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ABSTRACT

Brief discussions of general vs. special ability and of mathematical reasoning ability form the introduction of this paper on the education of mathematically gifted students. The second section of the paper describes the annual mathematics talent searches conducted by the Study of Mathematically Precocious Youth (SMPY). The third section covers SMPY's special educational provisions for the mathematically talented, including the basic components of the program, importance of fast pace, and other aspects of the offerings (skipping grades, part-time college study, credit by examination, early college entrance, college graduation in less than four years, and by-passing the bachelor's degree). Two illustrations of how selected students progressed through the program comprise the fourth section of this paper, while the final section summarizes SMPY's position concerning the education of mathematically precocious youth.  
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OUTLINE

The Study and Facilitation of Talent for Mathematics \*

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## The Study and Facilitation of Talent for Mathematics

Julian C. Stanley

### I. Introduction

#### A. General vs. Special Ability

In their preoccupation with IQ's as the principal criteria for defining intellectual giftedness, those who study or help the gifted have not given much attention to special mental abilities. For them the approach of Galton, Binet, Terman, and Wechsler has triumphed over that of Spearman and Thurstone.<sup>1</sup> The assessment of global IQ that Terman established so firmly on the basis of Binet and Simon's 1904-11 work<sup>2</sup> is one of psychology's greatest contributions to education, but like all single indices it has sharp limitations. The deliberate averaging of various abilities produces a generally useful average but does not highlight special abilities. The Stanford-Binet Intelligence Scale is indifferent, for example, to whether one excels on memory and is less strong on mathematical reasoning, or vice versa. Either can compensate for the other. A high IQ, even 140 or more, does not guarantee any particular special ability.

This creates a problem in parts of the U.S.A. where giftedness is defined by the state department of education as a minimum overall IQ such as 130 or 132. Because intellectually gifted students are identified in this way, there is a strong tendency to group them for instruction in most subjects according to IQ instead of determining their actual readiness for each subject separately. The outcome is usually inefficiency and frustration for both student and teacher. If David, whose IQ is 140, does not do as well in mathematics as Bill, who is his

age and whose IQ on the same test is also 140, it is likely to be said that David is poorly motivated, inattentive, or lazy. Actually, there is no reason to suppose that the two are anywhere near equal in mathematical reasoning ability. Grouping on that special ability first, with some consideration also being given to IQ as a measure of learning rate, will produce much more homogeneous classes than IQ as the primary grouping variable possibly could.<sup>3</sup>

Underlying the above discussion is a basic difference in philosophy between two approaches to identifying intellectual talent. If youths are required to average high by earning a high IQ, some with excellent special abilities will be missed. If they are chosen entirely via a test of special ability such as mathematical reasoning, nonverbal reasoning, mechanical comprehension, or spatial relationships, some will not have high IQ's (though if the criterion score is quite high, few are likely to have low IQ's). An obvious solution is to administer several tests of various special abilities such as the Differential Aptitude Test Battery<sup>4</sup> that cover a variety of abilities which reveal much about the youth's intellectual functioning. This approach has limitations when the examinee does not read well; for such persons an individually administered test of intelligence such as the Stanford-Binet or one of the Wechsler series will usually give more valid information about general ability than will group tests, which are often somewhat speeded and demand reading skills. This multi-aptitude group-test approach can be supplemented by other tests such as the Raven Progressive Matrices<sup>5</sup> that can be administered somewhat individually without a time limit.

A related approach is to test mainly for a particular special abil-

ity such as knowledge of general vocabulary and then test the high scorers on it further to see what they are like in other cognitive and affective aspects. One will inevitably lose any persons, however bright, who do not score well in the special area. The less the special-ability scores load factorially on general intelligence, the greater the loss of high-IQ individuals will be. For spelling, clerical speed and accuracy, certain types of spatial abilities, and many other abilities the loss might be considerable. If one's chief aim is to locate persons highly talented in a particular way, however, failure to identify those with high IQ's who are not especially talented in the desired field may not be important.

#### B. Mathematical Reasoning Ability

One of the most valuable types of intellectual talent for both society and the individual is mathematical reasoning ability. It undergirds much of current achievement in technology, science, and social science. Usually this ability is poorly assessed by in-school mathematics tests, because often they consist of a mixture of computation, learned concepts, and reasoning. Also, it is difficult to measure mathematical reasoning ability until the young student has acquired enough knowledge of elementary general mathematics with which to reason. The basic content of the test items must be fairly well known so that reasoning can be the chief trait measured.<sup>6</sup>

## II. SMPY's Annual Mathematics Talent Search

### A. The First Search

With these considerations in mind the Study of Mathematically Pre-

ocious Youth (SMPY) at The Johns Hopkins University began in 1971 a large-scale, systematic attempt to identify at some optimum age students who reason extremely well mathematically as determined by their scoring high on a test of mathematical reasoning ability quite difficult for youths their age. Various studies of the problem were made with a broad perspective in order to choose the appropriate age level, test, and testing conditions. These led in March of 1972 to SMPY's first Annual Mathematics Talent Search among seventh, eighth, and under-age ninth graders in the Baltimore vicinity. A total of 396 students, most of who had already scored in the top 5% of national norms on an in-grade mathematics test from an achievement-test battery, volunteered to take two tests designed primarily for above-average eleventh and twelfth graders. These were the College Entrance Examination Board's Scholastic Aptitude Test, mathematical part (SAT-M), and its Level I Mathematics Achievement Test.<sup>7</sup>

The staff of SMPY, consisting then of Lynn H. Fox, Daniel P. Keating, and the writer, was surprised and pleased at how high a number of the contestants scored on these two difficult tests. It was found that 49 percent of the boys and 30 percent of the girls already exceeded the average college-bound male twelfth grader's SAT-M score of 497. The top score earned on SAT-M was 790, only 10 points below the highest possible score for this test.<sup>8</sup> The Math I scores added little to the information provided by the SAT-M ones, so because the latter are less affected by differences among the mathematics curricula of schools in the grades kindergarten through seven, we settled on SAT-M for future

use. In light of our cumulative findings about the validity of SAT-M, this was probably a fortunate decision.

#### B. Summary of SAT-M statistics

Table 1 shows data concerning SAT-M scores in each of SMPY's four

(To the printer: Please put Table 1 about here.)

mathematics talent searches thus far. The first three groups were roughly comparable in age, being composed mostly of seventh and eighth graders, whereas the fourth consisted only of seventh graders and <sup>a few</sup> underage students in higher grades. There have been some fluctuations from year to year, such as in the highest SAT-M scores obtained (790, two 800's, 760, and two 780's during the respective four years), but considering the variety of recruiting methods used and the increasing geographical area covered each succeeding year that variation is not great. It is clear that a large reservoir of virtually untapped mathematical reasoning ability exists all around the region, though it is much greater in some places than in others.

For further statistics from the 1976 search see Figures 1-4.

#### C. Other Tests Used in the Searches

Besides administering SAT-M each of the four years, the SMPY staff varied from year to year the other aptitude and achievement tests used in the competition. In 1972 there was also a general-science talent search; it involved taking college-level Forms 1A and 1B of the Sequential Tests of Educational Progress (STEP).<sup>9</sup> A total of 192 students entered it, 138 of whom were also in the mathematics competition. In 1973 all 953 contestants took both parts of the SAT, mathematical and verbal. In 1974 the 1519 contestants took only SAT-M. Besides these

ability tests, self-report interest and values inventories were used during some of the years.

#### D. The Model for the Searches

The model for SMPY's talent searches is the same as the subtitle of its Mathematical Talent book, d<sup>3</sup>: discovery (finding the talented), description (testing the highest scorers a great deal more), and development (facilitating their education, especially in mathematics and related subjects). After the mathematically talented youth is identified and studied, it is feasible for someone to devise a smorgasbord of educationally accelerative options from which he/she may choose ad lib. This flexible counseling approach, adapted to the abilities, interests, motivations, and individual circumstances of each youth, does not constitute a program in the same sense that the usual procedures for helping gifted children do. Some highly talented students choose little or nothing from the bountiful possibilities, whereas others gorge themselves almost to the point of having to be restrained. No two tend to do exactly the same things at the same time.

### III. Special Educational Provisions for the Mathematically Talented

#### A. Basic Components of the Smorgasbord

What are the educational opportunities that constitute the smorgasbord? The chief theme is getting along faster and better with mathematics from Algebra I through the first or second year of college mathematics (usually Calculus I-III, linear algebra, and differential equations). An able youth may take the first year of high-school algebra a year earlier than usual, or complete two years of algebra in



one, or study both second-year algebra and plane geometry the same year, or learn the first year of algebra on his/her own and move rapidly into the second year of algebra and/or into geometry, or if able enough do even more than that. For example, one brilliant 11-year-old earned credit by means of the Advanced Placement Program Level BC examination in mathematics, with a grade of 4 on a 5-point scale, for the freshman year of calculus at Johns Hopkins. As mentioned above, another 11-year-old simply skipped the first year of college calculus entirely and was among the best students in the third semester. These are extreme examples indeed, of course, but indicative of how incredibly slow-paced 180 50-minute periods of introductory high-school algebra would have been for these students when they were the "normal" age for taking them, 13 or 14, if SMPY had not intervened strongly on their behalf!

#### B. Importance of Fast Pace.

The boredom and frustration of even the average-scoring contestants when incarcerated in a year-long introductory algebra class is difficult for an adult to appreciate. Often, highly able youths themselves are not aware of the extent of the slow-down, because it has been their lot from kindergarten onward. Actually, because of its more abstract nature and the abler-than-average students who enroll for it in the eighth or ninth grade, the already-bored student may actually experience a "lift" in spirits when entering beginning algebra. Only by being given opportunities to move ahead at a more appropriate rate and on a better level of rigor can the student realize how much time was being wasted and how much more enjoyment can be gained from studying mathematics.

That is why SMPY's various special fast-math classes, usually taught

by a college professor, have been so spectacularly successful for a considerable percentage of the students who entered them.<sup>10</sup> Being paced by their intellectual peers rather than merely by their age-mates and being moved ahead fast by an excellent teacher make a world of difference in the students' progress and their sense of accomplishment. Often they "take off" intellectually like rockets when allowed to do so. Several brief case studies later in this paper will illustrate this point.

### C. Other Aspects of the Offerings

#### 1. Skipping Grades

There are other features to the smorgasbord. One is skipping whole grades in school, especially the grade at the end of the middle school or the junior high school so that the student can more rapidly get into a senior high school, where the intellectual fare is usually more varied and appropriate for able youths.

#### 2. Part-Time Study in College

Another opportunity is the taking of college classes for credit part-time before becoming a full-time college student. An occasional student may be able to do this at age 9 or 10. For example, the boy who earned credit for college calculus by examination at age 11 also had been a highly successful college student of both the introduction to computer science and mini-computers while still 10. Other highly able students need to wait until they are 12-14 years old before taking a college course. Others who are somewhat less able should wait until they are 15-17, but not necessarily until they have completed high school. Nothing seems to boost level of aspiration more than making

an "A" in a regular college course in competition with regular college students. The SMPY participants' favorite such course is Introduction to computer science. Other courses often taken are college algebra and trigonometry, descriptive astronomy, and the calculus, but the range of courses taken by an appreciable number of the students is quite great.

In a five-year period we at SMPY have seen approximately 107 boys and 24 girls take a total of 277 college courses for credit before becoming full-time college students. In sheer number, this is the equivalent of  $7 \frac{1}{4}$  bachelor's degrees. The cumulative grade-point average of the group is 3.59, above the requirement for being placed on the Dean's List at Johns Hopkins.<sup>11</sup>

### 3. Credit by Examination

Another excellent way to move ahead is via college-credit examinations, particularly those of the College Board's Advanced Placement Program (APP). Many high schools do not facilitate or encourage this until the twelfth grade, if at all. The staff of SMPY has been working hard calling to the attention of highly able youths this splendid method of cutting both educational time and cost. It makes little sense for a student who at age 12 or 13 reasons extremely well mathematically, better than the typical male college-bound twelfth grader does, to enter college later without having already gained credit for the first year of calculus and probably also of several other subjects such as physics, chemistry, biology, and whatever else he/she finds feasible. In a time

of rapidly rising college costs, this type of acceleration can make an important difference in the institution a student's parents can afford, because it may well mean three years of undergraduate study rather than four.

The SMPY contestant who in January of 1973 as a 12 year old public school seventh grader scored 800 on SAT M managed to earn credit for two semesters of college calculus while still 13 years old and two semesters each of biology, chemistry, and physics while still 15 by making the highest possible grade (5) on each of four AP examinations. Also, while 14 he earned an "A" from a major university by correspondence study in a third-semester college calculus course. And he still has another year in which to take several more AP courses before going off to MIT or Harvard a year early, having skipped the eighth grade.

#### 4. Entering College Early

Quite a few of SMPY's participants come to college early by simply leaving high school before completing the last grade(s) or by doubling up in their subjects and thereby being graduated early. During the 1976-77 school year there were at Johns Hopkins itself 26 such students whom SMPY had sponsored for admission, and even more elsewhere whom it helped to become "radical accelerants." Six of those 26 plan to be graduated during the 1976-77 academic year three to five years ahead of schedule, three of them while still 17 years old, one at 18, and two at 19. All have done well and greatly enjoyed the intellectual and social stimulation of college as compared with the inappropriate pace and level of the earlier grades. Again, we must note that these are exceptionally

able youths; perhaps one in 200 or less of their age group, but there are many like them across the country who can hardly get permission to move even one year ahead of the age-in-grade lockstep. SMPY's efforts in this direction are virtually in their infancy, because the students in even the earliest of its talent searches were at most accelerated ninth graders during the school year 1971-72. Nearly all of them were only seventh or eighth graders then, whom one would "expect" to be in the twelfth grade or freshman year in college during the 1976-77 academic year. When in the fall <sup>of 1971</sup> or spring of 1972 we first met the six early college graduates mentioned above, only two of them had already skipped even one school grade. None had yet done anything else that accelerated their progress through the grades.

##### 5. College Graduation in Less Than Four Years

There are several ways to go through college in less than four years, as five of the six students mentioned above are doing. Perhaps the most straightforward is by entering with sophomore standing attained by some combination of college courses taken while still in high school and credits by examination.<sup>12</sup> Another is by taking heavier-than-required course loads and/or attending summer school. With the comparatively recent advent of intersessions--periods between terms--it has become possible in many colleges to get extra credits then via courses or work experiences. A third method, feasible at Johns Hopkins and a few other institutions, is taking a master's degree concurrently with the last year or two of the baccalaureate. SMPY's first radical accelerant did this. He entered Johns Hopkins in the fall of 1969 as a regular

freshman at age 13 after completing the eighth grade of a public school. By May of 1973 he had received the B.A. degree, with major in the area of quantitative studies, at age 17 years 7 months. Only the thesis was needed to complete, three months later, the M.S. Engineering degree with major in computer science. Had he done it during the senior year his two degrees could both have been awarded in May.<sup>13</sup>

#### 6. Bypassing the Bachelor's Degree

Some students simply shun the baccalaureate and go directly into graduate work after two years or so of undergraduate study. Most major universities will permit this for an occasional meteoric student, albeit perhaps reluctantly. Of course, many colleges and the undergraduate divisions of many universities are so rigidly addicted to the "Class of \_\_\_\_" concept that a student wishing to accelerate his/her educational progress much must be extraordinarily planful and persistent. The ablest and best-motivated intellectually precocious youths can usually find ways, however, especially including being careful which collegiate institution they choose. The facts about how a school's accelerative policies actually operate, as often contrasted with what their promotional literature says, should be obtained in writing before enrolling.

#### D. Few Major in Mathematics Itself

The reader will have perceived that this discussion has moved away from mathematics per se into more general considerations of educational acceleration. Few students who at an early age score startlingly well on SAT-M will become mathematicians at a highly selective college. Most of them will go instead into fields which mathematics undergirds or even

into those, such as the practice of medicine, where great mathematical ability is not essential or even very helpful. Often this is appropriate, because the need for persons holding the Ph.D. degree in mathematics itself is being rather fully met (somewhat more than a thousand such degrees annually)--except, of course, that probably there will never be enough of the ablest, most creative mathematicians. As in most specialties, considerable room exists at the very top.<sup>14</sup> Also, a bachelor's or master's degree in mathematics can be excellent background for doctoral work in a number of fields.

#### IV. Illustrations of How the Smorgasbord Works

##### A. A's Progress

To see how the smorgasbord of educationally accelerative opportunities is used by its most ravenous partakers, let us trace the progress of two similar boys, A and B, from the fall of 1971 through May of 1977.

A was born on 4 December 1959, so he became 17 years old in the late fall of 1976. In October of 1971, when we first met him, he was an 11-year-old sixth grader in a public school. In June of 1972 he entered SMPY's first fast-mathematics class. By August of 1973, after about 60 two-hour periods of rapidly paced instruction, he had completed at a high level two years of high-school algebra, college algebra, plane geometry, trigonometry, and analytic geometry. Before then he had skipped the seventh grade and also made "A" in a regular college course in computer science at Johns Hopkins taken on released time from the eighth grade. After that one year in the middle grade of a junior high school he skipped the ninth and tenth grades and entered the middle

grade of a rather selective senior high school. There he took Advanced Placement calculus and studied physics on his own, besides taking a regular schedule of eleventh-grade courses. Also, he enrolled at night and during the summer in several college courses. (This left him time to win a varsity letter in wrestling, be the math and science expert on the school's TV academic-games team, tutor a brilliant young friend in mathematics, and play an excellent game of golf. In spare moments he directed the successful campaign of his barely 14-year-old sophomore friend for the presidency of the student council!)

After just this one year in junior high and one in senior high, A entered Johns Hopkins at age 14 as a sophomore, with 14 credits by APP examinations (calculus and physics) and 20 credits from the college courses he had already taken. In January of 1977 he completed all requirements for the B.A. degree in mathematical sciences at age 17 years 1 month, perhaps the youngest student ever graduated from Johns Hopkins. Although a full-time college student only five semesters and no summers, he had taken advanced work in a number of different fields, including political science, economics, astronomy, and management.

To recapitulate: A skipped grades 7, 9, 10, 12, and 13. He utilized SMPY's fast-mathematics class, the APP examination opportunity, and college courses taken part-time. He completed college in only five-eighths of the usual time.

#### B. B's Progress

By contrast, B (born 10 July 1959) simply skipped grades 2, 11, 12, and 13. He took a college course each semester and summer term from



the second semester of the eighth grade at age 12 onward, entered Johns Hopkins with 39 college credits, and in six semesters is finishing his B.A. degree in electrical engineering with emphasis on computers.

During the two summers while in college he did high-level research in major industries. His B.A. degree will be due when he is 17 years 10 months old.

Note that B was not in any special-mathematics classes, but instead in regular college ones after completing only the first year of high-school algebra. Also, he did not attempt any of the APP examinations, but had completed two years of college chemistry before enrolling at Johns Hopkins. He skipped the second grade and spent three years in junior high school, compared with A's one, but got considerable released time from those studies in order to pursue college courses.

Both of these young men are highly promising, though for different fields. Both are intellectually brilliant and powerfully motivated, of course, but they are by no means unique, even in SMPY's experience.<sup>15</sup> What they did, or variations on it, can be accomplished by a considerable number of mathematically highly apt youths anywhere who also have excellent overall intellectual ability.

#### V. Conclusion

Specialists in the education of mathematically talented youths do not have the resources with which to develop mathematics courses and curricula. Instead, they must help such students use the best available courses in wisely accelerative ways. Special mathematics curricula such as SMSG (School Mathematics Study Group) and CEMREL (Central Mid-

western Regional Educational Laboratory, Inc.) are designed for above-average students, so at their regular pace they do not meet the needs of the mathematically most precocious youths well. Private schools are not in themselves a solution, either, though the increased pace and stimulating competition within a selective school--whether private or public--can be helpful.

Except for Table 1,

no attention has been given in this article to sex differences in mathematical aptitude and achievement, because that topic falls within the scope of Lynn H. Fox's chapter in this volume. Utilizing the mathematical potentialities of women better is an important topic that deserves far more research and development than it has yet received.

One can sum up the situation by reaffirming that great mathematical reasoning ability at an early age is a resource of inestimable value to individuals and society, but only to the extent that its expression is facilitated through the various subject-matter fields for which it is relevant. The highly precocious mathematical reasoner does not need anything like the Procrustean fit of five or six school years of 180 periods each in which to progress from first-year high-school algebra through the high-school calculus. In fact, he/she is likely to be hurt severely in mathematics by being forced to do so. That persistent finding by SMPY is not contradicted by any other studies of which we are aware.

It is high time that appropriate candidates for substantial acceleration in mathematics and related subjects be found, studied, and helped to move ahead fast and well. Doing this requires far more in-

genuity, determination, and persistence than it does money. The procedures that SMPY recommends actually tend to save a school certain costs of instruction. A little special effort on behalf of the mathematically talented can easily yield large outcomes. Precocious students, their parents, and educators genuinely concerned with the students' educational and personal welfare can spark local efforts to provide the smorgasbord of educationally accelerative opportunities from which each such youth needs freedom to choose over the years of school and college.<sup>16</sup>

#### Addendum Concerning Mathematics Education

SMPY is not a curriculum-development project. Though I am a former high-school teacher of science and mathematics and a Fellow of the American Statistical Association who has done postdoctoral work in statistics and mathematics at two great universities, it was obvious from the beginning of SMPY that for us to become concerned with the development of special materials for the top one in 200 or so young mathematical reasoners would go far beyond our resources of personnel and money. We fully appreciate the importance of the various unified mathematics curricula that hard-working, insightful mathematics educators have devised and tried out. Nothing in the philosophy or practice of SMPY prevents our helping youths who reason extremely well mathematically to move through a particular curriculum faster than most of the other students in it do. Even in a mathematics program designed for the upper third or fourth of the age group, the mathematically brilliant student is almost certain to move too slowly for his or her abilities unless special provisions are made. Perhaps in the standard algebra-geometry-trigonometry-analytic geometry-calculus sequence it is somewhat easier to determine what the exceptional student does and does not know, but the method of diagnostic testing followed by prescriptive teaching is just as applicable to "modern math" as it is to the older version.

## Footnotes

1. Florence L. Goodenough, Mental Testing: Its History, Principles, and Applications (New York: Rinehart & Co., 1949).

2. For a translation of these early articles from the French, see Binet Alfred/and Th. Simon, The Development of Intelligence in Children (The Binet-Simon Scale), tr. by Elizabeth S. Kite (Vineland, N.J.: The Training School, May 1916). For theoretical contributions that began in 1904 see Charles E. Spearman, The Abilities of Man (New York: Macmillan, 1927).

3. Lynn H. Fox, "A Mathematics Program for Fostering Precocious Achievement," in Mathematical Talent: Discovery, Description, and Development, Proceedings of the Third Annual Hyman Blumberg Symposium on Research in Early Childhood Education, ed. Julian C. Stanley, Daniel P. Keating, and Lynn H. Fox (Baltimore: Johns Hopkins University Press, 1974), pp. 101-25.

William C. George and Susanne A. Denham, "Curriculum Experimentation for the Mathematically Talented," in Intellectual Talent: Research and Development, Proceedings of the Sixth Annual Hyman Blumberg Symposium on Research in Early Childhood Education, ed. Daniel P. Keating (Baltimore: Johns Hopkins University Press, 1976), pp. 103-31.

Julian C. Stanley, "Special Fast-Mathematics Classes Taught by College Professors to Fourth- Through Twelfth-Graders," ibid., pp. 132-59.

Julian C. Stanley, "Rationale of the Study of Mathematically Precocious Youth (SMPY) During its First Five Years of Promoting Educational Acceleration," in The Gifted and the Creative: Fifty-Year

Perspective," Proceedings of the Seventh Annual Hyman Blumberg Symposium on Research in Early Childhood Education, ed. Julian C. Stanley, William C. George, and Cecilia H. Solano (Baltimore: Johns Hopkins University Press, to appear in 1977).

4. Published by The Psychological Corporation, 757 Third Avenue, New York, New York 10017, and usually called the "DAT."

5. Distributed by The Psychological Corporation, ibid.

6. Richard R. Skemp, The Psychology of Learning Mathematics (Baltimore: Penguin Books, 1971).

V. A. Krutetskii, The Psychology of Mathematical Abilities in Schoolchildren, trans. Joan Teller (Chicago: University of Chicago Press, 1976).

7. Stanley, Keating, and Fox, op. cit., esp. pp. 23-46. For results from the second and third talent searches, see Keating, op. cit., esp. pp. 23-31 and 55-89.

8. The reported score scale runs from 200 through 800, but the chance-score level is represented by a score of about 260-280, and 800 does not necessarily represent a perfect score. For comparison, the entering class at The Johns Hopkins University in the fall of 1976 was composed of persons who as eleventh or twelfth graders had averaged 674 on SAT-M, with a standard deviation of about 80 points. The young man who in March of 1972 while still 13 years old scored 790 plans to be graduated from Johns Hopkins in May of 1977 at age 19 years 1 month, three years ahead of schedule, having majored in engineering science. The top science scorer from the March 1972 contest, who was then a 12-year-old seventh-grader, also plans to receive his B.A. degree

from Johns Hopkins, he four years ahead of schedule at age 17 years 9 months. His major field is electrical engineering, with emphasis on computer science.

9. Published by the Educational Testing Service, Princeton, New Jersey 08540.

10. For documentation of this statement, see the references listed in Footnote No. 3.

11. Cecilia H. Solano and William C. George, "College Courses for the Gifted," Gifted Child Quarterly 20 (Fall 1976), 274-285

12. The main national programs by means of which high-school students may earn college credit by examination are the Advanced Placement Program (APP) and the College Level Examinations Program (CLEP), both sponsored by the College Entrance Examination Board and administered by the Educational Testing Service of Princeton, New Jersey 08540. Also, of course, it is possible--though often not easy or convenient--to get college credit for a subject by taking an examination in the relevant department of the college or university one attends or plans to attend.

13. Julian C. Stanley, "Use of Tests to Discover Talent," in Keating, op. cit., p. 9. This young man became 21 years old in November of 1976 while in his fourth doctoral year in computer science at a major university, where he has continued to do well.

14. Eric Temple Bell, Men of Mathematics (New York: Simon and Schuster, 1937).

Norbert Wiener, I Am a Mathematician, M.I.T. Paperback Series No. 20 (Cambridge, Mass.: Massachusetts Institute of Technology Press,

1956).

S. M. Ulam, Adventures of a Mathematician (New York: Scribner's, 1976).

15. Stanley, "Use of Tests to Discover Talent," in Keating, op. cit., pp. 19-21.

16. Some supplemental references not cited above are the following:  
Catharine M. Cox, The Early Mental Traits of Three Hundred Geniuses, Genetic Studies of Genius, vol. 2 (Stanford, Calif.: Stanford University Press, 1926). Some of these were markedly talented in mathematics.

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Julian C. Stanley, "The Student Gifted in Mathematics and Science," Bulletin of the National Association of Secondary School Principals 60 (March 1976), 28-37; "Test Better Finder of Great Math Talent Than Teachers Are," American Psychologist 31 (April 1976), 313-14; "Identifying and Nurturing the Intellectually Gifted," Phi Delta Kappan 58 (November 1976), 234-37; and "Concern for Intellectually Talented Youths: How It Originated and Fluctuated," Journal of Clinical Child Psychology 5 (Winter 1976), 38-42.

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Table 1

Mean Scores and Standard Deviations By Sex on SAT-M  
For the Four Talent Searches Held by SMPY to Date

Statistics	1972*		1973*		1974*		1976**	
	Girls	Boys	Girls	Boys	Girls	Boys	Girls	Boys
Number	173	223	416	537	591	928	366	507
Mean	442	501	461	518	479	513	422	459
Std. Dev.	84	110	82	94	77	90	65	88

\* Seventh and eighth graders and a few students in higher grades who were no older than age-in-grade eighth graders.

\*\* Seventh graders, and a few seventh-grade-age students in higher grades.

1976 TALENT SEARCH  
BOYS  
N = 507

SAT-M  $\geq$  416  
AND  
SAT-V  $\geq$  368  
N = 210  
41.4%

SAT-V  $\geq$  368  
BUT  
SAT-M  $<$  416  
N = 55  
10.9%

SAT-M  $\geq$  416  
BUT  
SAT-V  $<$  368  
N = 139  
27.4%

SAT-M  $<$  416  
AND  
SAT-V  $<$  368  
N = 103  
20.3%

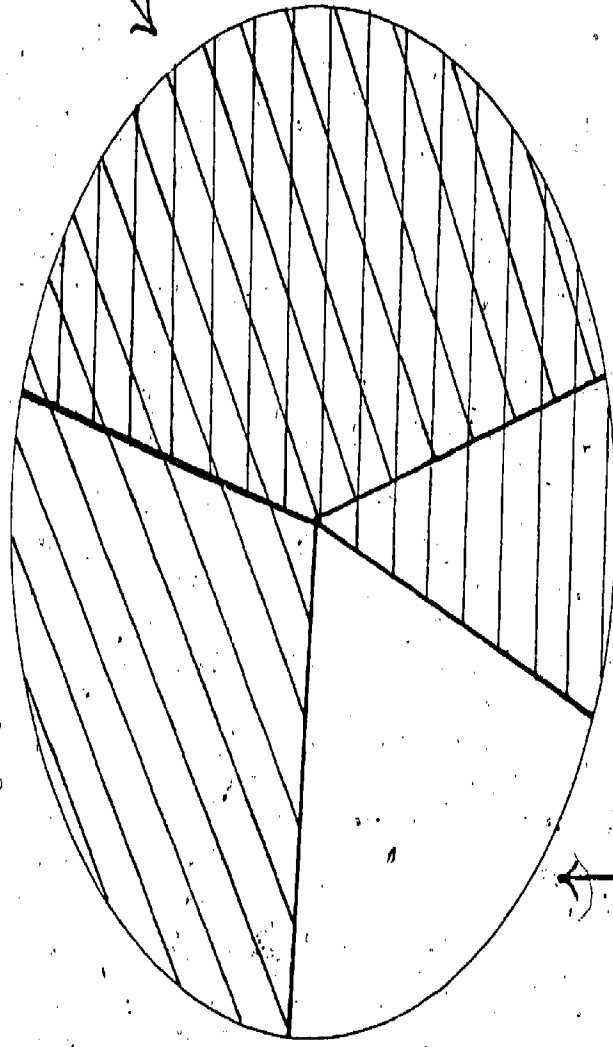


FIG. 1: MALES IN THE 1976 TALENT SEARCH WHO AS 7TH GRADERS (OR UNDER-AGE 8TH GRADERS) SCORED AS WELL AS OR BETTER THAN THE AVERAGE RANDOM SAMPLE 11TH AND 12TH GRADE MALE ON SAT-M AND AS WELL AS OR BETTER THAN THE AVERAGE RANDOM SAMPLE 11TH AND 12TH GRADER IN GENERAL ON THE SAT-V.

1976 TALENT SEARCH  
GIRLS  
N = 366

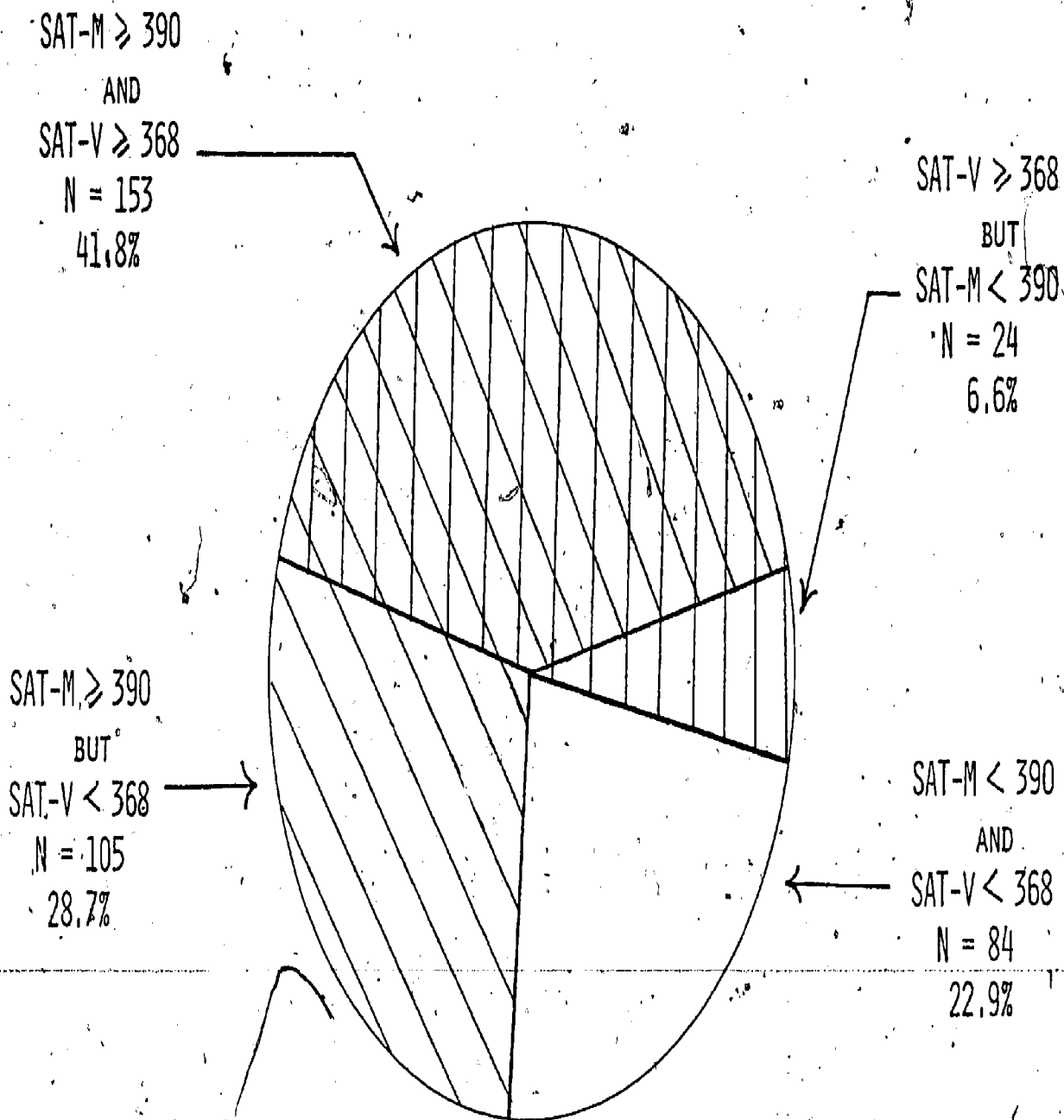


FIG. 2: FEMALES IN THE 1976 TALENT SEARCH WHO AS 7TH GRADERS (OR UNDER-AGE 8TH GRADERS) SCORED AS WELL AS OR BETTER THAN THE AVERAGE RANDOM SAMPLE 11TH AND 12TH GRADE FEMALE ON SAT-M AND AS WELL AS OR BETTER THAN THE AVERAGE RANDOM SAMPLE 11TH AND 12TH GRADER IN GENERAL ON SAT-V.

1976 TALENT SEARCH  
BOYS  
N = 507

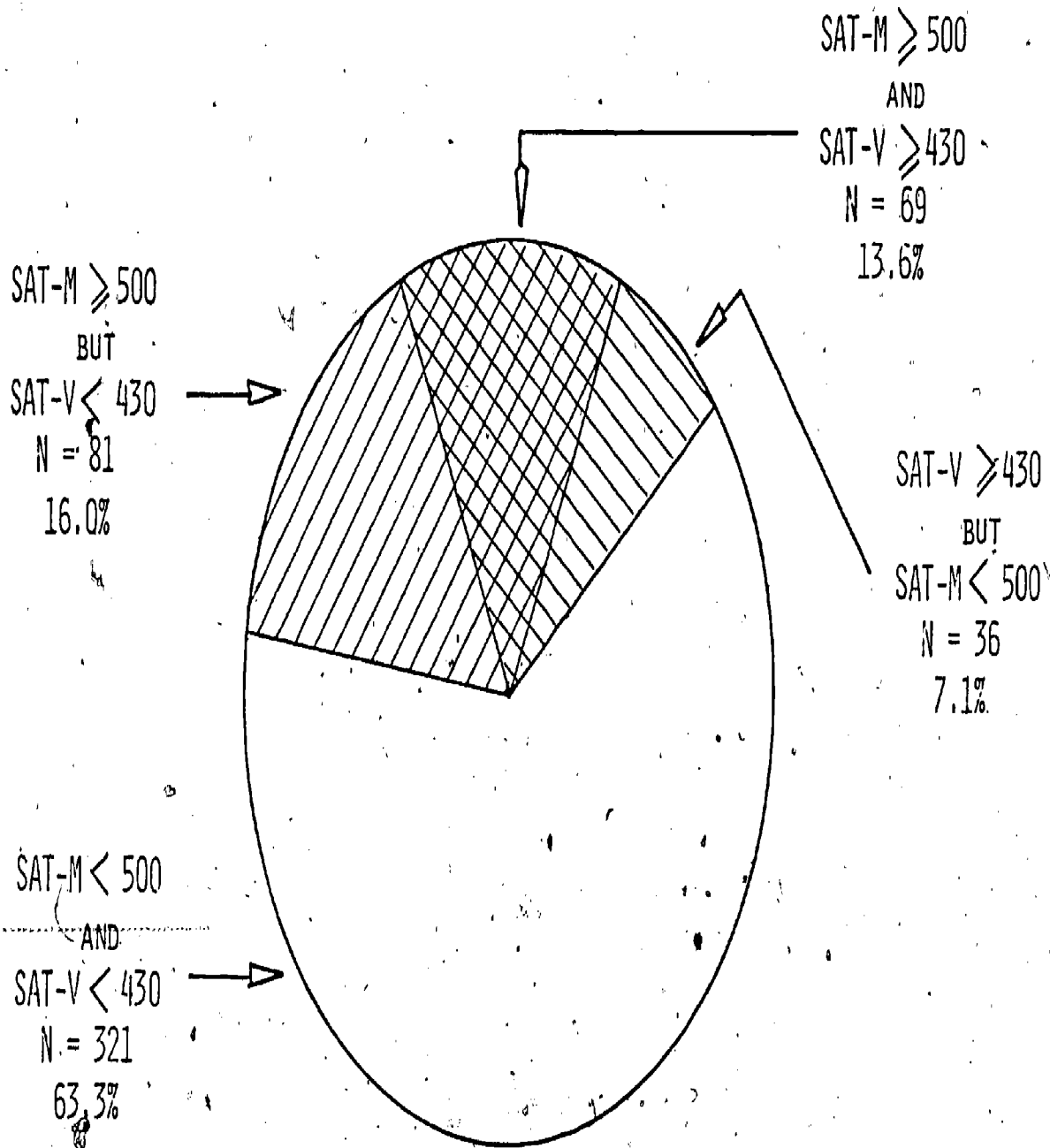


FIG. 3: MALES IN THE 1976 TALENT SEARCH WHO AS 7TH GRADERS (OR UNDER-AGE 8TH GRADERS) SCORED AS WELL AS OR BETTER THAN THE AVERAGE COLLEGE-BOUND 11TH AND 12TH GRADE MALE ON SAT-M AND AS WELL AS OR BETTER THAN THE AVERAGE COLLEGE-BOUND 11TH AND 12TH GRADER IN GENERAL ON SAT-V;

1976 TALENT SEARCH

GIRLS

N = 366

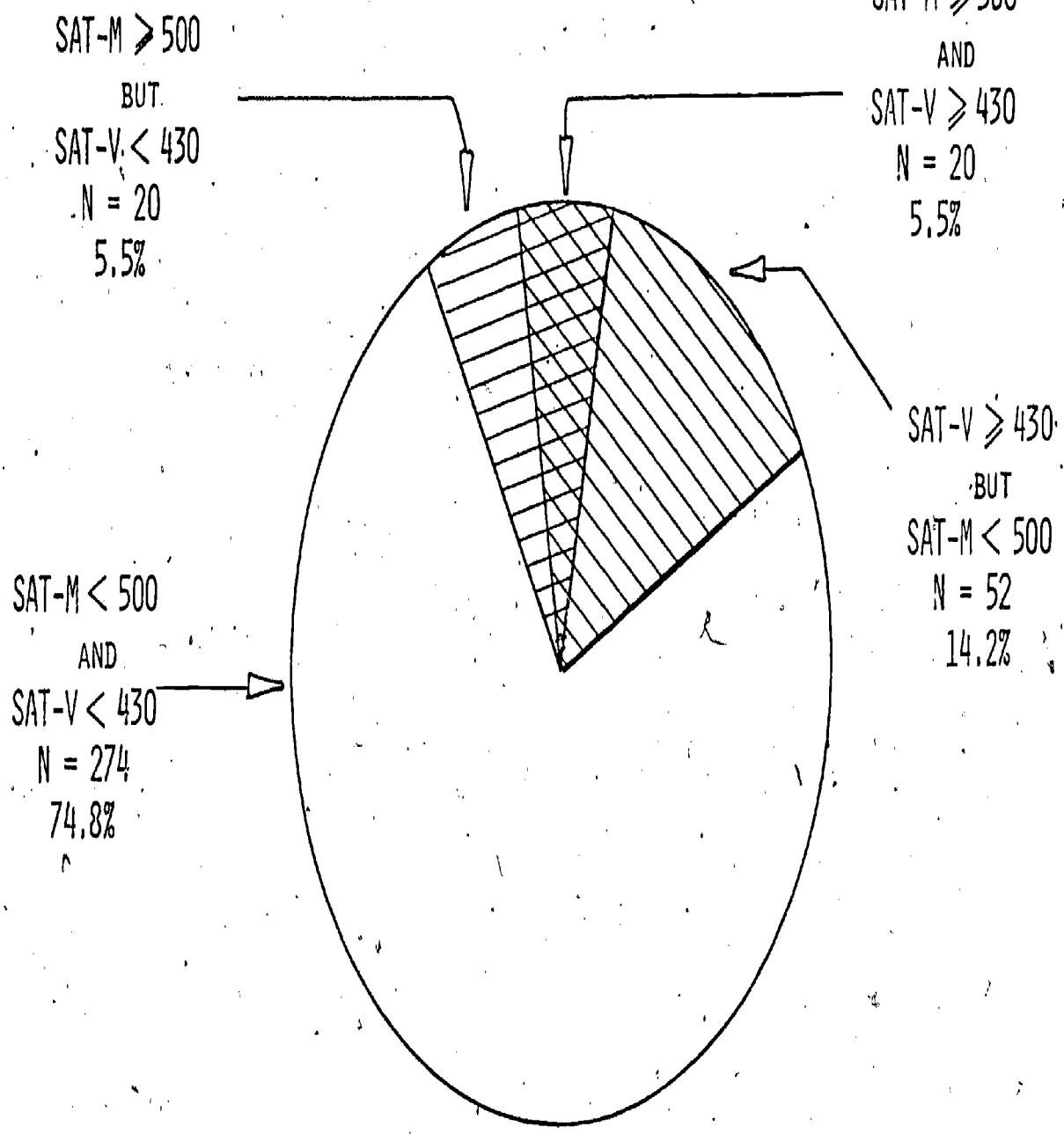


Fig. 4: FEMALES IN THE 1976 TALENT SEARCH WHO AS 7TH GRADERS (OR UNDER-AGE 8TH GRADERS) SCORED AS WELL AS OR BETTER THAN THE AVERAGE COLLEGE-BOUND 11TH AND 12TH GRADE MALE ON SAT-M AND AS WELL AS OR BETTER THAN THE AVERAGE COLLEGE-BOUND 11TH AND 12TH GRADER IN GENERAL ON SAT-V. (THE MALE NORMS FOR SAT-M WERE USED BECAUSE THEY ARE MORE STRINGENT.)