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WHAT DIFFERENCE WOULD IT MAKE IF CANCER WERE ERADICATED? AN EXAMINATION OF THE TAEUBER PARADOX

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Abstract—The immediate effect of discovering a way to cure cancer would be a reduction in the number of deaths in the United States by the number of people now dying from that cause. Within a short time, however, deaths from other causes would increase, and the net long-term effect would be relatively small. A parameter is derived that measures how much the expectation of life is increased by a marginal reduction in any cause of death. That parameter is additive in the several causes and has other advantages, though it does not avoid the assumption of independence.

The effect of a medical improvement is often measured as the difference between the expectations of life before and after the improvement. Thus, if $\mathring{e}_0$ is the expectation of life before an improvement consisting of the elimination of the ith cause of death, the difference $\mathring{e}_0^{(-i)} - \mathring{e}_0$ is taken as the effect, where $\mathring{e}_0^{(-i)}$ is calculated omitting the ith cause. The calculation allows for the increased mortality from other causes within each age group, and supposes throughout that the several causes act independently.

Unfortunately, on this approach the effects of the several causes are not additive. Because of the way life tables are constructed, one gets a different result by eliminating causes i and j separately (i.e., calculating $(\mathring{e}_0^{(-i)} - \mathring{e}_0) + (\mathring{e}_0^{(-j)} - \mathring{e}_0)$ than when one eliminates them together (i.e., calculating $\mathring{e}_0^{(-i-j)} - \mathring{e}_0$). The result thus depends on whether one adds the effects of the elimination of the several kinds of cancer, say, separately or assumes all kinds eliminated simultaneously.

A further and more important objection is that the measure is not well suited to finding the effect of marginal improvements. We are much more likely to want to know the effect of eliminating 1 percent of the cases of breast cancer than of eliminating all cases. Because of nonlinearity, one cannot take $(\mathring{e}_0^{(-i)} - \mathring{e}_0)/100$ for the effect of eliminating 1 percent of the ith cause. The present paper provides a parameter that applies linearly within a considerable range and is exactly additive as among causes. It does vary among populations but in an understandable way. Unfortunately, neither it nor any other measure can escape the assumption of independence among causes, about which some qualitative observations will be made later.

The issue that the parameter resolves is of professional interest to demographers and equally important to the public. Our parameter, among other uses, leads to a criterion for the direction that medical research ought to take. Consider, as an example, the case of cancer and suppose, to begin with, its total eradication. The following discussion shows how different the conclusions with our parameter can be from those of a naive consideration of the issues.

If a cure for cancer were discovered and made available today, 350,000 cancer deaths would be avoided in the next year. The overall death rate would be lower by nearly 18 percent. If the cure were quick and inexpensive, a large fraction of the country's hospital beds and medical personnel would be released for treatment of other ailments. Patients would be spared untold suffering. Such an implicit analysis underlies government proposals for eradication of cancer. The argument is sound for first effects on mortality but wholly misleading for the long term.

The first effects would soon be offset by more mortality from diseases other than cancer. As a result of the cancer cures, the population would include a higher proportion of people subject to other causes of death. This would be partly because they would be of the right age for heart trouble and other degenerative diseases, partly because, even at younger ages, the people who die of cancer are more subject to other ailments than the general population of their age.

At the extreme, it might be said that everyone dies of something sooner or later, so that, when the effects of the eradication of cancer had shaken down, the same number of deaths would occur as before, and the only benefit would be the substitution of heart and other diseases for cancer. A cure for cancer would only have the effect of giving people the opportunity to die of heart disease.

Thus, on the one hand, it can be argued that, since cancer deaths are 18 percent of all deaths, their elimination would lower deaths by 18 percent; on the other hand, it can be said that since everyone dies eventually all that this particular medical advance would do would be to increase the options: one could choose to die of heart disease rather than cancer. The issue will be designated the Taeuber paradox, after Conrad Taeuber, who drew attention to it. Cause of death has been discussed recently on a probability model by Manton et al. (1976), and Preston (1974) has taken up the effect of mortality change on the stable population, both articles appearing in this journal. Demetrius (1976 and earlier papers) has done important work applying information theory in biology and demography. All of these treatments are more sophisticated than the simple measure presented here.

The problem is different for the person and the community. The person wants to know how much longer he can expect to live if the risk of cancer is eliminated, and we will see that he can be given an exact statement of the increased expectation, so long as it is supposed that those dying of cancer are in average health otherwise. The community wants to know how its total deaths (and the associated burden on hospitals) will be reduced. This is a somewhat less definite question, since, with rates given, the number of deaths depends also on how many people there will be in the population.

To argue that the cure of cancer would make no difference to mortality, since those saved from cancer would die of something else, is plainly too strong a statement; everything depends on *how soon* they would die of other things. The elimination of tuberculosis did appreciably lengthen life, and elimination of puerperal mortality lengthened it even more. These cases suggest the answer: the effect of eliminating any cause of death depends on the average time that elapses before the persons rescued will die of some other cause. From this viewpoint, the successive causes of death are in a sequence; the advantage of disposing of any one depends on how young the people are that it affects. Among the youngest are victims of motor vehicle deaths and infectious diseases other than tuberculosis; heart disease deaths are on the average the oldest. Cancer strikes some ten years younger than heart disease in the United States.

But all this is unnecessarily intuitive and crude. It is much better to start with the expectation of life at birth and go on from this to find the effect of a discovery that would eliminate 100δ percent of the deaths from the ith cause, where δ is an arbitrary or hypothetical number like 0.01. As an introduction to the technique, let us disregard cause of death and examine the effect on $\mathring{e}_0$ of a fractional improvement in the age-specific death rates at all ages. If the chance of dying in the next dx of a year for a person who has reached age

x has been $\mu(x)dx$, suppose that this is changed to $\mu^*(x) = \mu(x)(1 + \delta)dx$. In our application, δ will be a small negative quantity, typically -0.01, representing a 1 percent improvement in all causes at all ages. If the death rate $\mu(x)$ was constant at all ages, say μ, then the question is immediately answered: a 1 percent improvement at all ages increases the expectation of life by 1 percent; in that case $\mathring{e}_0 = 1/\mu$, and $\mathring{e}_0^* = 1/[\mu(1 + \delta)]$. Insofar as $\mu(x)$ is not constant but increasing, the 1 percent improvement at all ages increases $\mathring{e}_0$ by less than 1 percent. If $\mu(x)$ was increasing with age just fast enough to make $l(x)$ a straight line, as in Graunt's life table, then it can be shown that the 1 percent increase in $\mu(x)$ at all ages gives 0.5 percent increase in $\mathring{e}_0$. Insofar as $\mu(x)$ is increasing rapidly, the $\mathring{e}_0$ is increased by much less than 1 percent, in some instances as little as 0.12 percent. These statements require demonstration.

The function of the life table that measures the change in $\mathring{e}_0$ consequent on a proportional change in the age-specific rates will be called H, defined as

$$H = \frac{-\int_0^\omega l(a) \ln l(a)\, da}{\int_0^\omega l(a)\, da}.$$

If everyone lived to age 75, say, and then died, H would be 0. If the survivorship, $l(x) = \exp\,[-\int_0^x \mu(a)da]$ is a straight line, then H is one-half. These are mathematical facts. That, for the United States, H is about 0.2 for males and 0.15 for females may be verified by calculation (Keyfitz, 1977, chap. 3). Note that the numerator of the constant H is the quantity of information contained in the l_x column, as defined by Shannon. The following shows how it can be derived in a demographically natural way.

The mathematics underlying the use of H start with the fact that a changed mortality $\mu^*(x) = \mu(x)(1 + \delta)$ translates into a power of the old survivorship curve,

$$l^*(x) = \exp\left[-\int_0^\omega \mu(a)(1 + \delta)da\right]$$

$$= [l(x)]^{1+\delta},$$

and this, in turn, translates into the new expectation of life:

$$\mathring{e}_0^* = \int_0^\omega [l(a)]^{1+\delta}da.$$

In order to make something of the expectation of life, we have to expand the $[l(a)]^{1+\delta}$ as a function of δ. The derivative of $[l(a)]^{1+\delta}$ is

$$\frac{d[l(a)]^{1+\delta}}{d\delta} = [l(a)]^{1+\delta} \ln l(a).$$

By Taylor's theorem, we have

$$[l(a)]^{1+\delta} \doteq l(a) + \delta l(a) \ln l(a)$$

in the neighborhood of $\delta = 0$, where we may stop at the first derivative. The first derivative seems to be all that is needed for most purposes, since for small δ, $l(a)$ is effectively a straight-line function of δ. Entering this value of $[l(a)]^{1+\delta}$ into the expression for expectation of life, we get for the ratio of the new expectation to the old,

$$\frac{\mathring{e}_0^*}{\mathring{e}_0} = \frac{\int_0^\omega [l(a)]^{1+\delta}da}{\int_0^\omega l(a)\, da}$$

$$= \frac{\int_0^\omega [l(a) + \delta l(a) \ln l(a)]\, da}{\int_0^\omega l(a)\, da}$$

$$= 1 - \delta H,$$

where H is as defined above.

The same H may be used to compare male and female mortality; it is found that, in the United States and other countries of low death rates, males average something like 50 percent excess mortality over females at the several ages, but the excess of the female expectation of life $\mathring{e}_0$ over the male is only about 10 percent. Apparently H is a convenient parameter for translating an average ratio of age-specific rates into a ratio of expectations.

The use of a constant violates the fact that, whether comparing males and females or thinking of a health improvement, the incidence of the difference is by no means the same over all ages. If we consider all causes together, the ratio of male to female mortality is over 2 in some ages and less than 1.1 in others. But the variation by age is likely to be less in a particular cause of death; for instance, a treatment that will cure 1 percent of cases of breast cancer might well have a relatively uniform proportional impact at the several ages.

Suppose that the age-specific rates from the ith cause go from $\mu^{(i)}(a)$ to $\mu^{(i)}(a)(1 + \delta)$; the probability of living to age a then goes from $l(a)$ to $l(a)[l^{(i)}a]^{\delta}$; the expectation of life at birth goes from $\mathring{e}_0 = \int_0^\omega l(a)da$ to

$$\mathring{e}_0{}^* = \int_0^\omega l(a)[l^{(i)}a]^{\delta}da.$$

Here $l^{(i)}(a)$ is the probability of living to age a in the face of risks from the ith cause alone.

Once we are given $l(a)$, $l^{(i)}(a)$, and δ, the new expectation of life is readily calculated. But we can make use of the fact that $\mathring{e}_0{}^*$ is a linear function of δ for a considerable range of values, and usually we are interested in knowing what will be the effect of a marginal improvement, say eliminating 1 percent of deaths, i.e., $\delta = -0.01$. This linearity suggests that we find a coefficient, $H^{(i)}$ defined below, that is in a sense a measure of the length of time from the nondeath from the ith cause up to the time when the person dies from the next thing that will hit him.

We have

$$\mathring{e}_0{}^* = \int_0^\omega l(a)[l^{(i)}a]^{\delta}da,$$

and, expanding $[l^{(i)}a]^{\delta}$ by Taylor's theorem around $\delta = 0$, this becomes

$$\mathring{e}_0{}^* \doteq \int_0^\omega l(a)[1 + \delta \ln l^{(i)}(a)]da$$

$$= \mathring{e}_0 \left[1 + \delta \frac{\int_0^\omega l(a) \ln l^{(i)}(a)\, da}{\int_0^\omega l(a)\, da}\right]$$

$$= \mathring{e}_0[1 - \delta H^{(i)}],$$

where $H^{(i)}$ is the average of the $-\ln l^{(i)}(a)$ weighted by the $l(a)$.

The increase of life expectation at birth is important for individuals; but how does it translate for the community? The simplest translation is for the stationary population, where the overall death rate is the reciprocal of the expectation of life. Hence, we can say that, with the elimination of 100δ percent of deaths from the ith cause, the crude death rate would go from $1/\mathring{e}_0$ to

$$\frac{1}{\mathring{e}_0[1 - \delta H^{(i)}]} \doteq \frac{1}{\mathring{e}_0}[1 + \delta H^{(i)}].$$

In short, a drop of 100δ percent of deaths from the ith cause would lower the crude death rate in the stationary population by $100\delta H^{(i)}$ percent (Table 1).

The change can include the same population extant at any one moment as before, but now based on fewer deaths and, therefore, on fewer births, and of course with a different age distribution. What may be called the turnover has declined. From one point of view, this decline of turnover is the main effect of all sanitary and medical advances from the beginning of time; in the stationary condition, Stone Age man with 25 years of expectation of life had three times the turnover of a modern population approaching an expectation of life of 75 years. One might compare a modern population having 15,000 deaths per year with a primitive population having the same number of

Table 1.—$H^{(i)}$, Perçent Increase in Expectation of Life at Birth Associated with 1 Percent Drop in Age-Cause-Specific Mortality: United States, 1930 and 1964

Causes of Death	1930		1964	
	Male	Female	Male	Female
Respiratory tuberculosis	0.0194	0.0190	0.0012	0.0005
Other infectious and parasitic	0.0253	0.0211	0.0018	0.0015
Neoplasms	0.0186	0.0289	0.0302	0.0308
Cardiovascular renal	0.0636	0.0622	0.0840	0.0650
Influenza, pneumonia, bronchitis	0.0370	0.0308	0.0078	0.0059
Diarrheal	0.0141	0.0116	0.0012	0.0011
Certain degenerative	0.0252	0.0261	0.0090	0.0080
Maternal	0.0000	0.0089	0.0000	0.0007
Certain diseases of infancy	0.0311	0.0239	0.0170	0.0125
Motor vehicle	0.0120	0.0043	0.0129	0.0049
Other violence	0.0327	0.0116	0.0192	0.0077
Other and unknown	0.0482	0.0448	0.0232	0.0184
Total	0.3272	0.2932	0.2073	0.1571

Source: Calculated from data in Samuel H. Preston, N. Keyfitz, and R. Schoen, Causes of Death: Life Tables for National Populations (New York: Seminar Press, Studies in Population Series, 1972).

deaths; the modern population would have about 1 million people, the primitive one about 2 million. Something of this kind is involved in comparing life tables, for all life tables have 100,000 deaths on the usual convention, and a corresponding number of exposed population from 2.5 million to 7.5 million.

What is different about the contemporary condition is that the residual causes crowd one on the other, and, hence, there are diminishing returns in increased life from successive discoveries. Past achievements toward eliminating disease, especially those related to infectious diseases, have involved causes of death that were followed by long intervals to the next cause; the causes that now remain are unfortunately clustered together.

Yet one would also like to evaluate the effect of a decline of 100δ percent in the deaths from cancer from the viewpoint of the person who is on the threshhold of the age interval subject to cancer. After all, early mortality is very low in contemporary advanced groups, and 85 to 90 percent of infants live to old age, say 60 years. Hence, we might like to do the entire calculation in disregard of mortality under age 60 as largely irrelevant to the question to which this paper is addressed. The mathematical argument above goes just as well with the number 60, rather than zero, as the lower limit of all integrals, the only difference being that the level of $H_{60}^{(i)}$ is now very different from what we called $H^{(i)}$. In general, one would expect $H_{60}^{(i)}$ to be larger than $H^{(i)}$, since the effect of a drop in the rates as a fraction of the expectation at age 60 would be greater than

Table 2.—$H_{60}^{(i)}$, the Effect of 1 Percent Change in Age-Cause-Specific Death Rates on $\mathring{e}_{60}$ Expressed in Percent: United States and Mexico, 1964

Causes of Death	United States, 1964		Mexico, 1964	
	Male	Female	Male	Female
Respiratory tuberculosis	0.0045	0.0014	0.0291	0.0177
Other infectious and parasitic	0.0044	0.0030	0.0390	0.0368
Neoplasms	0.1192	0.0969	0.0460	0.0673
Cardiovascular renal	0.3484	0.2163	0.1010	0.0959
Influenza, pneumonia, bronchitis	0.0213	0.0127	0.0901	0.0803
Diarrheal	0.0025	0.0023	0.0457	0.0454
Certain degenerative	0.0332	0.0248	0.0688	0.0495
Maternal	0.0000	0.0011	0.0000	0.0121
Certain diseases of infancy	0.0170	0.0125	0.0396	0.0320
Motor vehicle	0.0264	0.0097	0.0103	0.0028
Other violence	0.0456	0.0167	0.0918	0.0180
Other and unknown	0.0644	0.0413	0.2647	0.2466
Total	0.6869	0.4387	0.8261	0.7044

Source: See Table 1.

the fraction at age zero, at least for those causes that mainly affect the old. Table 2 shows that H_{60} for all causes together ranges from about 0.4 to 0.8, being lower for females in both Mexico and the United States. This says that a 1 percent improvement in mortality at all ages over 60 raises $\mathring{e}_0$ by 0.7 percent for U.S. males; its percentage effect is much greater than the 0.20 or so that we found for all ages.

Breaking down the effect for those 60 and over by cause gives the most easily understood expression of the result of a 1 percent diminution of mortality from cancer or heart disease. Table 2 shows that, for U.S. males, a 1 percent drop in deaths from cancer at ages over 60 raises the expectation of life at age 60 by 0.1192 percent; the corresponding figures for heart disease and motor vehicle accidents are 0.3484 and 0.0264 percent, respectively. These numbers combine both the prevalence of the several causes of death at ages over 60 and the length of time between the mean age of each and the mean age of subsequent deaths.

The constants $H^{(i)}$, when complemented by cost information, provide a solution to the allocation problem in medical research. Suppose that with the same expenditure one could eliminate either the fraction δ_i of the ith cause or δ_j of the jth cause. Which project should be undertaken? If $\mathring{e}_0$ is the right criterion, one would examine whether the relation

$$H_i\delta_i > H_j\delta_j$$

holds, and, if it does, the resources would be put into the ith cause; if the inequality runs the other way, the resources would go into the jth cause.

Yet any such criterion for choosing the direction of medical research has the limitation of choosing among predefined causes, like those of the International List of Causes of Death. Insofar as it does this, it is vulnerable to the "causes" specified being mere variants of the same underlying disease entity. If cancer, heart disease, etc., are merely alternative ways in which the aging of body cells makes itself manifest, then eradicating any one of them may make little difference. The proper entity to attack is the process of aging itself.

More realistically, suppose the "real" or direct causes of death are $a, b, c, \cdots$, and $\alpha, \beta, \gamma, \cdots$ are the statistically recognized causes. Suppose also that each of $\alpha, \beta, \gamma, \cdots$ is a combination, linear or not, of $a, b, c, \cdots$. Then, acting on α could help diminish a, but only indirectly and inefficiently.

To sum up with an example, the effect of total elimination of any cause of death has long been estimated by finding the expectation of life with and without that cause. The difference for cancer (using United States males at 1964 rates, for the example) amounted to 2.265 years, or 3 percent of $\mathring{e}_0$. By our alternative method, the effect on the expectation of life of eliminating 1 percent of cancer deaths would be to raise $\mathring{e}_0$ by $H^{(\text{cancer})} = 0.03$ percent. Multiplying by 100 to find the effect of eliminating 100 percent of cancer deaths, we find that the expectation of life is increased by just $100H = 3$ percent. (Since we are dealing with a curvilinear effect, this agreement is a coincidence; it does not apply to heart disease, for example.) Eliminating 1 percent of all causes of death raises $\mathring{e}_0$, the expectation of life, by $H = 0.20$ (or one fifth of 1 percent), all on 1964 male data. The constants $H^{(i)}$ and their sum H are useful for translating percent changes in age-specific mortality into percent changes in expectation of life. Insofar as a marginal reduction of deaths from a particular cause is more likely to occur than total elimination of that cause, $H^{(i)}$ is a more useful constant than $\mathring{e}_0^{(-i)} - \mathring{e}_0$.

All this is from the viewpoint of the individual. The death rate in the stationary population is the reciprocal of the expectation of life, so all of the above can be directly translated into death rates. For instance, the elimination of 1 percent of all age-specific death rates would decrease the overall stationary death rate by one-fifth of 1 percent, and the elimination of 1 percent of cancer death rates would decrease the overall stationary death rate by one-thirtieth of 1 percent. On the pure death model, reduction of death rates reduces the turnover of a population. After all, the life table is based on a fixed absolute number of deaths, usually 100,000; the life table population increases in number when age-specific death rates go down; the fixed number of deaths divided by the increased population gives a lower crude death rate.

The short-run effect of a reduction—partial or total—in cancer deaths is almost exactly equal to the number of deaths averted. If the 30,000 deaths from cancer expected to occur in the next month were averted, the general death rate at the relevant ages being something like 0.01 per year or 0.0008 per month, only 24 deaths would occur during the first month among the lives saved from cancer. Hence, 24,976 of the 25,000 would survive the month, always supposing the cancer patients to be in the same state of health as the general population age by age. However, as masses of people moved up into the older ages past the former cancer barrier, they would be subject to the high rates of heart disease, etc., for those ages. Ultimately, there would be enough of these that the situation would shake down to the small net effect of eliminating cancer that is measured by the constant $H^{(i)}$.

All calculations of the type here discussed inevitably suppose independence of the several causes of death. They show a surprisingly small effect of eliminating any one cause, and if the independence assumption does not hold the effect would be smaller yet. For the most common kind of dependence must be a positive one—people saved from cancer would be more susceptible to heart and other diseases. In view of the existance of positive dependence, the values of $H^{(i)}$ here calculated must be thought of as giving the upper limit on the improvement in mortality due to the elimination of a given cause of death (Shepard and Zeckhauser, 1975).

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