A Visit to Hungarian Mathematics

Reuben Hersh and Vera John-Steiner

In July 1988, we visited Budapest to participate in the Sixth International Congress on Mathematical Education. We decided to use this opportunity to try to shed some light on the legendary reputation of Hungarian mathematics. One of us (V.J.-S.) is a native of Budapest and is familiar with the city and its language.

Our investigation focused on historical, pedagogical, and social-political aspects of Hungarian mathematical life. We did not attempt to survey Hungarian mathematical research of the present. Even so, our time proved too short for our ambitions. The important Hungarian mathematicians whom we missed are certainly more numerous than those we interviewed.

We spoke in depth to a dozen people, and carried out formal interviews with eight: in Hungary, Béla Szőkefalvi-Nagy, Pál Erdős, Tibor Gallai (recently deceased), István Vincze, and Lajos Pósa; in the United States, Agnes Berger, John Horváth, and Peter Lax. (While we were in Budapest, two of the leading newspapers carried major articles honoring Szőkefalvi-Nagy's 75th birthday.)

We asked all our interviewees the question, what is so special about Hungarian mathematics? What made possible the production of so many famous mathematicians in such a small, poor country, in the period between the two Wars?

In our interviews, and also in our reading, we got two quite distinct kinds of answer. Type 1 was internal. It related to institutions and practices within the world of mathematics. The other kind, type 2, was external. It related to trends and conditions in Hungarian history and social life at large. Perhaps one contribution of this article is to point out the importance of both types of answer. One could conjecture that favorable conditions of both types—within mathematical life and within socio-politico-economic life at large—are necessary to produce a brilliant result such as Hungarian mathematics of the 1920s and 1930s. In the terminology used by Mihály Csikszentmihályi and Rick Robinson [5] in their study of creativity, perhaps conditions have to be right both in the "domain"—the area of creative work—and in the "field"—the ambient culture.

Bolyais, Father and Son

Hungarian mathematics began, in a sense, with János Bolyai (1802–1860), one of the creators of non-Euclidean geometry, and his father Farkas (1775–1856), also a creative mathematician of importance. In their lifetimes, they were totally ignored, both at home and abroad. "It is a widely accepted opinion that Farkas Bolyai was the first mathematician in Hungary to have

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original results" [4], page 222. He studied at Göttingen from 1796 to 1799 and established a lasting friendship with fellow student Carl Friedrich Gauss [4]. He and Gauss were both interested in the "problem of parallels" (independence of Euclid's fifth postulate). Farkas returned to Hungary and, in 1804, became mathematics professor at the Reformed College of Marosvásárhely in Transylvania.

In 1832–1833, he published a two-volume textbook in Latin entitled *Tentamen juventutem studiosam in elementa matheseos introducendi.* It was reprinted in 1896 and 1904.

János (1802–1860) inherited his father's interest in the problem of parallels. In fact, with one single exception, Farkas was the only human being who understood and appreciated János's discovery of non-Euclidean "hyperbolic" geometry. When Farkas sent his son's discoveries to Gauss, Gauss replied, "I cannot praise this work too highly, for to do so would be to praise myself." Gauss had anticipated János's discoveries by decades. His decision to withhold his own work from publication made it impossible for János to attain the recognition he knew he deserved.

A few years after János Bolyai died in 1860, foreign mathematicians began to get interested in him. In 1868, Eugenio Beltrami in Italy published his discoveries on the pseudosphere. He found that this surface is a model for the Bolyai–Lobatchevsky hyperbolic geometry, and so provides a relative consistency proof for it. In 1871, Felix Klein and, in 1882, Henri Poincaré published their models of the hyperbolic plane. In 1891, C. B. Halsted of the University of Texas published an English translation of János Bolyai's work on hyperbolic geometry, called the *Appendix.* He visited János's grave and made strenuous efforts to gain recognition for him.

By this time, Hungary began to realize that one of its most illustrious sons was a mathematician. The Hungarian Academy of Sciences established the Bolyai Prize: 10,000 gold crowns, to be awarded every five years, to the mathematician whose work in the previous 25 years had given most to the progress of mathematics. The first prize committee was made up of Gyula König (1849–1913), Gusztáv Rados (1862–1942), Gaston Darboux, and Felix Klein. The first Bolyai Prize went to Henri Poincaré in 1905; the second, to David Hilbert in 1910. Unfortunately, one consequence of the First World War was the devaluation of the fund from which the prize was to be given. It was never awarded again.

Ausgleich and Emancipation

After losing her independence to the Turks in 1526, Hungary was for centuries occupied, first by the Ottoman and later the Habsburg Empires. In 1848, there was a revolution and feudalism was abolished. In 1848–1849, an unsuccessful war for independence was waged against the Austrian Empire. This was followed by years of passive resistance. Then, in 1866, the Austrian Emperor Franz Joseph suffered a humiliating military defeat by Prussia. Faced also with rising nationalism among Czechs, Ruthenians, Romanians, Serbs, and Croatians, the Emperor granted the Hungarians a large measure of economic and cultural independence. In return, the Magyars renewed their allegiance to him. This pact became known as the *Ausgleich,* "the compromise." A year later, non-Hungarian minorities were granted civil rights. In particular, the Hungarian Jews, 5% of Hungary's population, were emancipated. For the first time, they were permitted to work for the state, including teaching in its schools. Laura Fermi writes [7], "From peasants and peddlers they turned into merchants, bankers, and financiers; they moved into independent businesses and the professions. Soon they entered all cultural fields, giving themselves at last to the intellectual pursuits that are the highest aim of the Jewish people."

The *Ausgleich* was followed by 40 boom years. Along with the commercial and industrial development of Budapest came the creation of an educational system, including universities, college-preparatory schools (gymnasiums), and a technical college. Many of the gymnasiums were denominational—Catholic, Protestant, or Jewish. Most were for boys, but there were some for girls. All this led to the appearance of mathematics teachers and professors. And some of them were brilliant, creative people.

Laura Fermi's informants give a vivid picture of intellectual life in Budapest [7]. (See also the recent book [69] of John Lukács.)

> Budapest intellectuals, most of them individualists with no desire to conform, threw ideas at each other in cafes, expounded progressive or eccentric theories in the newspapers, turned their thumbs down in theaters at artists acclaimed in other countries, or made stars of unknown artists . . . Many students belonged to the Galilei Club of progressive undergraduates founded in 1908 by the philosopher Gyula Pikler and the future sociologist, Károly Polányi (George Pólya was a member.) . . . Most future emigrés lived in Budapest or went there for their education . . . In Budapest, they had to keep mentally alert, to emulate and compete, and in order not to be submerged, they had to develop their capabilities to the full.

She goes on:

> The flowering of Hungarian talent in the generation of the cultural wave was due to the special social and cultural circumstances obtaining in Hungary at the turn of the century. By then a strong middle class had emerged and asserted itself. Having risen in response to needs that the nobility did not feel inclined to fill and the peasants could not fill, it was largely Jewish and was animated by the intellectual ambitions of the Jews. The intellectual portion of this middle class converged upon the capital where it

> created a peculiarly sophisticated atmosphere and kept its members under continuous stimulation. The political anti-Semitism of the early twenties hit this segment of the population with great vehemence and gave the intellectuals a further reason for striving to excel and stay afloat. Under these circumstances, talent could not remain latent. It flourished.

This must definitely be classified as a type 2 (field) explanation.

By the time of the First World War, economic strains were affecting Budapest life. Then defeat in the war destroyed the Austro-Hungarian Empire. In Hungary, it was succeeded by a Soviet Republic that survived for only 4 months. The Bolsheviks were overthrown by an invading Romanian army. They were succeeded by Admiral Horthy's clerical authoritarian regime, which in time became one of Hitler's allies.

The Allies treated Hungary not as a captive country like Slovakia and Croatia, but as a defeated power like Austria and Germany. The Treaty of Trianon gave two-thirds of Hungary to Romania, Czechoslovakia, Austria, and Yugoslavia. Hungary had been primarily agricultural; now it had to live by exporting manufactured goods. But the world market had shrunk, new competitors were busy. Hungary never regained the comfortable prosperity of Franz Joseph's time. Yet, in mathematics its standing after the war would become even more impressive than before.

John Horváth offers a somewhat similar type 2 explanation.

> You can name the day in 1900 when Fejér sat down and proved his theorem on Cesàro sums of Fourier series. [This work is described later. R.H.] That was when Hungarian mathematics started with a bang. Until then, there were just a few people who did mathematics. But from then on, every year somebody appeared who became a major mathematician on the international scene. A similar emancipation of the Jews happened in Prussia in 1812. And there you immediately had people like Jacobi, who became a professor in Königsberg. In Klein's *History of Mathematics in the 19th Century*, he has a little remark, that with the emancipation a new source of energy was released. There is one other thing which I sometimes mention. It's quite surprising how many of the mathematicians who came into the profession in Hungary after World War II are sons of Protestant ministers: Szele, Kertész, Papp, there's quite a number. And I guess the reason is much the same. Those kids would have become Protestant ministers just as the old ones would have become rabbis.

[Note: In Horváth's analogy between potential ministers and potential rabbis, there is, of course, no suggestion that the social–legal positions of Protestants and of Jews were equivalent or even similar. Peter Lax points out that György Hajós (see below) started out by studying for the priesthood.]

Another type 2 explanation, from John von Neumann: "It was a coincidence of cultural factors: an external pressure on the whole society of this part of Central Europe, a feeling of extreme insecurity in the individuals, and the necessity to produce the unusual or else face extinction" [59].

Contest and Newspaper

When George Pólya (1887–1985) was asked [1] to explain the appearance of so many outstanding mathematicians in Hungary in the early twentieth century, he gave two sorts of explanations. First, the general one: "Mathematics is the cheapest science. Unlike physics or chemistry, it does not require any expensive equipment. All one needs for mathematics is a pencil and paper. (Hungary never enjoyed the status of a wealthy country.)"

Then three specific type 1 explanations:

1. *The Mathematics Journal for Secondary Schools* (*Középiskolai Matematikai Lapok*, founded in 1894 by Dániel Arany). "The journal stimulated interest in mathematics and prepared students for the Eötvös Competition."
2. The Eötvös Competition. "The competition created interest and attracted young people to the study of mathematics." (This comment is more remarkable because Pólya himself, when a student, refrained from handing in his paper in the Competition!)
3. Professor Fejér. "He himself was responsible for attracting many young people to mathematics, not only through formal lectures but also through informal discussions with students."

We say more about Professor Fejér later. As to *Középiskolai Matematikai Lapok* and the Eötvös Competition, it is virtually impossible to talk to or read about any Hungarian mathematician without hearing tribute to the stimulation and inspiration of these two institutions.

In [1], Pál Erdős was asked: "The great flowering of Hungarian mathematics—to what do you attribute this?"

"There must be many factors. There was a mathematical journal for high schools, and the contests, which started already before Fejér. And once they started they were self-perpetuating to some extent. [Domain, type 1.] Hungary was a poor country—the natural sciences were harder to pursue because of cost, so the clever people went into mathematics. [Field, type 2.] But probably such things have more than one reason. It would be very hard to pin it down."

In our own interview with Erdős, we pursued this remark.

> RH: Do you feel that your mathematical development was affected by the high school mathematics newspaper (*Középiskolai Matematikai Lapok*)?
> *Erdős*: Yes, of course. You actually learn to solve problems there. And many of the good mathematicians realize very early that they have ability.

Our interviewee Agnes Berger, a retired statistics professor at Columbia University, has vivid memories of *Középiskolai Matematikai Lapok*. "The paper came once a month. It had problems grouped according to difficulty. The solutions were published in the following way: everybody who sent in a correct solution was listed by name, and the best solution or solutions were printed. So here you were taught right away to value, not only the solution, but the best solution, the most beautiful solution. It was called the model solution (*minta válasz*). It was a tremendous entertainment. Also, those people who did well, submitting many solutions, the frequent solvers, had their pictures published at the end of the year!"

We asked Tibor Gallai about *Középiskolai Matematikai Lapok*.

> *Gallai:* Nowhere else in the world is there this kind of high school paper, and this more than anything else is responsible for the excellence of Hungarian mathematics.
> RH: Do you have any idea why this took place in Hungary? What was it in this country that made this possible?
> *Gallai:* For part of 1894 and 1895 the Minister of Education was Loránd Eötvös (1848–1919), after whom the University is named. He was deeply committed to the development of Hungarian culture and science. While he was in office there was founded the Eötvös Collegium, with the purpose of improving the training of high school teachers. So he is part of what stimulated our development.
> RH: How do you feel about present-day competitions and students compared to years ago?
> *Gallai:* The quality is much higher now. When I first participated 60 years ago, the names of the students who solved the problems could easily be published, because there were only 30 or 40 of them. Now there are 600. It's impossible to publish all the names.
> *Vera Sós:* Now the problems are more difficult and demanding. There is a whole range of mathematically-oriented young people who have a more effective foundation.

While mathematics education in Hungary for the gifted and talented looks enviable from the perspective of the United States, not all Hungarian mathematics educators are satisfied with their situation. Lajos Pósa, who once was one of Erdős's most promising discoveries, has devoted himself in recent years to mathematics education for the normal or everyday student, not just the brilliant. He feels that the system does not do justice to these students, that the teachers, although supposed to teach by the problem-solving method, often do not feel sure or comfortable about problem-solving, and that many students fail to master mathematics as they could and should.

The Eötvös competition was established in 1894, the same year as *Középiskolai Matematikai Lapok*. The competition was established by the Mathematical and Physical Society of Hungary, at the motion of Gyula König, under the name of "Pupils' Mathematical Competition." This was done in honor of the Society's founder and president, the famous physicist Baron Loránd Eötvös (mentioned earlier by Tibor Gallai), who became Minister of Education that year. König was a powerful personality who dominated Hungarian mathematical life for several decades. His most famous deed in research seems to have been an incorrect proof of Cantor's continuum hypothesis. (He used a false lemma of Felix Bernstein. Except for Bernstein's lemma, König's argument was correct. König's own contribution to the proof survives as an important theorem in set theory.) König wrote an early book on set theory, but its impact was diminished because Hausdorff's famous book on that subject appeared at about the same time. König's son, Dénes (d.1944), is remembered as the father of graph theory (more details later).

Between the two wars, the competition continued under the name, "Eötvös Loránd Pupil's Mathematical Competition." At present, it carries the name of József Kürschák (1864–1933), who is remembered in particular for his extension of the notion of absolute value to a general field. He was professor at the Polytechnic University in Budapest and a member of the Hungarian Academy. In 1929, he compiled the original Hungarian edition and wrote the preface to *Problems of the Mathematics Contests*. In 1961, it was published in English as the *Hungarian Problem Book* [38]. The publication of the original *Problem Book* honored the tenth anniversary of Eötvös's death. Winners before 1929 who later became famous include Lipót Fejér (1880–1959), Dénes König, Theodore von Kármán (1881–1963), Alfréd Haar (1885–1933), Ede Teller (later known in the U.S. as Edward), Marcel Riesz (1886–1969), Gábor Szegő (1895–1985), László Rédei (1900–1980), and László Kalmár (1900–1976).

The English edition [38] contains a preface by Gábor Szegő. He wrote:

> [For a successful mathematics competition] some sort of preparation is essential to arouse public interest. In Hungary, this was achieved by a [high-school mathematics] Journal . . . I remember vividly the time when I participated in this phase of the Journal (in the years between 1908 and 1912). I would wait eagerly for the arrival of the monthly issue and my first concern was to look at the problem section, almost breathlessly, and to start grappling with the problems without delay. The names of the others who were in the same business were quickly known to me, and frequently I read with considerable envy how they had succeeded with some problems which I could not handle with complete success, or how they had found a better solution (that is, simpler, more elegant or wittier) than the one I had sent in.

We get an impressive picture of Hungarian secondary mathematics education early in the twentieth century, including the Eötvös Competition, from Theodore von Kármán, one of the preeminent founders of modern aeronautics. In his autobiography [65], he tells about his high school, the Minta, or Model Gymnasium, which

> became the model for all Hungarian high schools. Mathematics was taught in terms of everyday statistics. We looked up the production of wheat in Hungary, set up

tables, drew graphs, learned about the "rate of change" which brought us to the edge of calculus. At no time did we memorize rules from a book. Instead, we sought to develop them ourselves . . . The Minta was the first school in Hungary to put an end to the stiff relationship between the teacher and the pupil which existed at that time. Students could talk to the teachers outside of class and could discuss matters not strictly concerning school. For the first time in Hungary a teacher might go so far as to shake hands with a pupil in the event of their meeting outside of class.

Each year the high schools awarded a national prize for excellence in mathematics. It was known as the Eötvös Prize. Selected students were kept in a closed room and given difficult mathematics problems, which demanded creative and even daring thinking. The teacher of the pupil who won the prize would gain great distinction, so the competition was keen and teachers worked hard to prepare their best students. I tried out for this prize against students of great attainments, and to my delight I managed to win. Now, I note that more than half of all the famous expatriate Hungarian scientists, and almost all the well-known ones in the United States have won this prize. I think that this kind of contest is vital to our educational system, and I would like to see more such contests encouraged here in the United States and in other countries.

After the liberation of Hungary from the Nazis in 1945, the system of contests was greatly enlarged. The Kürschák competition attracts around 500 contestants every autumn. The top 10 contestants are admitted to the university without an admission exam.

For seventh- and eighth-graders there is a special 3-session competition. (If they want to, they may also enter the competition for older students.) For first- and second-year high-school students, there is the "Dániel Arany" competition. There are special competitions at teacher-training institutes.

Apart from all these prize competitions, the Bolyai Society is aware that some mathematically talented youngsters do not do well under test conditions. Publication in *Középiskolai Matematikai Lapok* is another path to recognition. In addition to the problem section, it contains papers by students and young researchers. Erdős told us, "I did not do terribly well at these competitions," yet a few years later his discoveries in number theory were internationally recognized.

At lower age levels, a rich variety of extracurricular activities are offered. For elementary pupils, there is the "Young Mathematicians Friendship Circle," part of the Society for the Popularization of Science. For high-school students, the Mathematical Society organizes monthly "High School Mathematical Afternoons," and for the best (around 60 of them), the "Youth Mathematical Circle." The "Circle" holds a national meeting at Christmas and at Easter.

The highest level in the contest hierarchy is the "Miklós Schweitzer Memorial Mathematical Competition." This is open to both university and high-school students. It consists of 10 or 12 "very hard" problems, which may be worked at home.

"The Schweitzer competition is an important event in our mathematical life. The problems are discussed for days. It is accepted that those who win a prize, or whose results in the competition are published, have proved their wide knowledge of mathematics and their ability to do research. The award ceremony is not just a handing out of prizes. It is a regular scientific session of the Bolyai Society. All the problems are solved at this session" [33].

But who was Schweitzer? Here are some sentences from *Commemoration* [72], a lecture Pál Turán gave in March 1949 to the Bolyai Mathematical Society, in memory of Hungarian mathematicians lost in the war and in the Holocaust:

"Miklós Schweitzer graduated from secondary school in 1941, and in the same year won second prize in the Loránd Eötvös mathematics competition. In 1945, on January 28, near the Cog Railway, he received a German bullet in his body, just a few days before the liberation he so longed for. At that moment he knew that his greatest desire, to be a full-time university student, would never come true. He was granted only a short time to live—a stormy, uncertain time—but he availed of it well."

Then Turán goes on for three pages, presenting Schweitzer's discoveries in classical analysis. The Cog Railway is in Budapest. It carries people up and down Freedom Hill.

Hungarian Specialties

Hungarian mathematics included many of the major trends and specialties of the twentieth century. But three fields have been characteristically Hungarian: classical analysis in the style of Lipót Fejér; linear functional analysis in the style of Frigyes Riesz (1880–1956); and discrete mathematics in the style of Pál Erdős and Pál Turán.

Fejér and Riesz were born in 1880. Each was famous for many important discoveries, and even more for an elegant style, a knack for using simple, familiar tools to obtain far-reaching, unexpected results.

Fejér was born in the provincial town of Pécs. His father, Samuel Weisz, was a shopkeeper. (In Hungarian, "white" is "fehér." "Fejér" is an archaic spelling.) The family had deep roots in Pécs; Fejér's maternal great-grandfather, Dr. Samuel Nachod, received his medical degree in 1809. In high school, Lipót Fejér became a faithful worker of the problems in *Középiskolai Matematikai Lapok*. It is reported that László Rácz, a secondary school teacher who led a problem study group in Budapest, often opened his session by saying, "Lipót Weisz has again sent in a beautiful solution." [This same Rácz later identified János Neumann (1903–1957) as an outstanding mathematical talent!] In 1897, Fejér won second prize in the Eötvös competition. Then he studied at the Polytechnic University in Budapest. König, Kürschák, and Eötvös were among his teachers.

In December 1900, while a fourth-year student, he published his most famous work. This was the use of Cesàro sums (averages of partial sums) to sum the Fourier series of functions which are continuous but not smooth. This method permits one to solve Dirichlet's problem in a disc for arbitrary continuous boundary data. (The use of ordinary partial sums can fail if the boundary data are not piecewise smooth.) This result of Fejér's is still important wherever Fourier analysis is practiced. It was the core of his Ph.D. thesis. Fourier analysis and summation of series continued as his lifelong interests. For the next 5 years, Fejér did not find a permanent, full-time job. Among the odd jobs he picked up was one in an observatory, watching for meteors.

In 1905, Poincaré came to Budapest to accept the first Bolyai prize. When he got off the train he was greeted by high-ranking ministers and secretaries (possibly because he was a cousin of Raymond Poincaré, the politician who later became President and four times Premier of the Third Republic). According to the still-current story, he looked around and asked, "Where is Fejér?" The ministers and secretaries looked at each other and said, "Who is Fejér?" Said Poincaré, "Fejér is the greatest Hungarian mathematician, one of the world's greatest mathematicians." Within a year, Fejér was a professor in Kolozsvár, in the region of Transylvania. Five years later, mainly by Loránd Eötvös's intervention, he was offered a chair at the University of Budapest.

Our interviewee Agnes Berger was one of Fejér's students.

RH: Can you describe Fejér's teaching?

Berger: Fejér gave very short, very beautiful lectures. They lasted less than an hour. You sat there for a long time before he came. When he came in, he would be in a sort of frenzy. He was very ugly-looking when you first examined him, but he had a very lively face with a lot of expression and grimaces. The lecture was thought out in very great detail, with a dramatic denouement. It was a show.

RH: What did you work on?

Berger: Interpolation. Turán was in fact my real advisor. The way a professor was expected to behave there was very different from the way it is here. I was greatly amazed when I saw that in America a professor would sit down with a graduate student. Nothing like that ever happened in Budapest. You would say to the professor, "I'm interested in this or that." And then eventually you would come back and show him what you did. There was none of the hand-holding that goes on here. I know people here who see their students every week! Have you ever heard of such a thing? Well, I did have Turán, who acted for me like an advisor. I don't think of Fejér as a college teacher. There was only one Fejér in all of Hungary. And in Szeged there was Riesz. Only two in the whole country. That is a very exalted position.

Pál Turán wrote: "A coherent mathematical school in Hungary was created first by Fejér" [55]. George Pólya said, "Almost everybody of my age group was attracted to mathematics by Fejér." Besides Pólya, Fejér's students included Marcel Riesz, Ottó Szász, Jenő Egerváry, Mihály Fekete (1886–1957), Ferenc Lukács, Gábor Szegő, Simon Sidon, later Pál Csillag (1896–1944), and still later Pál Erdős and Pál Turán. "Fejér would sit in a Budapest cafe with his students and solve interesting problems in mathematics and tell them stories about mathematicians he had known. A whole culture developed around this man. His lectures were considered the experience of a lifetime, but his influence outside the classroom was even more significant" [2].

Of course, this brilliant career was not without its shadows. "Naturally, World War I had an impact on him, to which a serious illness added in 1916. The effect of counterrevolutionary times was shown by a three year gap in the list of his papers. He never did overcome the effect of those times, as could be perceived again and again from his hints" [55]. Turán's reference to "those times" is clear to Hungarians who lived through them. He means, the "white terror," the early years under Horthy, following the suppression of the Hungarian Soviets.

At some time between the two wars, Fejér was visited in his office at the University of Budapest by a professor seeking Fejér's assistance in some academic matter. After polite conversation, to be sure Fejér remembered to do whatever service he wanted, the visitor pressed into Fejér's hand his "professional card," and left. Presumably, he had forgotten that on the reverse side of the card he had written a reminder to himself: "Go see the Jew." Fejér kept the card, and showed it to John Horváth, our informant.

It is reported that for some reason Fejér was not on the best of terms with Béla Kerékjártó (1898–1946), the topologist who, with Frigyes Riesz and Alfréd Haar, dominated the mathematical scene at Szeged until he moved to Budapest in the late 1930s. Presumably, it was after some unsatisfactory encounter with Kerékjártó that Fejér produced his still remembered cutting remark, "What Kerékjártó says is only topologically equivalent to the truth."

In 1927, due to the political climate of the time, Fejér did not get enough votes to enter the Hungarian Academy of Science. In 1930, after being elected to societies in Göttingen and Calcutta, he was finally admitted to the Hungarian Academy.

The politics of this period are difficult to grasp today. Horthy accepted the role of Jewish capital in Hungary. He was even on social terms with some upper-class Jews. Nevertheless, he instituted a quota system against Jews seeking to enter a university. No more than 5% of the students could be Jews. As for faculty positions, they became virtually out of the question, even for someone like Erdős.

The twenties were a time when talented, ambitious Jewish young people in Budapest knew that if they were to achieve what they were capable of, they must leave. Von Neumann went to Berlin, and then to Princeton; Pólya to Zürich and then to Stanford; Szegő to Berlin, Königsberg, and then Stanford; von Kármán to Göttingen to Aachen and then to Cal Tech; Marcel Riesz to Lund; Mihaly Fekete to Jerusalem; and so on, through Teller, Eugene Wigner, Leo Szilárd, Arthur Erdélyi, Cornelius Lánczos, and Ottó Szász (1884–1952). Fejér and Riesz, older men with tenured positions, remained in Hungary.

Most of these emigrés left in the 1920s, before the Nazi onslaught. They had time to move in an orderly way, without disrupting their careers or their creativity.

In 1944, Fejér was pensioned off as an alien element to the nation. Late one December night, the residents in his house on Tátra Street were lined up by Arrow Cross "lads," to be marched to the bank of the Danube. They were saved by the phone call of a brave officer. Other Budapest Jews did meet death from a gunshot there by the river bank. After the liberation, Fejér was found in an emergency hospital on Tátra Street "under hardly describable circumstances." But with the end of the war he again received honors, both from Hungary and abroad.

Erdős reports that in his later years Fejér was no longer the bubbling, convivial wit of his youth. "He once told Turán, 'I feel I was burned out by thirty.' He still did very good things, but he felt that he didn't have any significant new ideas. When he was 60 he had a prostate operation, and after that he didn't do very much. He kept on an even keel for 15 or 16 years more, and then he became senile. It was very sad. He knew he was senile, and he would say things like, 'Since I became a complete idiot. . . .' He was happy when he didn't think about it. He continued to recognize my mother and me. In the hospital he was well cared for, till he died of a stroke in 1959."

Frigyes Riesz

The other major figure in Hungarian mathematics between the two wars was Frigyes Riesz. His younger brother Marcel was also a famous mathematician, but he lived most of his life away from Hungary.

The Riesz brothers were born in the town of Győr, where their father, Ignácz, was a physician. In 1911, Marcel received an invitation from Gosta Mittag-Leffler to give three lectures in Stockholm. He stayed on and became one of Sweden's most influential mathematicians, holding a chair at Lund from 1926 until 1952 and again from 1962 to 1969. Two of his most famous pupils were Lars Gårding and Lars Hörmander.

For most of his life, Frigyes was professor at Szeged,

Frigyes Riesz in 1925 (from Riesz's *Collected Works*, Akádemiai Kiadó, Budapest, 1960).

a city about 100 miles from Budapest, near the southern border with Yugoslavia. Mainly because of his presence, the University of Szeged became a recognized center of mathematical research. He was known to post-war students of my generation for his great book, *Functional Analysis* [44], co-authored with his famous student and colleague, Béla Szőkefalvi-Nagy. The first part of their book is modern real analysis, and the second part is linear operators. Both parts are written with a truly intoxicating elegance. The basic principle is, "Much with little." Results both general and precise, using elementary, concrete tools—trigonometry, plane geometry, first-semester calculus—the true Hungarian style.

Ray (Edgar R.) Lorch spent the year 1934 in Szeged working with Riesz. We are indebted to him for an account [26] of how this book came to be.

> Riesz was a dangerous man with whom to collaborate in writing a paper or a book. He was constantly having new ideas on how to proceed, and the latest brain-child was the favorite. This would lead to disconcerting results for the collaborator, who was perpetually out of step. An example was told me by Tibor Radó, his ex-assistant. During the academic year, Riesz would lecture on measure theory and functional analysis. Radó would take copious notes. When summer arrived, Riesz would depart for a cooler spot (Győr). Radó would sweat it out for three months, writing

up at Riesz's request all the material, to be in publishable form in the fall. At the end of September Riesz would put in his first day at the Institute, and Radó would come to the library to greet his superior, proudly carrying a stack of eight hundred pages, which he placed in Riesz' lap with great satisfaction. Riesz glanced at the bundle, recognized what it was, and raised his eyes with a mixture of kindness and thankfulness, and at the same time with a spark of merriment, as if he had pulled off a fast one. "Oh, very good, very good. Yes, this is very nice, really nice. But let me tell you. During the summer I had an idea. We will do it all another way. You will see as I give the course. You will like it." This took place many years in a row. The book was not written until Riesz, probably under the pressure of advancing age, wrote the book in collaboration with Béla Szőkefalvi-Nagy some 18 years later. As we all know, the book, *Leçons d'Analyse Fonctionnelle,* was an international best-seller for decades.

Frigyes did his university studies at the Polytechnic in Zürich and at the University of Göttingen, and then earned his Ph.D. at Budapest. At Göttingen, he was influenced by Hilbert and Hermann Minkowski, and at Budapest by König and Kürschák. He did post-doctoral study in Paris and Göttingen and taught high school in Lőcse (now Leviče, in Slovakia) and in Budapest.

In 1911, he was appointed to the University of Kolozsvár, which was founded in 1872. It was an important center of scholarship, in some ways more progressive than the university at Budapest. In 1920, in accord with the Treaty of Trianon, Transylvania was ceded to Romania. The town of Kolozsvár was renamed Cluj. A new university was established in Hungary, at Szeged. The Hungarian-speaking students and faculty of Kolozsvár were invited to Szeged. Riesz first went to Budapest in 1918, and then in 1920 to Szeged, along with Alfréd Haar, who had also been a professor at Kolozsvár. Lipót Fejér had gone from Kolozsvár to Budapest in 1911.

In Szeged, Riesz and Haar created the Bolyai Institute, and in 1922 the journal, *Acta Scientiarum Mathematicarum,* which quickly attained international standing. His greatest research achievement was the theory of compact linear operators. One must also mention the Riesz representation theorem, the re-creation of the Lebesgue integral without use of measure theory, and the introduction of subharmonic functions as a basic tool in potential theory. He introduced the function spaces L^p, H^p, and C and did the basic work on their linear functionals. He proved the ergodic theorem. He proved that monotone functions are differentiable almost everywhere. The Riesz–Fischer theorem is a central result about abstract Hilbert space. It is also an essential tool in proving the equivalence between Schrödinger's wave mechanics and Heisenberg's matrix mechanics.

We quote István Vincze [63].

As a lecturer Riesz was somewhat unpredictable. He was not always perfectly prepared for the lecture. When that happened he would ask his assistant, László Kalmár, for help. But Kalmár wasn't always available. [László Kalmár (1900–1976), like Riesz, was of Jewish ancestry and Calvinist persuasion. A universal mathematician, he was remembered by many as also a superb teacher. R.H.] Nevertheless, we found Riesz a first-class interpreter of science. In his lectures everything appeared naturally in historical perspective. That was highly instructive. When he was not well prepared, he often spent time on very interesting digressions. Once he gave a brilliant explanation of why scientific work is easy. "Everyone has ideas, both right ideas and wrong ideas," he said. "Scientific work consists merely of separating them."

Lipót Fejér was born only three weeks after Frigyes Riesz (on February 9, 1880; Riesz was born on January 22). There was constant teasing between them. For instance, Fejér would claim that he actually was older than Riesz, because Riesz was born a month prematurely.

Riesz loved a quiet, balanced life. He liked order. He was jovial, even a bit aristocratic. Much of his social life took place in a few fashionable rowing and fencing clubs, where empty-headed "notables" from the city and the military could also be found. He belonged to the most exclusive rowing club in Szeged, and would go there from early spring to late autumn. In the evening he would go to the fencing club and play bridge.

He backed László Kalmár very strongly, and hoped Kalmár would become an outstanding mathematician (which he did). But he expected Kalmár to remain a bachelor and devote all his life to science. (As Riesz did himself, and as also did Marcel Riesz, Alfréd Haar, Lipót Fejér, Dénes König, and Pál Erdős.) However, Kalmár did get married. This made Riesz lose his temper to some extent. For a while he was nervous and impatient to Kalmár. Then he calmed down. Kalmár's wife was also an able mathematician, and Riesz liked her, as all of us did. Riesz could see that Kalmár's scientific goals had not been hurt by marriage.

When reading a mathematics journal, he sometimes would heave a sigh: "At last he also understands it." (Meaning, the author at last understands what Riesz and others discovered earlier.) Once Riesz said that a good mathematics book—while of course proving all the theorems—should be more than just a sequence of theorems and proofs. It should discuss the significance of the theorems, clarify them from different viewpoints, explain their connections to other parts of mathematics.

Fortunately, Riesz did not suffer any injury or imprisonment during the war. Some of his fellow faculty members petitioned to the government that he be exempted from the deportation of the Jews which took place starting in 1943. On advice of friends, he went to Budapest early in 1944. While deportation of the Jews was being enforced in the provinces, he was in Budapest. He returned to Szeged the following summer, and on October 11 Szeged was lucky enough to fall, almost without combat, into the hands of the Soviet Army. (Budapest was not to be so fortunate.) Soviet troops had crossed the Tisza River above and below Szeged and encircled it. So the Germans abandoned Szeged and blew up its bridges. Their Hungarian allies were stranded on the east side of the river.

A few years later, a decade-long desire of Riesz was fulfilled: to hold a chair at the University of Budapest. In Budapest Riesz lived a quiet, contented life. He was not completely satisfied with his new social standing, which was much different from what he had enjoyed between the two World Wars. But the changes did not disturb him too much. His new sport became swimming in Gellért

Bath or in Palatinus Bath on Marguerite Island. He liked to read crime stories, and smoke cigars occasionally.

He did not have many personal students. Edgar R. Lorch, Béla Szőkefalvi-Nagy, Tibor Radó, and Alfréd Rényi (1921–1970) all became well known. He never refused anyone who came to him for help, but such a thing rarely happened. Nevertheless, he taught every mathematician in the world. Even today, all mathematicians learn from his elegant demonstrations and penetrating ideas.

In addition to Riesz, Haar, Szőkefalvi-Nagy, and Kalmár, two other mathematicians whom we have already mentioned played important parts at Szeged: Kerékjártó and Radó. Kerékjártó was a topologist. Radó was an analyst, best known for his research on surface area. He was an early mathematical emigrant to the United States. He became a professor at Ohio State in 1931. In 1932, he published an article in the *American Mathematical Monthly* [37] on the Eötvös competition in Hungary.

An anecdote about the Riesz brothers is told by both Szőkefalvi-Nagy and John Horváth. (Horváth was a long-time friend and colleague of Marcel Riesz.) It seems that Marcel once submitted a paper to the Szeged *Acta*, where Frigyes was founder and editor. It was certainly a good paper, but Frigyes wrote to his brother, "Marcel, you have written also better things."

To be fair, Marcel did publish in the Szeged *Acta*. In Volumes I and II, 1921–1923, he had four papers. As a new journal, *Acta* may have been actively seeking papers in those years. Since these papers of Marcel Riesz are on Fourier series, he probably had written them years before, while still in Hungary and perhaps under Fejér's influence.

Here is another story Horváth heard from Marcel Riesz. When Hilbert wrote his paper on the integral-equation solution to Dirichlet's problem, he very much wanted Fredholm to read it. But Fredholm never read it. Then, when Frigyes Riesz wrote his papers, he very much wanted Hilbert to read them. But Hilbert never read them. And finally, when Marcel wrote his big paper on the hyperbolic Cauchy problem, all the time he was working on it he tried to write it so that his brother would understand it. But Frigyes never read it.

(Unfortunately, this story is all too typical in mathematics.)

I had always wondered why the Riesz–Szőkefalvi-Nagy *Functional Analysis* was first published in French. To this question Professor Szőkefalvi-Nagy was able to give a simple answer.

Szőkefalvi-Nagy: We published in French because we had written it in French. First of all, both of us knew French. At least, for writing mathematics. Riesz wrote French very well. Both of us did know German too. But it was just after the war, and Germany was very much compromised by fascism.
RH: Sure.
Szőkefalvi-Nagy: Of course we had nothing against the great mathematicians in Germany.
RH: I understand.
Szőkefalvi-Nagy: English? Well, the Cold War already began to. . . .
RH: I see.
Szőkefalvi-Nagy: Russian? Neither of us knew Russian.
RH: So it had to be French. Anyhow, it was translated very quickly into English.
Szőkefalvi-Nagy: It was translated into German, English, Russian, Japanese, even into Chinese.
RH: How did Riesz survive the war? How did he get through those years, '44, '45?
Szőkefalvi-Nagy: It wasn't easy. He was very tolerant. He was greatly esteemed and respected by all kinds of people. During the last year of the war, Hungary was occupied by Hitler. On March 19, '44, from one day to the next, German troops were here in Szeged. After this came bombing by the Allies. Szeged was bombed by British bombers from the north and the south. And then the Jewish people lost a whole population.

Although Riesz was of Jewish origin, he was not arrested. But it was not safe for him to leave his apartment until October, when the Red Army surrounded Szeged. Of course, Riesz had a number of very good friends who were not Jewish. I visited him every second or third day. He kept himself ready for a journey, he had his rucksack packed.
RH: How did he get food?
Szőkefalvi-Nagy: I told you, he had friends. One was a young lady, the daughter of a medical school professor. The janitor at the Institute came every other day to fix his bath.
RH: Was there any risk in bringing him food?
Szőkefalvi-Nagy: That problem existed. Not physically, but mentally. It was very bad to know that your existence depended on some crazy people.
RH: Was he able to do mathematical work at home?
Szőkefalvi-Nagy: Yes, but lower in intensity. He listened as much as possible to radio broadcasts, and he received plenty of books and periodicals. He could survive, but under pressure of uncertainty. The period from the beginning of April, '44, till the following October was difficult. Then when the Red Army came in, the professors elected him rector of the university.

I was in Budapest during the siege. There it was much worse. My wife's mother and father lived in Budapest, and she was afraid of losing contact with them. Fortunately, we didn't lose anyone. But for several months we had to hide in a cellar with many other people, under conditions far from pleasant.
RH: How long did the siege go on?
Szőkefalvi-Nagy: From the middle of December, '44, until February 12th. Some fighting continued even after that.
RH: How did people keep from starving?
Szőkefalvi-Nagy: That was a problem which everybody had to solve for himself. I thought ahead of time of storing some potatoes and lard. Even during the siege, if you got up just before midnight and went to a certain place early in the morning, before sunrise, and stood and waited till they opened, then perhaps you had some chance to get a kilogram or two of bread. That was possible almost until the last day. But then there was nothing. The shops were neither open nor shut: their entrances had been bombed out. Many people were starving. It was a war! But in a war there are fallen horses. No doctor had inspected them, but nevertheless, in the morning many people tried to take away a kilogram or so of horse meat. It was very difficult.

In the middle of March I came back to Szeged by myself. Partly by train, partly by carriage, partly by horse car, partly just walking. I found Szeged taken over by Soviet troops. Peace banners were on the street and the market was open. And in Szeged I found Riesz. He didn't hate people. He had some sharp, critical words, but he never was too hard.

RH: Do you think that was partly why he later decided to go to Budapest, because he had bad feelings about some people in Szeged?

Szőkefalvi-Nagy: No. I think it was because he had never married, and he was getting older. There was a third Riesz brother in Budapest, a lawyer, married. Frigyes lived with him. And he had students in Budapest. Horváth was one. So was János Aczél, do you know him? He's in Canada, at Waterloo University. And Ákos Császár, who is now the president of the János Bolyai Mathematical Society, and was president of the ICME Congress in Budapest.

Riesz died in a hospital early in 1956, possibly of blood-vessel problems which had troubled him for some time.

It is strange that Hungary's greatest mathematician waited for years for an invitation from his country's leading university. Under Horthy, and much more under Hitler, it was not acceptable to have more than one Jew in an academic department at the Péter Pázmány University (as the Loránd Eötvös University of Budapest was called before 1952). Fejér had been there since 1911. After the war, such rules no longer applied.

Erdős and Turán

The two major streams of Hungarian mathematical research which Fejér and Riesz inspired were joined in the 1930s by a third—"discrete" mathematics, including combinatorics, graph theory, combinatorial set theory, number theory, and universal algebra.

This development began with Dénes König, son of Gyula König. Erdős and Turán attended his seminar. König wrote the first book about graph theory, *Theory of Finite and Infinite Graphs,* published in 1936, and until 1958 the only text on the subject. It has recently been reprinted in German and translated into English. According to *Mathematical Reviews,* "It can truly be called a classic of graph theory . . . a sound introduction to many branches of the subject, and a valuable source book."

In the late twenties and early thirties, a small group of friends met to do mathematics, informally and privately, even after they had left the university. They were interested in combinatorics, graph theory, and other kinds of discrete mathematics.

Often they met in Budapest's Liget Park, near a certain statue depicting "King Béla's Anonymous Historian." So they called themselves "the Anonymous Group." None of the group had jobs; there were no jobs in the early 1930s. Like other unemployed Budapest mathematicians, they put some bread on the table by tutoring gymnasium students. (To mention three others, not part of the Anonymous Group—Rózsa Péter tutored Peter Lax, and Mihály Fekete and Gábor Szegő tutored János Neumann—known later in the United States as John von Neumann.)

The leader of the Anonymous Group, by virtue of his originality, productivity, and total devotion to mathematics, was Pál Erdős. Erdős won his first fame by an elegant new proof of Chebychev's theorem: "Between any number and its double lies at least one prime." He shared with Atle Selberg the glory of finding the first elementary proof of the prime number theorem. He has led in creating the field of mathematics known as "extremal combinatorics" or "extremal graph theory": "Given some function of a finite set system on n elements, what is the largest value the function can take?" Usually one finds the answer, if at all, only asymptotically for large n. Erdős left Hungary for England in 1934. He says that by that year it was obvious that Hungary was unsafe.

Other members of the Group were Márta Wachsberger, Géza Grunwald (1910–1943), Anna Grünwald, András Vázsonyi, Annie Beke, Dénes Lázár, Esther (Eppie) Klein, Tibor Gallai, György Szekeres, László Alpár, and Pál Turán. Esther Klein is credited [10] with first bringing to the group (and solving) a problem on finite sets, of the type considered earlier (as they later learned) by Frank Ramsey in England. "Ramsey theory" became one of the recurrent themes in the work of Erdős, Turán, Szekeres, and others. Szekeres and Klein married and escaped by way of Shanghai to Australia. There they have helped inspire Hungarian-type problem competitions. Gallai became famous both as a researcher and as a teacher. Like Erdős, he was one of our interviewees. Alpár became a communist, and was imprisoned in France until the end of World War II. Then he returned to Hungary, to be imprisoned again by the Stalinist Hungarian regime. When released from jail for the second time, he for the first time took up mathematics full time. Turán served in a Fascist labor camp during World War II. Before and after that, he had a brilliant research career. At the time of his death in 1976 he had become a major figure in international mathematics.

By the inspiration of leaders such as Erdős, and by its mutually stimulating relationship with computer science, discrete mathematics has become a recognized part of contemporary mathematics. Discrete mathematics is now the largest mathematics research specialty in Hungary. Hungary is preeminent in this field; it exports combinatorialists to leading mathematics departments in the United States.

Finale

In this sample of Hungarian mathematics we have had to neglect some important figures. Jenő Hunyadi (1838–1889) and Manó Beke (1862–1946) were pioneers who should be remembered. György Hajós (1912–1970) won fame by proving Minkowski's conjecture on the lattice-packing of unit cubes.

Lajos Schlesinger (1864–1933) became a professor at Leipzig, the first Hungarian mathematician to hold a chair at a German university. He wrote two important books on ordinary differential equations [70, 71]. Mathematicians working today on isomonodromy deformations use "Schlesinger transformations." Peter Lax writes, "Some of Schlesinger's results have become of interest recently because of renewed interest in Painlevé equations in connection with complete integrability. His books are in the spirit of Lazarus Fuchs, whose student Schlesinger must have been and whose son-in-law he was."

[For a detailed history of pre-twentieth-century mathematics in Hungary see [74].]

We cannot attempt a survey of Hungarian mathematicians since World War II, but there are some we must mention. László Fejes-Tóth (b. 1915) is famous for studying packings, coverings, and tessellations in two and three dimensions. He has created a mini-school on these topics.

Rózsa Péter (1905–1977), mentioned earlier as Peter Lax's tutor, was a very special figure. Morris and Harkleroad [32] call her "Recursive Function Theory's founding mother." She was the first to propose (at the International Congress in Zürich in 1932) that recursive functions warrant study for their own sake. She published important papers about them, and the first book on the subject [35]. Her little book *Playing with Infinity* [36] is a beautiful presentation of modern mathematics for the general reader. She was a poet, and a close friend of László Kalmár, whom we mentioned above as Frigyes Riesz's lecture assistant. A brief biography of her is in [32].

László Rédei (1900–1980) was an influential algebraist who worked on algebraic number theory and on Pell's equation. One of his favorite types of problem was to find the algebraic structures (groups, semigroups, rings) all of whose proper substructures possess some particular interesting property. Rédei earned his Ph.D. at Budapest in 1922, and taught high school in Miskolc, Mezőtur, and Budapest until 1940. While still a gymnasium teacher, he was recognized as part of Hungary's mathematics research community. In 1940, he became department head at Szeged, first in geometry, later in algebra and number theory. From 1967 to 1971 he headed the Department of Algebra at the Mathematical Institute of the Hungarian Academy of Sciences. He published nearly 150 research papers and 5 books, including *Lacunary Polynomials over Finite Fields* and *The Theory of Finitely Generated Commutative Semigroups*.

"The main feature of the whole career of László Rédei is hard, stout work; in this he can give an example to every mathematician. Maybe this explains why he was able to go on working even beyond 75. Several times he attacked seemingly hopeless problems, running the risk of complete failure. His efforts were often crowned with success only years later. He had several problems on which he worked continuously for about ten years. He often considered problems in a highly original way, contrary to the expectations of all the other mathematicians . . . He always felt his pupils were his collaborators, and he never refused to learn from them" [68].

Finally, it will be our pleasure to describe a memorable giant whose name is not well enough known among American mathematicians—Alfréd Rényi.

Alfréd Rényi

Rényi was born in Budapest, the son of an engineer "of wide learning," and the grandson, on his mother's side, of Bernát Alexander, a "most influential" professor of philosophy and aesthetics at Budapest. His uncle was Franz Alexander, the famous psychoanalyst. He attended a humanistic (rather than scientific) gymnasium and maintained a lifelong interest in classical Greece. In 1944, he was brutally dragged to a Fascist labor camp, but he managed to escape when his company was transported to the West. For half a year he hid with false papers [39]. At that time Rényi's parents were captives in the Budapest ghetto. Rényi "got hold of a soldier's uniform, walked into the ghetto, and marched his parents out . . . It requires familiarity with the circumstances to appreciate the skill and courage needed to perform these feats" [60].

After the Liberation, he received his Ph.D. at Szeged with Frigyes Riesz. He did postgraduate work in Moscow and Leningrad, where he worked with Yu. V. Linnik on the Goldbach conjecture. There he discovered a method which, according to Turán, is "at present one of the strongest methods of analytical number theory."

If I feel unhappy, I do mathematics to become happy. If I am happy, I do mathematics to keep happy.

From 1950 on, he was director of the Mathematical Institute of the Hungarian Academy of Science. In 1952, he founded the chair of probability theory at Loránd Eötvös University in Budapest. Under his leadership, the Mathematics Institute became an international center of research and the heart of Hungarian mathematical life. He had the rare ability to be equally at home in pure and applied mathematics. He was a leading researcher in probability theory. He was also one of the important number theorists of our time, and he contributed to combinatorial analysis, graph theory, integral geometry, and Fourier analysis. He produced more than 350 publications, including several books. "Once when a gifted young mathematician told him that his working ability strongly depended on external circumstances, Rényi answered: 'If I feel unhappy, I do math to become happy. If I am happy, I do math to keep happy' " [57].

Three of his books are accessible to everybody, including, of course, all mathematicians, regardless of their field or their level. The *Dialogues on Mathematics* [39] is a remarkable work of philosophy and literature. It contains three dialogues—with Socrates, Archimedes, and Galileo. They deal in profound and original ways with fundamental issues in the philosophy of mathematics, yet their light touch and dramatic flair make them readable by anyone. "For Zeus's sake," asks Rényi's Socrates, "is it not mysterious that one can know more about things which do not exist than about things which do exist?" Socrates not only asks this penetrating question, he answers it.

The *Letters on Probability* [40] contain four warm personal letters from Blaise Pascal to Pierre Fermat, communicating Pascal's enthusiastic opinions and ideas about the origins and foundations of probability theory. The letters are composed in complex sentences, in the literary style of Pascal and Fermat's day, and display easy familiarity with their lives and work. Nevertheless, as Rényi makes clear in a "Letter to the Reader," the actual author is Rényi, not Pascal. This *jeu d'esprit* must be unique in the writings of modern mathematicians. The fourth letter especially will repay any reader interested in the foundations of probability. Here Pascal, who (like Rényi) holds the frequentist interpretation of probability, reports in novelistic detail a dispute in the salon of Madame d'Aiguillon with his foppish friend "Damien Miton," an upholder of the subjectivist view.

The *Diary on Information Theory* [41], like the two earlier books, is also written "behind a mask." The diary is kept by one "Bonifac Donat," and contains Bonifac's "lecture notes" on five of "Professor Rényi's" lectures, plus Bonifac's preparation for a talk of his own. The last diary entry says, "The professor doesn't look too well. I hope it's nothing serious." In fact, the professor was not well enough to finish that last chapter. It had to be completed by one of Rényi's old pupils, Gyula Katona. Rényi died on 1 February 1970, at the age of only 49.

In view of their hardships, it is amazing how Hungarian mathematicians have been able to persist and create, in poverty and unemployment, in labor camps or under siege. We close with an unforgettable quote from Pál Turán:

> It sounds incredible, but it is true. The story goes back to 1940, when I received a letter from my friend George Szekeres in Shanghai. He described an unsuccessful attempt to prove a famous Burnside conjecture (which was disproved later). The failure of his attempt could have been obtained from a special case of Ramsey's theorem, but Ramsey's paper, beyond its mere existence, was then unknown in Hungary.
>
> At that time, most of my income came from private tutoring, and I had to teach my pupils at their homes. While traveling between two pupils, I pondered the contents of the letter. My train of thought soon led me to finite forms, and then to the following extremal problem: What is the maximum number of edges in a graph with n vertices, not containing a complete subgraph with k vertices? Though I found the problem definitely interesting, I postponed it, being then mainly interested in problems in analytical number theory.
>
> In September 1940 I was called for the first time to serve in a labor camp. We were taken to Transylvania to work on building railways. Our main work was carrying railroad ties. It was not very difficult work, but any spectator would have recognized that most of us did it rather awkwardly. I was no exception. Once one of my more expert comrades said so explicitly, even mentioning my name. An officer was standing nearby, watching us work. When he heard my name, he asked the comrade whether I was a mathematician. It turned out that the officer, Joseph Winkler, was an engineer. In his youth he had placed in a mathematical competition; in civilian life he was a proofreader at the print shop where the periodical of the Third Class of the Academy (Mathematical and Natural Sciences) was printed. There he had seen some of my manuscripts.
>
> All he could do for me was to assign me to a wood-yard where big logs for railroad building were stored and sorted by thickness. My task was to show incoming groups where to find logs of a desired size. This was not so bad. I was walking outside all day long, in the nice scenery and the unpolluted air. The problems I had worked on in August came back to my mind, but I could not use paper to check my ideas. Then the formal extremal problem occurred to me, and I immediately felt that this was the problem appropriate to my circumstances.
>
> I cannot properly describe my feelings during the next few days. The pleasure of dealing with a quite unusual type of problem, the beauty of it, the gradual approach of the solution, and finally the complete solution made these days really ecstatic. The feeling of some intellectual freedom and of being, to a certain extent, spiritually free of oppression only added to this ecstasy.

This beautiful memory appeared in Turán's "Note of Welcome" in the first issue of the *Journal of Graph Theory* [58]. When writing it, he was already battling his last illness. He died on 26 September 1976. The *Journal*'s first issue appeared in 1977.

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Which Is to Be Master—III

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Imagine you are reading a mathematical manuscript (say, a paper submitted to *The Intelligencer*), and you come upon the following sentence: "The eventual halting of this algorithm leaves an output which includes a solution of equation (5)." The meaning is clear, isn't it? Only it is not reader-independent.

To Reader A it may be clear that the sentence means, "If the algorithm halts, it will have yielded a solution of (5)."

To Reader B it may be clear that it means, "The algorithm is sure to halt at some time, and when it does, it will have yielded a solution of (5)." Rather different!

What makes the difference? Reader A—surely not a native of an English-speaking country—has taken the English word "eventual" to have the sense of French "éventuel": "occurring contingent on some condition; or, possibly occurring and possibly not occurring." This notion needed a good word to express it, and in the eighteenth century the French word "éventuel" was coined to fill the need. It spread to neighbouring languages: German ("eventuell"), English, Romanian, . . . Everyone used it. Only in English, over time, the original meaning was lost.

Reader B learned English as a native, or picked up its eccentric usage of this word somehow. In English, to say something will happen eventually, as in the advertising slogan "Eventually—Why Not Now?", means it will happen inevitably, sooner or later. English even has a verb "to eventuate" fitting this meaning, corresponding to French "aboutir."

BUT, alas, having lost the French-derived meaning of "eventual," we lack any word to express the concept for which "éventuel" was created. Sometimes we manage with articles: "A misstep would be fatal" does not predict that the misstep will occur. For our author to write

A solution (a,b,c) of (5) must satisfy $a^n + b^n = c^n$

makes no commitment as to whether there exists such a triple; but to say

The solution (a,b,c) of (5) must satisfy $a^n + b^n = c^n$

implies the solution's existence.

Now that I have explained how much better off English would be if it had kept the international meaning of "eventual," I ought to follow by saying I will permit it in *The Intelligencer*, oughtn't I? No, sorry. But why not? I permit the international meaning of "hopefully" (= "it is hoped" or German "hoffentlich"), don't I? No, not that either. Maybe eventually, but not now.