 until the time for adding it in occurs. As De Morgan remarks, "don't *say* 'carry 3' but *do* it." Moreover it is not absolutely necessary to distinctly mark the end figure of each partial product; these products can be kept in memory and added up afterwards. The above example would be carried out by a person who had good command of such a power in somewhat the following manner (the same figures denote that they were operated upon before they entered full consciousness:)

$^{3},\ ^{6},\ 18;$
$^{3},\ ^{3},\ 9,\ ^{1}\ 10;$
$^{3},\ ^{1},\ 3,\ ^{1},\ 4;$
$408;$
$^{4},\ ^{6},\ 24;$
$^{4},\ ^{3},\ 12,\ ^{2},\ 14;$
$^{4},\ ^{1},\ 4,\ ^{1},\ 5;$
$544;$
$5848.$

This shortening can in adding be carried to such an extent that only the results are noticed; *e. g.*, as soon as a person catches a glimpse of 405 and 540 he knows the sum. In the above case the time for associating the numbers with their products has become exceedingly small; as Bidder says most of the time is required for the registration of the results on the memory, and this as was shown above can in exceptional cases be very small.

Finally an enormous shortening can be made if the adding, subtracting, multiplying, etc., can be done *before the numbers themselves come into full consciousness.* Münsterberg has shown that associations made in such a low degree of

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[1] Ibid, p. 163.

consciousness require comparatively little time.[1] A few years ago I made the attempt to acquire this ability and after considerable practice I was able on the sight of two figures to add or subtract them before they had attracted my full attention; in other words while they were yet in the field of consciousness they aroused the proper association and the result entered the focus of consciousness first.

We might be tempted to carry the process of "cutting-off" consciousness still further, and to say in just the same manner as on the sight of the figures 136×43 the partial products spring at once of themselves into the mind of a mathematician so in exceptional cases these partial products might be added before they became fully conscious, so that nothing but the result appears; a further application of the "cut-off" would bring the final answer to the whole problem instantly into mind. To be sure the testimony of the elder Bidder is against this, but it is only an extension of the principle and seems necessary to explain the difficulty of Colburn in telling how he did his examples. In his early years all he could say was that the problem was given and the answer was almost at once there. It would also help to explain such cases as are furnished by Dase; he had been given the number

935173853927;

Schumacher mentioned 7. "As soon as he heard 7 he repeated the number

6546216977489."[2]

This performance of processes before the factors became fully conscious would show itself in the popping of the answer into the mind before the person has thought out clearly how it was obtained. Upon the involuntary answers the rapid calculator would have to rely; if he stopped to make sure of each step time would be lost; he must always go ahead without a question as to whether he is right or not. The younger Bidder says, "I am certain that unhesitating confidence is half the battle. In mental arithmetic it is most true that he who hesitates is lost."[3]

[1] Münsterberg, Beiträge zur experimentalen Psychologie, Freiburg i/B 1890, Heft 1.
[2] Briefwechsel zw. Gauss und Schumacher, V 302.
[3] Spectator, 1878, p. 1634.

In still another way it would be possible to save time. It is common practice to extend the multiplication table beyond ten at least to 12×12. Here two figures are multiplied by two figures. In like manner it is easy to learn the table of 15 or 25 and with an effort we could undoubtedly learn complete tables of addition, subtraction, multiplication, division up to perhaps 30.[1] In an example like the following we would divide the number into periods of two figures each and operate directly with them:

2419 × 3017
19, 17 323
24, 17 408
30, 17 570
30, 24 720

7,298,123

To get an idea of the wonderful ease and rapidity with which examples can be done in this way make use of a multiplication table reaching to 100×100.[2] When moreover no time is lost in turning the leaves of such a table, in running down a column and recording the results on paper, then a person who could hold such a table in his head ought to be able to answer many problems in less time than even Dase required.

It is really not so difficult to obtain such a table of the products of two figures. "I formerly knew an instructor whose scholars of 8 to 12 years of age, for the most part knew the Pythagorean table extended to 100×100, and who calculated rapidly in the head the products of two numbers of four figures, in making the multiplication by periods of two figures."[3] Did any of the prodigies possess such a table? Considering their enormous powers of memory it would be almost unexplainable if they did not. Although Bidder asserts that he really had no such table, yet Mondeux actually possessed part of such a table, and I think we can pre-suppose it in the case of Colburn, Buxton and even Dase.

[1] "In my opinion, all pupils who show a tolerable capacity should slowly commit the products to memory as far as 20 times 20." DeMorgan, Elements of Arithmetic, London, 1857, p. 25.

[2] For example, Waldo's Multiplication and Division Table for Accountants, Computers and Teachers in the Primary Schools; New York, 1880.

[3] Lucas, Le calcul et les maschines à calculer, Assoc. française pour l'avancement des sciences, Paris, 1884, p. 2.

A number of other rules by which the processes of addition, subtraction, etc., can be shortened are given by DeMorgan in his Elements of Arithmetic, Appendix I; also in Companion to the Almanac 1844 and Supplement to the Penny Cyclopædia, article Computation.

There are also little "kinks" put in practice by many people of which the ready reckoners were not slow to avail themselves, *e. g.*, multiplying by two easy numbers and taking the difference instead of multiplying by an awkward number. In regard to the example given above p. 36 Bidder says "I should know at a glance, that

$$400 \times 173 = 69{,}200$$

and then $3 \times 173 = \quad 519$

the difference being 68,681."[1]

Now that we have some idea of how the mind works in solving arithmetical problems, and of how it shortens the time required, let us see how the prodigies actually worked.

Dase, on a test before Schumacher, divided "each number into two parts, of which one contains the highest figures and three zeros, and the other the three lower figures, reckoned the 4 partial products in his head, and *noted down every time* separately the results with pencil, which he afterwards added mentally."[2] As Gauss said, Dase seems to have depended on his remarkable accuracy of memory and to have possessed powers of calculation which at best were not equal to those of many mathematicians. "When he needs $8\frac{3}{4}$ hours to multiply two numbers each of 100 figures in his head, this is really a foolish waste of time, for a moderately practised reckoner could do the same on paper in *much* shorter, in less than half the time."[3] Gauss, however, was himself such a wonderful reckoner that judging from the standpoint of an ordinary person, he underestimated Dase's powers.

Buxton was much slower, as is seen from the following: "Admit a field 423 yards long and 383 wide, what was the area? After I had read the figures to him distinctly, he gave me the true product, viz., 162009 yards, in two minutes, for

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[1] Proceedings Civ. Eng. XV 260.
[2] Briefwechsel zw. Gauss und Schumacher, V, 32.
[3] Briefwechsel zw. Gauss und Schumacher, V, 300.

I observed by my watch how long every operation took him."[1] On paper this is easily done in twenty seconds. "Allowing the distance between York and London to be 204 miles, I asked him how many times a coach-wheel turned round in that distance, allowing the wheel's circumference to be six yards! In 13 minutes he answered 59,840 times."[2] On paper this requires 35 seconds. The clumsiness of Buxton's methods is phenomenal. He was required to multiply 456×378. This he did as follows:

$456 \times 5 = 2280$, which $\times 20 = 45600$;
$45600 \times 3 = 136800$.
$2280 \times 15 = 34200$;
$136800 + 34200 = 171000$;
$456 \times 3 = 1368$;
$171000 + 1368 = 172368$.[3]

When Mondeux had to multiply two entire numbers he often divided them into portions of two figures; he recognized that in a case where the factors are equal the operation is simpler, and the rules used by him for obtaining the product, or rather the power demanded are precisely those given by Newton's binomial formula.[4] That is to say, in an example, 2419×3017, he would proceed as we have done above, and in a case like 2419×2419 (or 2419^2) or 2419 to any power he worked according to the formulas $x^2 + 2\,xy + y^2$ (*i. e.*, $24^2 \times [2 \times 24 \times 19] + 19^2$), $x^3 + 3\,x^2 y + 3\,xy^2 + y^3$, etc. "Guided by these rules he could give on the instant the squares and cubes of a multitude of numbers; for example, the square of 1204 or the cube of 1006. As he knows almost by heart the squares of all the entire numbers less than 100, the division of the greater numbers into periods of two figures enables him to obtain their squares more easily."[5]

A partial account of Bidder's method's of multiplication has already been given; here it is necessary only to add a few facts left untouched and an explanation of his ways of extracting roots and finding factors. Most important is the contrast between his multiplication table, understood and

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[1] Gent. Mag. XXI, 347.
[2] Gent. Mag. XXI, 347.
[3] Gent. Mag. XXIV, 251.
[4] Comptes rendus, XI, 953.
[5] Comptes rendus, XI, 954.

made part of himself, and the mechanical associations of most people.

"In order to multiply up to 3 places of figures by 3 figures, the number of facts I had to store in my mind was less than what was requisite for the acquisition of the common multiplication table up to 12 times 12. For the latter it is necessary to retain 72 facts; whereas, my multiplication up to 10 times 10 required only 50 facts. Then I had only to recollect, in addition, the permutations among the numbers up to a million, that is to say, I had to recollect that 100 times 100 were 10,000; 10 times 10,000 were 100,000, and that ten hundred thousands made a million. . . . . Therefore, all the machinery requisite to multiply up to 3 places of figures was restricted to 68 facts. . . . . If you ask a boy abruptly, "what is 900 times 80," he hesitates and cannot answer, because the permutations are not apparent to him; but if he had the required facts as much at his command as he had any fact in the ordinary multiplication table, viz., that 10 times 10 = 100, and that 900 times 80 was nothing more than 9 times 8 by 100 times 10, he would answer off hand 72,000; and if he could answer that, he would easily say 900 times 800 = 720,000. If the facts were stored away in his mind so as to be available at the instant he would give the answer without hesitation. If a boy had that power at his command he might at once with an ordinary memory proceed to compute and calculate 3 places of figures."[1]

The following gives an insight into the rapidity of Bidder's associations: "Suppose I had to multiply 89 by 73, I should say instantly 6,497; if I read the figures written out before me I could not express a result more correctly or more rapidly; this facility has, however, tended to deceive me, for I fancied that I possessed a multiplication table up to 100 times 100, and when in full practice even beyond that; but I was in error; the fact is that I go through the entire operation of the computation in that short interval of time which it takes me to announce the result." The velocity of the

[1] Proceedings Civ. Eng., XV, 259.

mental processes cannot be adequately expressed; the utterance of words cannot equal it. . . . . Were my powers of registration at all equal to the powers of reasoning or execution, I should have no difficulty in an inconceivably short space of time in composing a voluminous table of logarithms."[1]

The least intelligible of all the explanations given by ready reckoners is that of Colburn. His friends tried to elicit a disclosure of the methods by which he performed his calculations, but for nearly three years he was unable to satisfy their inquiries. He positively declared that he did not know how the answers came into his mind.[2] In London he made a couple of explanations. "In one case he was asked to tell the square of 4395: he at first hesitated, ... but when he applied himself to it he said it was 19,316,025. On being questioned as to the cause of his hesitation, he replied that he did not like to multiply four figures by four figures; but, said he, I found out another way: I multiplied 293 by 293, and then multiplied this product twice by the number 15 which produced the same result. On another occasion, when asked the product of 21,734 multiplied by 543, he immediately replied, 11,801,562; but, upon some remark being made on the subject, the child said that he had, in his own mind, multiplied 65,202 by 181 [$21734 \times (181 \times 3) = (21734 \times 3) \times 181$]."[3]

Finally, it is worthy of remark that the attempt has been made to teach the performance of long multiplications without writing more than the problem and the answer. Although the method proposed is undoubtedly not the best, yet it suggests the possibility of inventing a practicable school-method. For example, multiply in one line 2681475 by 93165. Number the figures with indices as shown:

$$\begin{array}{r} {\scriptstyle 7\,6\,5\,4\,3\,2\,1} \\ 2681475 \\ {\scriptstyle 4\,3\,2\,1\,0} \\ 93165 \\ \hline {\scriptstyle 11\,10\,9\,8\,7\,6\,5\,4\,3\,2\,1} \\ 249819618375 \end{array}$$

[1] Proceedings Civ. Eng. XV, 255.
[2] Memoirs, p. 39.
[3] Analectic Mag. I, 1813.

Place the product of any two figures in that place of the result which has an index equal to the sum of the indices, of course adding in any carried numbers. Thus, $\overset{0}{5} \times \overset{1}{5} = 25$; the sum of their indices being 1 the 5 goes in the first place. $(\overset{0}{5} \times \overset{2}{7}) + (\overset{1}{6} \times \overset{1}{5}) + 2 = 67$; the 7 goes in the second place. $(\overset{0}{5} \times \overset{3}{4}) + (\overset{1}{6} \times \overset{2}{7}) + (\overset{2}{1} \times \overset{1}{5}) + 6 = 73$; the 3 goes in the 3rd. place, etc. Such a method of multiplication would undoubtedly be of assistance in training the ability for mental calculation. Of course we do not advocate an attempt at doing such enormous problems wholly in the mind, but shorter ones can be easily learned to great advantage. We should, however, take a few hints from Bidder, Safford, Colburn, *et al.* Suppose we had 379 × 42. Let us mentally index the numbers as above; then 4 × 3 = 12, which belongs

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 321
 379
  10
  42
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4321
12
126
154
1554
1590
15918
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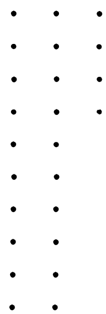
in the 4th place. $3 \times 2 = \overset{3}{6}$; add this on to the other, 126. $4 \times 7 = \overset{3}{28}$; add to 126=154. $7 \times 2 = \overset{2}{14}$; add on to 154=1554. $4 \times 9 = \overset{2}{36}$; add, = 1590. 9 × 2=18; add on, =15918, Ans.

Mentally the figures would not be repeated, but as Bidder explains, the first obtained result would be modified. As a help in learning to keep the correct places, a card with several numbered compartments might be placed before the eyes, at first actually, then mentally; thus,

8	7	6	5	4	3	2	1

For paper or slate this of course requires more time and figures; but mentally such a process is quite possible, whereas the ordinary way of multiplying 3 figures by 2 figures is absolutely impossible to an ordinary person. With practice boys of advanced classes could undoubtedly be taught to multiply even 3 figures by 3 figures in the head.

R. A. Proctor, using Colburn as an illustration, explains the feats of calculating boys by an increased power of picturing a number as so many things and of modifying this picture according to the operation to be performed.[1] 24 would be presented as



If 24 were to be multiplied by three all that is necessary is to picture three sets of dots; then to conceive the imperfect columns brought together on the right, giving six columns of ten and three columns each of four dots; and these three give at once (by heaping them up properly) another column of ten with two over: in all seven columns of ten and one of two,—that is, seventy-two. Proctor, who remarks that "all good calculators have the power of picturing numbers not as represented by such and such digits, but as composed of so many things,"[2] and who once possessed this power in no inconsiderable degree, says of this example that it "takes long in writing, but as pictured in the mind's eye, the three sets representing 24 formed themselves into the single set representing 72 in the twinkling of an eye."[3]

The suggestion is ingenious but it is only a suggestion. Unfortunately for Proctor's attempt to explain how the ready-reckoners reckoned, several of them have given extended accounts of the processes employed by them. The appendices to Colburn's Memoirs (of which Proctor did not know, for he says, "if Colburn had retained his skill until he had acquired power to explain his method, etc.") give an account of his methods of multiplying, extracting roots, etc., which flatly contradicts Proctor's explanation. In regard to Bidder Proctor afterwards admits that there was no room to doubt that his processes of mental arithmetic were commonly only

[1] Cornhill Mag. XXXII, p. 163; Science Byways, p. 349.
[2] Belgravia, XXXVIII, p. 451.
[3] Cornhill Mag., XXXII, p. 163; Science Byways, p. 350.

modifications of the usual processes.[1] Proctor arrived at this opinion on the evidence furnished by persons who had known Bidder (Bidder's own account was unknown to him). It is quite a confirmation of the theory of rapid calculation I have proposed, to find that the explanation of Bidder's powers advanced by Proctor is contained as one of the parts of my explanation, which is founded on the first hand evidence of Colburn, Bidder, Dase, etc.

Among the other mathematical operations in which the prodigies distinguished themselves more or less is the extraction of roots of numbers. In the first place it is to be remarked that Buxton knew nothing of this operation, and on the one occasion on which such a problem was given him he succeeded only approximately after a long time, apparently by running over the squares of various numbers till he found the one nearest to the given square.[2] Dase liked to extract the 5th root, "because he had noticed that in the fifth power the units are the same as in the root. I saw that with our system of numbers the $(4n + 1)$ power has the same units as the root, a rule of which his is only a single case (for $n=1$)"[3]

In an appendix to his Memoirs, Colburn attempts to explain his methods of finding square and cube roots and of factoring. His rule, first formulated two years after he began, was as follows: Find a number whose square ending with the last two figures of the given square; then, when the given square consists of five places, what number squared will come nearest under the first figure (when 6 places, then the first two figures, when 7, the first 3, etc.) of the given square. For example:

What is $\sqrt{92,416}$?
1. What number squared ends in 16? Ans. 04.
2. What number squared comes next to 9? Ans. 3.
   Square root, 304.

What is $\sqrt[3]{321,489}$?
1. What number cubed ends in 89? Ans. 67.
2. What number cubed comes nearest to 32? Ans. 5.
   Cube root, 567.

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[1] Belgravia, XXXVIII, p. 456.
[2] Gent. Mag. XXIII, p. 557.
[3] Briefwechsel zw. Gauss und Schumacher, V, 382.

Colburn gives a table of the numbers which squared produce any given termination; to each termination there are four possible roots (to 25 there are 10) from which he must choose; *e. g.* a number ending in 16 can have one of the roots 04, 54, 46, or 96. "It is obvious that it requires a good share of quickness and discernment, in a large sum, to see which of the four roots . . . . is the right one."[1] The table for cube roots is very much simpler. These methods are of use only when the given number is an exact square or cube. Both depend on the last two figures, and a person would "probably greatly confuse the calculator by merely adding a small number to the square or cube."[2] Nothing ever excited so much surprise as the facility with which Bidder extracted square and cube roots. "Yet there is no part of mental calculation for which I am entitled to less credit. In fact, it is a mere slight of art." "Nearly every example proposed to me was a true square or cube; hence I hit upon the following expedient. . . . ."[3] He then gives a method exactly like that of Colburn.

It is not necessary to enter into the question how the prodigies found the factors of numbers. Colburn's process is found *in extenso*, on p. 183 of his Memoirs. It is clumsy and involved; he himself allowed it to be a "drag of a method."[4] Bidder's methods are explained on pages 272, 273 and 274 of Vol. XV of the Proc. Civ. Eng.

There is one other characteristic of the association of numbers that meets us in some of the persons under consideration, namely, the firmness with which long series of arithmetical associations cling together. This is seen in the independence of a process of reckoning among other activities and other processes of reckoning. Of Mondeux we read that his thoughts were as strongly directed to the arithmetical operation he had to perform as if he were completely isolated from his whole environment.[5] Buxton would talk freely whilst doing his questions, it being no molestation or

[1] Memoirs, p. 181.
[2] Hamilton's letter in Graves' Life of Sir Wm. R. Hamilton, p. 78.
[3] Proceedings Civ. Eng. XV, p. 266.
[4] Graves, Life of Sir Wm. R. Hamilton, p. 78.
[5] Comptes rendus, XI, p. 956.

hindrance to him.[1] "He would suffer two people to propose different questions, one immediately after another, and give each their respective answers without the least confusion."[2] In this not so very uncommon ability of doing two things at once the mathematicians seem to be specially favored. Dirichlet, for example, says "that he established the solution of one of the difficult problems of the theory of numbers, with which he had for a long time striven in vain, in the Sixtine Chapel in Rome while listening to the Easter music."[3]

MATHEMATICAL INCLINATION. The peculiar fascination for performing arithmetical calculations is sometimes a source of pleasure in itself; a distinguished savant during a public meeting undertook the multiplication of two long lines of figures and explained his action by "the pleasure it would give him to prove his calculation by division."[4] At the sight of figures, geometrical diagrams, and above all, algebraic formulas, young Galois was seized with a veritable passion for the abstract truths hidden behind these symbols.[5]

Even after Safford had lost his powers he continued to find pleasure in taking large numbers to pieces by dividing them into factors, or in satisfying himself that they were prime.[6] The younger Bidder remarks, "With my father as with myself the mental handling of numbers or playing with figures afforded a positive pleasure and constant occupation of leisure moments. Even up to the last year of his life my father took delight in working out long and difficult arithmetical and geometrical problems."[7]

In regard to special inclination to mathematics and its relation to ability for calculation, and also to other abilities, great diversity is shown by the persons we have considered. They can be variously grouped:

1. Those having strong mathematical inclinations with great powers of mental calculation (not necessarily rapid):

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[1] Gent. Mag. XXI, p. 347.
[2] Gent. Mag. XXI, p. 61.
[3] Kummer, Gedächtnissrede, etc., p. 34.
[4] Èloge d' Ampère, Smithsonian Report, 1872, p. 112.
[5] Magasin pittoresque, 1848, t. XVI, p. 227.
[6] Belgravia, XXXVIII, p. 456.
[7] Spectator, 1878, p. 1634.

here we should include nearly all arithmetical prodigies, although Colburn took no satisfaction in answering questions by the mere operation of mind; unless questioned, his attention was not engrossed by it at all; the study of arithmetic was not particularly interesting to him, but it afforded a very pleasing employment.[1] Nevertheless, the fascination for calculation was in some cases overpowering. Gauss considered mathematics the queen of the sciences and arithmetic the queen of mathematics; Buxton had neither eyes nor ears for anything else, and Mondeux and Dase greatly resembled him.

Corresponding to this class we might point out more than one distinguished mathematician who had not the ability to calculate; indeed, it would not be going too far to say that nine out of ten mathematicians have at least no liking for reckoning.

2. Those with inclination and ability for mathematics, including arithmetic: Nickomachos, Gauss, Ampère, Safford, Bidder.

3. Those with special inclination and ability for arithmetic alone;

a. having had no opportunities for other branches of mathematics: Fuller, Buxton, Mangiamele;

b. in spite of opportunities: Colburn, Dase, Mondeux.

c. where the talent disappears before opportunity for development is possible: Whately.

Mathematical Precocity. "There are children, I know," says Arago, "whose apathy nothing seems able to arouse, and others again who take an interest in everything, amuse themselves with even mathematical calculations without an object." There are still others more seldom than either of these classes, who confine their interest to mathematical calculations alone. Strange as the fascination for arithmetic seems, it becomes still more so when it is manifested at an age at which it is normally absent; strangest of all is the union of ability to the inclination.

Hamilton included calculation in an all-sided precocity;[2]

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[1] Memoirs, p. 69.
[2] Graves, Life of Sir Wm. R. Hamilton, Dublin.

Pascal's ability was for geometry,[1] as was also Clairaut's.[2] With Whately, Colburn, Bidder, Mondeux and Mangiamele the precocity showed itself alone in calculation; the same is true for Gauss's first years. Ampère and Safford, however, resemble Hamilton in showing inclination and ability for the most varied pursuits; the difference being that the mathematical side showed itself in Hamilton after the philological.

Special precocity in calculation showed itself (as far as our knowledge goes) at the following ages:

Gauss, 3,	Prolongeau, 6½,
Whately, 3,	Bidder, 10,
Ampère, between 3 and 5,	Mondeux, 10,
Safford, 6,	Mangiamele, 10,
Colburn, 6.	

It is remarkable that in nearly every case (possibly with the exception of Colburn and Whately) the arithmetical prodigies showed rather an extraordinary ability to learn calculation, not an ability to calculate before learning.

IMAGINATION. One peculiarity in the imaginative powers of the arithmetical prodigies is worthy of remark, namely their visual images. Bidder said, "If I perform a sum mentally it always proceeds in a visible form in my mind; indeed, I can conceive of no other way possible of doing mental arithmetic." This was a special case of his vivid imagination. He had the faculty of carrying about with him a vivid mental picture of the numbers, figures and diagrams with which he was occupied, so that he saw, as it were, on a slate the elements of the problem he was working. He had the capacity for seeing, as if photographed on his retina, the exact figures, whether arithmetical or geometrical, with which he was occupied at the time. This faculty was also inherited, but with a very remarkable difference. The younger Bidder thinks of each number in its own definite place in a number-form,[3] when, however, he is occupied in multiplying together two large numbers, his mind is so engrossed in the operation that the idea of locality in the series for the moment sinks out of prominence.[4] Is a number form injurious to calculating powers? The father seems to have arranged and used his

[1] Vie de Pascal par Mme. Perier.
[2] Nouv. Ann. de Math,. Paris, 1861, 1ière serie, XX, Bulletin, p. 50.
[3] Plate I, Fig. 20, in Galton's Inquiries into Human Faculty.
[4] Galton, Inquiries, etc.. p. 134.

figures as he pleased; the son seems to be hindered by the tendency of the figures to take special places. It would be interesting to know if the grandchild, who possesses such a vivid imagination and in whom the calculating power is still further reduced, also possesses a number-form. The vivid, involuntary visualizing seems to indicate a lack of control over the imagination, which possibly entends to figures, and this perhaps makes the difference.

Colburn said that when making his calculations he saw them clearly before him.[1] It is said of Buxton that he preserved the several processes of multiplying the multiplicand by each figure of the lower line in their relative order, and place as on paper until the final product was found. From this it is reasonable to suppose that he preserved a mental image of the sum before him.

Of the other calculators we have no reports. Children in general do their mental problems in this way. Taine relates of one, that he saw the numbers he was working with as if they had been written on a slate.

The well-known case of Goethe's phantom, the case of Petrie, who works out sums by aid of an imaginary sliding rule, the chess-players who do not see the board, etc., are instances of the power of producing vivid visual imaginations that can be altered at will.

III.

Can we learn anything of practical use from the prodigies? The following points suggest themselves for consideration:

1. Bidder, Safford and the African brokers all speak for the fact that under cultivation the power of mental calculation could be greatly developed; the immense saving of time in school and afterwards that would result from an ability to shorten the associations, to use a multiplication table of two figures, and above all to register mentally, is sufficient to justify a trial.

2. Fuller, Ampère, Bidder, Mondeux, Buxton, Gauss, Whately, Colburn and Safford learned *numbers and their values* before figures, just as a child learns words and their meanings long before he can read. Bidder declares emphatically, "The reason for my obtaining the peculiar power of

[1] Med. and Philos. Journal and Review, New York, 1811, p. 22.

dealing with numbers may be attributed to the fact, that I understood the value of numbers before I knew the symbolical figures. . . . . In consequence of this, the numbers have always had a significance and a meaning to me very different to that which figures convey to children in general."[1]

3. Ampère, Bidder and Mondeux learned their arithmetic from pebbles. Arago says of Ampère, "It may be he had fallen upon the ingenious method of the Hindoos, or perhaps his pebbles were combined like the corn strung upon parallel lines by the Brahmin mathematicians of Pondichéry, Calcutta and Benares, and handled by them with such rapidity, precision and accuracy."

The Roman *abacus*, the Chinese *swanpan* and the success of the numeral-frames used in our primary schools, seem to point to the fact that it is best to teach "calculation" (*i. e.*, "pebbling" from Lat. *calculi*, pebbles), before "ciphering." The Arabic *tsaphara*, cipher, means empty; Arabic numeration, however, was considered mysterious by the people of the middle ages, and remains mysterious to many a child of to-day; to the former (and also not seldom to the latter) "ciphering" meant a secret and unintelligible process. If we could do away with the mystery of calculation perhaps the values of numbers and the tables might become then so indelibly fixed in the minds of children and so easy of application that they also could do long "sums" mentally or even carry the two-figure multiplication tables in their heads.

4. Dase's power of quick apprehension suggests the extension of the training sometimes attempted in schools, in which a slate with letters or figures is shown for an instant to the scholars who are then required to tell how much they recognized.

In conclusion it is necessary to express my obligations to President Hall and to Dr. de Perrot. To Dr. de Perrot of the Mathematical Department of this University, credit must be given for proposing the subject, for a large part of the references, and for numerous valuable suggestions and points of information.

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[1] Proceedings Civ. Eng. XV, 256.