tree of a formula. As an additional bonus one has in this case Kreisel's correspondence between the paths (of given complexity) in such a tree with cut and the counter-models (of the same complexity) for the formula in question, which allows one to make a bridge to the senior author's topic of constructive models. Yet one finds in this book the indirect proof of the interpolation theorem containing, in fact, the direct construction using the normal form (not normalization) theorem for Gentzen-type derivations. If the authors had in mind to illustrate proof-theoretic transformations, the natural deduction system (used in this book only to ensure "The practical possibility of presenting all necessary formal proofs") is much more appropriate. In this case, of course, one has to stress the difference between normal form and normalization theorems.

To sum up, the book can be very useful as an aid for a lecturer in model theory, but not as a textbook. Both the notation and the formulations are too heavy for a beginner. The reader is invited to read the beginning of §24. Even a person with some experience should be constantly on guard against such surprises as an incorrect definition of the diagram of a model (p. 170) or the failure of induction in the proof of a model construction theorem (p. 188; formula $\Phi^{\neg}$ can be longer than $\neg\Phi$). The reviewer was puzzled by a remark on page 167 on the "finitarity" of the notion of elementary equivalence. The proof one page 176 (of the condition for a formula to be an $\forall$-formula) obviously uses the distinctness of $b_1, \ldots, b_k$, which can be false. This list can be extended further.

It should be added that in the English translation of this book (see the following review), most of the serious errors—especially in Chapter 6—have been corrected. G. MINC

YU. L. ERSHOV and E. A. PALYUTIN. ***Mathematical logic.*** Revised English translation by Vladimir Shokurov of the preceding. Mir Publishers, Moscow 1984, 303 pp.

The review above of the original Russian edition gives an adequate picture of the content of the English translation; note that the major errors have been corrected. The translator, however, has done a poor job. There are too many passages like the following: "The mechanism of compatibility is mainly of methodical importance. It allows us to recognize a common part in many theorems proving which involves construction of models." Moreover, since the Russian language has no indefinite and definite articles, their proper insertion in an English translation requires knowledge of the subject matter, something which the translator fails to display. Thus, this book is unsuitable for beginners. An experienced graduate student or logician will be able to glean interesting tidbits here and there. Especially valuable is Chapter 5 (*Model theory*), which contains concepts and results not found in other textbooks.

ELLIOTT MENDELSON

YU. I. MANIN. ***A course in mathematical logic.*** Translated from the Russian by Neal Koblitz. Graduate texts in mathematics, vol. 53. Springer-Verlag, New York, Heidelberg, and Berlin, 1977, xiii + 286 pp.

The range of material covered in Manin's book is impressive: a (standard) proof of the completeness of first-order logic; the undefinability of truth in the language of arithmetic, proved à la Smullyan; aspects of quantum logic, including results of Specker and Kochen; a more or less complete proof, via Boolean-valued models, of the independence of the continuum hypothesis; constructible sets and the consistency of the GCH; recursive functions; the solution by Davis–Matijasevič–Putnam–Robinson to Hilbert's tenth problem; Kolmogorov complexity; the Gödel incompleteness theorem; the arithmetical hierarchy; Gödel's observation concerning the length of proofs; and Higman's embeddability theorem.

I regret that I cannot be at all enthusiastic about the manner in which these topics are presented; my overall impression was of a book hurriedly worked up from course notes. Many textbooks in logic written by masters of exposition are now available; no one need rest content with a book from which craft is so noticeably absent. Manin asserts that his book is "above all addressed to mathematicians," but whether professionals or students is by no means clear. I dare say there are few mathematicians at home with Čech cohomology groups (p. 205), Serre's *Arbres, amalgames*, SL_2 (p. 265), and the fact that the solutions to the Pell equation form a cyclic semigroup (p. 216) who will need to see three verifications that $(P \rightarrow (Q \rightarrow P))$ is a tautology (pp. 31–32). Manin has ignored the first law of exposition: Pick an audience and stay with it.

The major drawback of the book is that a diligent reader cannot in general have sufficient confidence that after suitable effort, she or he will be able to see the truth of a theorem for which a proof is offered. Manin constantly violates Littlewood's law of exposition: Do not omit from the presentation of an argument *two* consecutive steps. And Manin omits more than small steps. In the proof of the independence of CH, he writes, "as in §3 of Chapter II, it can be verified that all the tautologies and logical quantifier axioms are 'true' and that the rules of deduction preserve 'truth.'" But when the next two pages contain the notoriously intricate proof that " = " behaves as it should in Boolean-valued models, a reader ought not to take logic for granted. Concerning his account of the independence results of Gödel and Cohen, Manin writes, "the number of omitted formal deductions does not exceed the accepted norm." The fears this ominous statement raises prove justified: his treatment of the independence results is far worse in this regard than any of the standard treatments, e.g. the spendid presentations of Kunen (LI 462) and Drake (XLIII 384). It is hard to know what principles govern the inclusion and exclusion of arguments and proofs. Manin includes a proof that any two well-orderings are comparable, but refers the reader to Rosser's ***Simplified independence proofs*** (XXXIX 328) for proof of the fact that the regular open sets of $\{0,1\}^I$ form a complete c.c.c. Boolean algebra. Why one and not the other? Much too much is omitted for the book to be read enjoyably.

On the other hand, too many pages that would be better spent on elaboration of the matter at hand are devoted to irrelevancies. The two most extravagant digressions concern the semantics of those formulas of ancient Icelandic poetry called *kennings*, e.g. "storm of spears" (meaning *battle*), and the light thrown on logic by the case of the solider Zasetsky, the aphasic patient of A. R. Luria and the subject of Luria's remarkable ***Man with a shattered world.***

The curious reader may wish to compare Manin's presentation of the solution to Hilbert's tenth problem with that given in Martin Davis's article *Hilbert's tenth problem is unsolvable*, first published in the ***American mathematical monthly***, vol. 80, (1973), pp. 233–269, and now included as an appendix to the 1982 Dover reprint of Davis's ***Computability and unsolvability***. It was for the thought, patience, and care displayed in his exposition that Davis won the Chauvenet Prize. It is these qualities that are absent from ***A course in mathematical logic***.

I do not wish to deny that the foregoing comments are niggling; but it is precisely because such objections can be raised to Manin's book that its usefulness as a *textbook* is severely limited.

GEORGE BOOLOS

ABRAM ARONOVICH STOLYAR. ***Introduction to elementary mathematical logic***. English translation edited by Elliott Mendelson of ***Eléméntarnoé vvédénié v matématičéskuú logiku.*** The MIT Press, Cambridge, Mass., and London, © 1970, pub. 1971, and Dover Publications, New York 1983, vii + 209 pp.

Introduction to elementary mathematical logic contains an introduction, three chapters, and three short appendices. Its introduction contains a brief history of logic, distinguishes object languages from meta-languages, and defines logical functions. The first chapter is on propositional logic, and includes normal forms (defined by using Boolean functions), and applications to switching circuits. The second chapter contains a careful development of the propositional calculus including proofs of consistency and independence. The final chapter on predicate logic includes the algebra of sets, a definition of the predicate calculus, proofs of traditional syllogisms, and the use of predicate logic with equality to formalize a simple geometric theory and the arithmetic of the natural numbers. The three appendices are the standard proofs of three theorems: the duality principle for propositional logic, the deduction theorem for the propositional calculus, and the completeness theorem for the propositional calculus.

In his introduction to the Russian edition of the book, Stolyar points out that as mathematical logic finds more applications in mathematics, linguistics, computer science, and other fields, a wide variety of people are looking for some familiarity with the subject. He proposes his book, not as a survey of the subject matter, but as basic reading, which will make further readings in the subject intelligible. He feels that this book will be suitable for advanced high school students and for high school teachers as well as for college students and others. Although most of the prerequisite content for the book (algebra, geometry, introductory calculus, and mathematical induction) would have been covered by advanced high school students in the United States, the level of mathematical sophistication required to read the book would be too high for any but exceptional students below the junior level in college. In particular, his use of functional notation would be unintelligible to most students in the United States before they had studied linear algebra.

Once students are comfortable with functional notation and reading mathematical texts on their own, it is a very suitable book for self study. After finishing this book one should have a sound basis for reading