



Which cognitive factors are necessary for the manifestation of mathematical talent? A necessary condition analysis approach

Natalia Colino ^a, Alejandro Maiche ^a and Mirko Antino ^b

^aFaculty of Psychology, Universidad de la República, Montevideo, Uruguay; ^bFaculty of Psychology, Universidad Complutense de Madrid, Madrid, España

ABSTRACT

In a science, technology, engineering, and mathematics (STEM)-focused world, understanding and nurturing mathematical talent are crucial for fostering innovation and meeting future demands. This study aims to deepen the understanding of mathematical talent by identifying the cognitive factors essential for its manifestation. The study involved an initial screening of 673 high school students aged 12–19 years from an Uruguayan educational institution. Of these, 76 were identified as gifted based on cognitive aptitude and creativity assessments. To pinpoint those with mathematical talent, these students subsequently undertook a Mathematics Olympiad test and additionally reported their flow state during problem solving. Using a necessary condition analysis (NCA) approach, we identified the critical factors required for the manifestation of mathematical talent. Our findings indicate that creativity is not essential for mathematical talent. However, NCA revealed that moderate levels of abstract reasoning, numerical aptitude, and flow state, along with high levels of spatial ability, are necessary for mathematical talent. These necessary conditions enable the initial emergence of mathematical talent and serve as critical foundations for its subsequent development. These findings highlight the need for evidence-based educational policies and strategies to support mathematically talented students and ensure their potential contribution to a stronger and more inclusive STEM workforce.

ARTICLE HISTORY

Received 11 August 2024
Accepted 1 October 2025

KEYWORDS

Giftedness; mathematical talent; cognition; spatial ability; creativity; flow state

Introduction

We live in a world increasingly shaped by advancements in science, technology, engineering, and mathematics (STEM). As a result, the demand for professionals with expertise in these areas continues to grow, driven by labor market trends and policy initiatives aimed at strengthening STEM education (Freeman et al., 2019). Identifying and nurturing young talent with high potential in STEM is essential for sustaining innovation and addressing future challenges.

CONTACT Natalia Colino  nataliacolinocouto@gmail.com  Faculty of Psychology, Universidad de la República, Dr. Tristán Narvaja 1674, Montevideo CP:11200, Uruguay

 Supplemental data for this article can be accessed online at <https://doi.org/10.1080/13598139.2025.2570654>

© 2025 European Council for High Ability

Research from the Study of Mathematically Precocious Youth (Benbow, 2012) demonstrates that early identification of mathematically gifted individuals combined with tailored educational interventions significantly enhances the likelihood of success in STEM-related academic and professional pathways. Csikszentmihalyi et al. (1993) highlight that a lack of adequate support, especially during adolescence, can result in early disengagement from mathematics and, likely, a long-term exit from the domain.

Mathematical talent offers a unique lens for exploring giftedness, bridging the gap between initial potential and its transformation into domain-specific expertise (Gagné, 2020). This perspective emphasizes the importance of defining giftedness within specific domains, such as mathematics, language, and sports, by observing individuals' natural behaviors and contexts to identify their aptitudes (Dai, 2018). However, as Assouline and Lupkowski-Shoplik (2021) pointed out, many educators lack professional training for identifying and effectively teaching mathematically talented students. Standard curricula often fail to provide students with sufficient challenges, highlighting the urgent need to leverage their strengths early and support their rapid cognitive development. Against this backdrop, a crucial question arises: What cognitive predispositions are necessary for the emergence of mathematical talent and how can they be cultivated?

Research on giftedness and talent is relatively new and remains largely confined to developed countries (Hernández-Torrano & Kuzhabekova, 2020). Within the mathematics domain, the number of studies is even smaller. Although some studies have focused on superior mathematics performance, many do not address giftedness and talent. Colino and Maiche (2022) systematized recent empirical studies on mathematical giftedness and talent within the cognitive dimension and concluded that these individuals typically possess strong working memory and enhanced visuospatial skills. Other studies linked to mathematical giftedness and talent have examined creativity (Haavold et al., 2020; Leikin, 2021; Roldán & Ferrando, 2021) and motivation (Knopik & Oszwa, 2023; Paz-Baruch & Hazema, 2023). However, most research in this area has focused on the predictors of mathematical talent rather than examining whether these factors are essential for its emergence and development. Addressing this gap is crucial as it limits our ability to effectively identify and support mathematically talented individuals.

To bridge this gap, our study was grounded in the theoretical frameworks of Gagné (1985, 2020) and Renzulli (1978, 2020), who provided complementary perspectives on giftedness and talent dynamics. In this regard, we structure the theoretical framework into three sections. The first explores general giftedness, tracing its conceptual evolution and analyzing Renzulli's Three-Ring Conception of Giftedness, which highlights the interplay between above-average ability, task commitment (a form of motivation), and creativity as essential components of gifted behavior. The second section focuses on mathematical giftedness and talent, beginning with Gagné's Differential Model of Giftedness and Talent, which emphasizes the transformation of exceptional potential into domain-specific talent through practice and learning. Following this theoretical distinction, we reviewed the key cognitive factors (e.g. spatial ability) identified in prior research on mathematical giftedness as essential for mathematical talent. The third and final section addresses the identification of students with mathematical talent, drawing on both an inclusive perspective – based on Renzulli's model of giftedness – and a domain-specific approach developed by Stanley and colleagues (Stanley & Brody, 2001). By integrating these frameworks, our study specifically investigates whether the

three-ring components of gifted behavior – above-average ability, creativity, and task commitment – are not only associated with mathematical talent but also serve as necessary conditions for its emergence and subsequent development. Moreover, this study seeks to determine the minimum level of each factor required for the manifestation of mathematical talent.

To achieve this, we employed the necessary condition analysis (NCA; Dul, 2016) approach, a novel methodology that goes beyond correlation and prediction, to identify the essential cognitive conditions underpinning mathematical talent. Our study began with a sample of 673 students, 76 of whom were identified as gifted, based on cognitive aptitude and creativity assessments. Following the perspective of Subotnik et al. (1996), who regarded achievement in Mathematics Olympiads as a benchmark for gifted students, the participants completed a Mathematics Olympiad test to identify those with mathematical talent. In addition, they reported their state of flow (as a measure of task commitment) after engaging in problem-solving tasks. Using NCA, we identified specific variables and threshold levels that constitute the essential conditions for the manifestation of mathematical talent.

This study aims to provide actionable insights for educators and policymakers by situating its findings within the broader context of STEM education and talent development. Understanding the conditions necessary for mathematical talent will enable more effective early identification and intervention strategies that support the academic and career development of gifted students. Given the strong link between mathematical abilities and success in STEM fields (Lubinski & Benbow, 2006; Wai et al., 2009), ensuring adequate educational support for mathematically talented individuals is crucial. Without targeted interventions, many high-ability students are insufficiently challenged, which can lead to disengagement and limit the full realization of their potential, ultimately resulting in a loss of prospective contributions to the STEM talent pool (Subotnik et al., 2011). Moreover, aligning talent development initiatives with global STEM education policies (Marginson et al., 2013) helps create inclusive pathways for students from diverse backgrounds. By helping identify and nurture these cognitive abilities early on, this study supports pathways that may facilitate students' future engagement in STEM fields.

Theoretical background

General giftedness

The concept of giftedness has evolved significantly since the early 20th century (Dai, 2018). Initially seen as an innate and static high intellectual capacity, contemporary models, such as those by Gardner (2003), Renzulli (1978, 2020), and Sternberg (1995, 2024), now understand giftedness as a complex and multifaceted phenomenon, such that it is no longer considered a fixed attribute but rather raw material of talent (Tourón, 2020).

Renzulli's Three-Ring Conception of Giftedness (Renzulli, 1978) defines gifted behavior as the intersection of three traits: above-average ability, task commitment, and creativity.

Above-average ability refers to individuals in the top 15–20% in terms of potential performance or achievement. This includes both general abilities (e.g. abstract reasoning and spatial ability) and specific abilities (e.g. leadership and photo-manipulation skills) (Renzulli & Reis, 2021). In the present study, we focused on three general abilities: spatial ability (the capacity to visualize, recall, and mentally transform images in two or three dimensions), numerical aptitude (the ability to reason inductively or deductively using mathematical concepts and relations), and abstract reasoning (the ability to solve novel problems by deducing the logical rules that underlie abstract patterns).

Task commitment describes focused motivation for a specific task, encompassing perseverance, hard work, dedication, and self-confidence (Renzulli & Reis, 2021). Deci and Ryan (1985) highlighted the natural capacity of some individuals to remain continuously focused on a particular aspect unless interrupted. Csikszentmihalyi (1996) introduced the concept of flow experience, which from our perspective, aligns closely with the state of consciousness that gifted individuals attain when they engage in activities that they are passionate about and excel in. Unlike Renzulli's (1978) traits of involvement and concentration, this complex state of consciousness is also characterized by a third dimension: enjoyment (Csikszentmihalyi, 2014). In our study, we measured these three flow components, dividing them into nine dimensions: merging actions and awareness, clear goals, concentration on the task at hand, unambiguous feedback, challenge-skill balance, transformation of time, sense of control, loss of self-consciousness, and autotelic experience.

Creativity can be observed in all fields of knowledge. However, the debate over whether creativity is a domain-specific or general trait remains unresolved. Acar et al. (2024) advocate for a moderate stance, as current research has not provided definitive conclusions. The relationship between creativity and cognition is another ongoing debate. Some authors argue that creativity has a cognitive basis. According to Guilford (1950, 1967), certain abilities span the creative process and can serve as indicators of creativity. In our study, we focused on three of them: fluency (the ability to generate a wide range of ideas), flexibility (the capacity to shift perspectives and explore alternative approaches), and originality (the production of statistically uncommon responses).

Three rings – above-average ability, task commitment (flow state), and creativity – are critical variables in our study. We examine whether they are necessary conditions for mathematical talent.

Mathematical giftedness and talent

Giftedness and talent are often used interchangeably, but Gagné (2020) differentiated them based on three assumptions: latent qualities, that is potentialities, are observable and measurable, unevenly distributed in the population, and transform into competencies through learning activities. In his Differential Model of Giftedness and Talent, Gagné (1985, 2020) suggests using the term giftedness exclusively for the presence of exceptional potential in a specific domain, while reserving the term talent for notable achievements in a particular field. He not only differentiates between the two, but also establishes a causal relationship, presenting talent as the gradual transformation of high natural abilities (giftedness) into high-level occupational skills through systematic learning, practice, and application in a specific field (Gagné, 2019). Considering a normal distribution of

aptitude and competency measures, the author defined the 90th percentile (top 10%) as the minimum threshold for prevalence in both gifted and talented populations.

The concept of mathematical giftedness has matured in recent years and is still being explored (Leikin, 2021). Mathematical talent is observed through high performance in problem-solving and corresponds to developed mathematical potential although it is also associated with creativity (Leikin, 2019).

Pioneering work by Krutetskii (1976) identified common traits in mathematically gifted and talented individuals such as precision, rapid generalization of mathematical concepts, curiosity, mental flexibility, and persistence. Krutetskii found that these individuals process and retain information more effectively (Kang, 2015). He concluded that high mathematical ability depends on the capacity for logical thinking in spatial, quantitative, and symbolic relationships. This framework guided the selection of the cognitive variables constituting the ability ring in our study: numerical aptitude, abstract reasoning, and spatial ability.

The Study of Mathematically Precocious Youth (SMPY), founded by Julian C. Stanley in 1971, has also significantly contributed to our understanding of mathematical giftedness and talent. Benbow (2012) and Lubinski and Benbow (2006) emphasized the importance of mathematical reasoning – conceptually related to the numerical aptitude assessed in our study – and spatial ability as a key predictor of long-term achievement and success in STEM fields. As early as 1987, they noted that mathematical reasoning was crucial to the performance of the U.S. Olympic team. Other studies have linked numerical aptitude to mathematical giftedness and talent. For example, a meta-analysis by Ridwan et al. (2023) revealed that numerical aptitude significantly improves learning outcomes and mathematical ability. Additionally, Kurnaz (2018) identified mathematical reasoning, spatial thinking, and problem-solving as significant predictors of math achievement in gifted students.

Spatial ability is another well-documented indicator of mathematical talent. Contemporary analyses emphasize that spatial ability complements mathematical reasoning and is essential for the abstract thinking required at advanced levels of mathematics (Uttal et al., 2013). In a systematic review, Mix and Cheng (2012) highlighted the role of pure spatial abilities, such as mental rotation, visuospatial working memory, and perspective taking, even when not embedded in mathematical contexts. Despite their apparent independence from mathematical content, these abilities are strongly associated with mathematical learning. Spatial skills during adolescence have also been found to predict later achievement in STEM fields (Wai et al., 2009). More recently, systematic reviews by Sipahi and Bahar (2024) and Colino and Maiche (2022) confirmed that mathematically gifted students consistently outperform their peers in spatial tasks.

Leikin et al. (2017) examined abstract reasoning in students with exceptional extra-curricular mathematical achievements (e.g. Math Olympiads). They found that these students performed better in working memory and pattern recognition tasks, which are components of abstract reasoning. These results could also explain their creativity in problem-solving. Based on these findings, the researchers distinguished mathematical giftedness and talent from general giftedness combined with curricular excellence in mathematics. Other studies have linked abstract reasoning to mathematical talent and giftedness. Green et al. (2017) found that fluid reasoning was the only significant predictor of future math achievement. Further supporting the idea that this cognitive

skill is a defining feature of mathematical giftedness are the findings of Hansen et al. (2022). A systematic review by Sipahi and Bahar (2024) supports this idea, reporting that mathematically gifted individuals significantly outperformed both art prodigies and control groups on fluid reasoning tasks (Ruthsatz et al., 2014; Zhang et al., 2014). Pattern recognition has also been linked to mathematical giftedness and talent in some empirical studies (Ramírez Uclés et al., 2018; Yan Kong & Benny, 2018).

Studying individuals with exceptional problem-solving skills has significantly contributed to the understanding of mathematical giftedness and talent. We believe that mathematical talent requires a foundation of outstanding cognitive aptitudes (giftedness), which enhance the resolution of complex problems involving strategy and creativity. Based on the main findings on mathematical giftedness and talent in the cognitive dimension, the variables that constituted the ability ring in our study were numerical aptitude, abstract reasoning, and spatial ability.

In line with Renzulli's Three-Ring Model, we considered creativity and task commitment as essential dimensions of giftedness and mathematical talent, alongside the ability ring. Creativity has been widely studied within the mathematical domain, and the notion of mathematical creativity has even contributed to the conceptual development of mathematical giftedness and talent (Haavold et al., 2020; Sriraman, 2005). Some authors have examined the connections between creativity and mathematical talent by comparing mathematically talented students with university-level mathematics experts. Roldán and Ferrando (2021) found that mathematically talented students outperformed university-level experts in problem solving, demonstrating greater flexibility in their strategies, generating more solutions, and producing more original responses. Similarly, Elgrably and Leikin (2021) demonstrated that international Mathematical Olympiad participants exhibited higher creative performance and more successful proving processes than university mathematics majors. However, the nature of this relationship is complex and still needs to be explored (Leikin, 2021). Motivational factors, such as flow experienced during problem-solving as a manifestation of task commitment, have also been associated with mathematical talent (Csikszentmihalyi et al., 1993; Knopik & Oszwa, 2023; Paz-Baruch & Hazema, 2023).

These core dimensions – ability, creativity, and task commitment – are consistently reported in the literature as indicators of mathematical talent and support the validity of our study design.

Identifying gifted students with mathematical talent

Our study examined gifted students with mathematical talent to understand the aptitudes that enable high achievement in this domain. This helped us identify these conditions early on in other individuals, thereby enhancing and promoting the natural development of talent. It also allowed us to continue nurturing these factors among those who already demonstrated mathematical talent.

Renzulli's Three-Ring Conception of Giftedness highlights the importance of considering three core dimensions in the identification process: ability, creativity, and task commitment. In addition, Renzulli Identification System for Gifted Programming Services (RIS/GPS), emphasizes the value of combining quantitative and qualitative measures from multiple sources. This inclusive and flexible framework aims to identify

approximately 15% of the student population who show high potential in academic and/or creative – productive domains (Renzulli & Gaesser, 2015). Consistent with this approach, our screening phase integrated teacher nominations (based on the three core dimensions) alongside standardized assessments of cognitive aptitudes and creativity in the second phase. This model guided both the selection of areas examined in our study and the structure of our initial identification procedure.

However, identifying students with exceptional mathematical talent requires a more domain-specific and rigorous strategy. To address this, we adopted a third assessment phase inspired by the Talent Search model developed by Julian Stanley and colleagues in the late 1960s, initially focused on identifying seventh- and eighth-grade students with exceptional mathematical reasoning (Stanley & Brody, 2001). Its effectiveness lies in the use of above grade-level assessments to capture advanced potential.

While the original model used the Scholastic Assessment Test (SAT), we used complex, unconventional problems from Math Olympiads. These problems, which were drawn from high-level competitions, exceeded grade-level expectations and avoided ceiling effects. This allowed for a more accurate measurement of mathematical competence. At the same time, they included a range of entry points to ensure an adequate floor, prevent frustration, and maintain engagement. These problems spark intellectual curiosity and develop robust mathematical thinking by addressing unfamiliar problems (Falk, 2001). Andreescu and Gelca (2008) note that, unlike typical exercises, Olympiad problems offer no immediate clues on where to start, requiring perseverance, inspiration, and mental effort to break down complexity into simpler steps. These problems capture the essence of mathematical creation, which require immersion in a problem, exploring approaches, and connecting seemingly unrelated areas of mathematics, leading to unexpected solutions and new perspectives.

In summary, Math Olympiads present unconventional challenges requiring creative problem-solving, original thinking, and the application of techniques in unknown contexts, which are characteristic skills of mathematically talented students (Zubova et al., 2021). Therefore, Olympiads and math competitions play crucial roles in detecting and developing mathematical talent (Leikin, 2019).

Furthermore, empirical studies have shown that Olympiad performance reveals cognitive traits that are not captured by IQ or school achievement scores, suggesting it should be considered a distinct and valid criterion for identifying giftedness (Boran et al., 2015). Since the second half of the 20th century, national Olympiads in Russia have functioned as an efficient system for identifying cognitively gifted children, forming part of the country's talent development infrastructure (Liashenko et al., 2017). Participation in Olympiads also enables the identification of mathematically gifted students who may not yet have distinguished themselves in conventional classroom settings, whether due to personal traits or contextual barriers (Zubova et al., 2021). The early detection of such students, even at the primary level, has been shown to facilitate their long-term mathematical development. This approach aligns with previous research employing Olympiad-style tasks (e.g. Wagner & Zimmermann, 1986) or selecting participants based on outstanding performance in such competitions, including members of national teams (Leikin et al., 2017) to identify mathematically gifted students. These studies confirm the utility of Olympiad problems not only for developing mathematical talent but also for detecting it effectively.

Based on this framework, this study aimed to determine whether cognitive aptitudes such as numerical aptitude, abstract reasoning, and spatial ability, as well as creativity and flow state experienced while solving mathematical problems, are necessary conditions for students who demonstrate mathematical talent, through Mathematics Olympiad test, compared with those gifted in other areas.

Materials and methods

Participants

To achieve the goal of this study, a convenience sample was used because of the difficulty in accessing gifted individuals identified in Uruguay. Initially, 673 high school students from an educational institution who were pioneers in working with gifted students from an inclusive education perspective participated in the research. From this group, 76 students were identified as gifted through cognitive aptitude and creativity tests and selected for the main study. The final sample included 36 girls (ages 12.3 to 17.8 years, $M = 15.4$, $SD = 1.6$) and 40 boys (ages 12.1 to 19.0 years, $M = 15.3$, $SD = 2.0$). All participants took a Mathematics Olympiad test, and immediately after completing it, they reported their flow state during the problem-solving process.

This study was part of the High Abilities Project at the participating institution and involved students from all grades of secondary school. For this reason, rather than restricting the sample to narrower cohorts, we included the entire age range of students identified as gifted in order to maximize the number of participants with exceptional mathematical talent. This age range aligns with international practices, such as the International Mathematical Olympiad (IMO), in which pre-university students under the age of 20 compete on the same level and solve the same mathematical problems.

Sampling and procedure

The study was conducted in three consecutive phases as part of the High Abilities Project at the participating educational institution: initial screening, cognitive-creative profile assessment, and measurement of mathematical talent and flow state during problem-solving. Consent was obtained from all families, and the students agreed to participate in three small-group meetings with the lead researcher and a team of psychologists and psycho-pedagogues. Individual results were later shared with participants and their family.

Initially, Raven's Advanced Progressive Matrices Test Series II (RAPM; Raven, 2006) was administered to 673 high school students in small groups. Teachers also completed a qualitative group screening questionnaire that included a checklist of high abilities/giftedness indicators (Pérez & Bendelman, 2018). Students were selected for further assessment if they scored at or above the 90th percentile on the RAPM or were nominated by at least two teachers on 12 or more of the 24 checklist items. This process identified 88 students as potentially gifted.

These students then underwent a cognitive-creative assessment. Cognitive aptitude was measured using the (Técnicos Especialistas Asociados (TEA)) Aptitude Battery (BAT-7; Arribas et al., 2013), and creativity was measured with the Creative Imagination Test-Youth Version (Prueba de Imaginación Creativa para

Jóvenes (PIC-J); Artola et al. (2008). This second phase confirmed 76 students as gifted, demonstrating high levels of cognitive aptitudes and/or dimensions of creativity.¹

In the final phase, these participants attempted a Mathematics Olympiad test and subsequently completed the Activity Flow State Scale (AFSS; Payne et al., 2011).

Measures

- (1) Raven's Advanced Progressive Matrices Test Series II (RAPM; Raven, 2006). A nonverbal screening test, administered within a 45-minute time limit that measures general intelligence.
- (2) Checklist of indicators for High Abilities/Giftedness (Pérez & Bendelman, 2018).
- (3) TEA Aptitude Battery (BAT-7; Arribas et al., 2013). The complete battery was administered over approximately two hours, in line with the institution's identification protocol. For the purposes of this study, only the scores for spatial ability, numerical aptitude, and abstract reasoning were used.
- (4) Creative Imagination Test-Youth Version (PIC-J; Artola et al., 2008). The complete test was administered over approximately 45 minutes. For the purposes of this study, we used the scores for fluency, flexibility, and originality mediated by written language. A global creativity score was calculated by summing these three subscales.
- (5) Mathematics Olympiad test. Completed without calculators or materials within two hours. The test comprised three problems of increasing difficulty: one puzzle-inspired problem, one from a regional Olympiad, and one from an International Olympiad.
- (6) Activity Flow State Scale (AFSS; Payne et al., 2011). Completed within five minutes, assessed nine dimensions of flow on a 5-point Likert scale: merging actions and awareness, clear goals, concentration on the task at hand, unambiguous feedback, challenge-skill balance, transformation of time, sense of control, loss of self-consciousness, and autotelic experience.

Operationalization of variables

The dependent variable was mathematical talent, operationalized categorically based on test performance. A jury of former university Math Olympians scored the Mathematics Olympiad test on a 30-point scale, divided into six performance categories. Students who scored in categories 5 or 6 were classified as demonstrating mathematical talent. For the NCA, this cutoff was used to determine the minimum level of each necessary condition required to reach at least category 5 of mathematical talent.

The independent variables were derived from the second and third phases. Numerical aptitude, abstract reasoning, spatial ability, and global creativity (sum of fluency, flexibility, and originality scores) were obtained from the second phase and expressed in percentile ranks. Flow state was measured in the third phase and included as a direct score (sum of the AFSS items).

Data analysis

To achieve the goal of this research, we employed NCA (Dul, 2016) using the NCA package version 3.3.3 in R version 4.3.1. As NCA is a relatively new statistical method, we outline its main features.

NCA identifies necessary but not sufficient conditions that determine the presence of a particular variable in a dataset. Unlike traditional correlational regression analyses that seek causal relationships, NCA focuses on identifying the prerequisites for a phenomenon to occur. Dul (2016) emphasized that NCA can also determine the minimum level of the necessary condition required for a certain result level. In our study, NCA helped us identify the levels of cognitive aptitudes, creativity, and flow necessary to achieve high performance in mathematics (achieving Categories 5 or 6 in the Mathematics Olympiad test). The analysis included the previously described independent variables expressed as percentiles (cognitive aptitudes, creativity) or direct scores (flow).

Graphically, NCA results are represented by drawing a ceiling line on a scatter plot, which delineates the area with observations from the area without observations. NCA focuses on the empty zone – located in the upper left corner above the ceiling line – to identify necessary conditions. This area represents the effect size (d), which ranges from 0 to 1. A larger value indicates a stronger necessary condition. Dul (2016) classifies effect sizes as small if $d < 0.1$, medium if $0.1 \leq d < 0.3$, large if $0.3 \leq d < 0.5$, and very large if $d \geq 0.5$. An effect size threshold of $d = 0.1$ is suggested for determining if a variable qualifies as a necessary condition. We also report the accuracy of the ceiling line, which is the percentage of observations that lie on or below it relative to the total number of observations. Dul (2016) recommends 95% accuracy as the desired benchmark; below this percentage, the interpretation of the ceiling line would not be appropriate.

NCA provides more actionable insights than simple linear models for identifying high mathematical abilities because it specifically identifies the minimum requirements necessary for exceptional mathematical performance. While linear models focus on average effects and correlations, NCA identifies ceiling lines and bottleneck factors that must be met to enable high achievement (Dul, 2016). This makes NCA particularly valuable to educational institutions, policymakers, and other stakeholders because it establishes clear thresholds that can be applied to admissions, curriculum design, and talent development programs. For example, if NCA identifies certain cognitive or creative skills as necessary prerequisites, schools can ensure that these skills are assessed and developed before allowing students to progress to advanced mathematical programs. This bottleneck-oriented approach offers practical advantages over traditional linear analyses as it helps institutions focus their resources on identifying and developing these essential conditions rather than spreading efforts across all potentially contributing factors. In addition, the logic of necessary conditions is consistent with the reality of talent development, where the absence of certain critical factors cannot be compensated for by the presence of others, regardless of their strength.

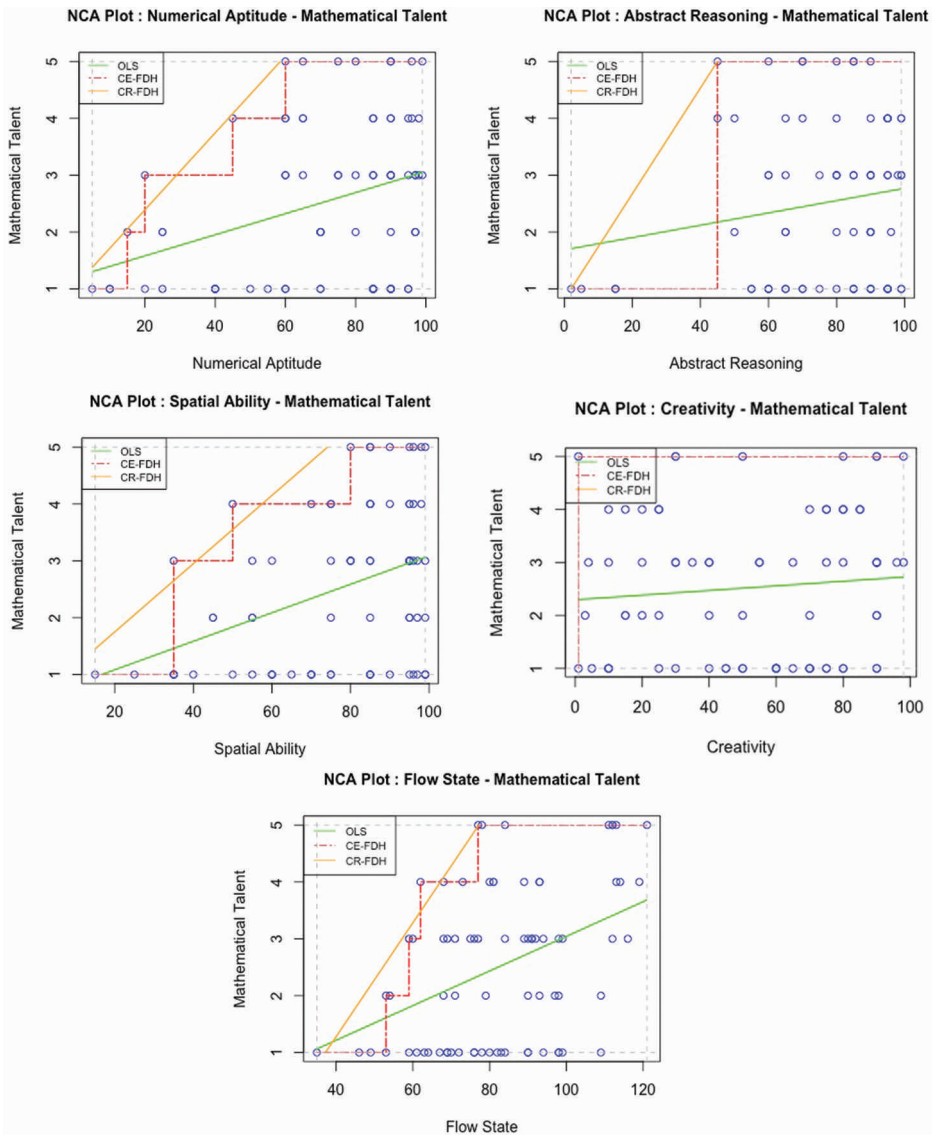


Figure 1. NCA scatter plots of mathematical talent for each independent variable. Mathematical talent is categorized according to the sample procedure. Numerical aptitude, abstract reasoning, spatial ability and creativity are presented by percentiles, while flow state is presented using direct scores. Green lines indicate the OLS (ordinary least squares) regression line, broken red lines represent the CE-FDH (ceiling envelopment - free disposal hull), and orange lines represent the CR-FDH (ceiling regression - free disposal hull). Note: although the mathematical talent scale ranges from 1 to 6, this sample does not include any students in the highest category.

Results

Scatter plots of mathematical talent for each independent variable (cognitive aptitudes, creativity, and flow state) were generated with the ceiling line (see Figure 1). In addition,

effect sizes (CR-FDH; Dul, 2016) and the accuracy of the ceiling line were calculated for each case.

In Figure 1, we observe that both cognitive aptitudes and flow state show a pattern where increases in these variables correspond to increases in mathematical talent, leaving an almost empty space in the upper left corner of the scatterplot. According to Dul (2016), this indicates a necessary condition. However, this pattern is not seen in the NCA plot of mathematical talent in relation to creativity.

Next, we analyze the effect sizes of these independent variables on mathematical talent. For variables whose effect on mathematical talent exceeds Dul’s (2016) threshold of 0.1, we will report the minimum value required to ensure the presence of talent and the accuracy.

Numerical aptitude has an effect size of $d = 0.258$ (CR-FDH), which is considered medium and close to the threshold for a large effect. A minimum score at or above the 60th percentile in numerical aptitude is a necessary condition for the presence of mathematical talent, with an accuracy of 98.6%.

Abstract reasoning is also a necessary condition for mathematical talent, with a medium effect size of $d = 0.222$ (CR-FDH). Being at least in the median percentile in this aptitude is necessary to achieve mathematical talent, with 100% accuracy.

The strongest relationship is observed between *spatial ability* and mathematical talent. Spatial ability not only acts as a necessary condition for the presence of mathematical talent but also has a large effect size of $d = 0.313$ (CR-FDH) on it, with the 80th percentile being the minimum necessary. The accuracy is 97.3%.

In contrast, the NCA plot of mathematical talent in relation to *creativity* shows no necessary condition, and the effect size is null ($d = 0.000$, CR-FDH). Looking at the effect

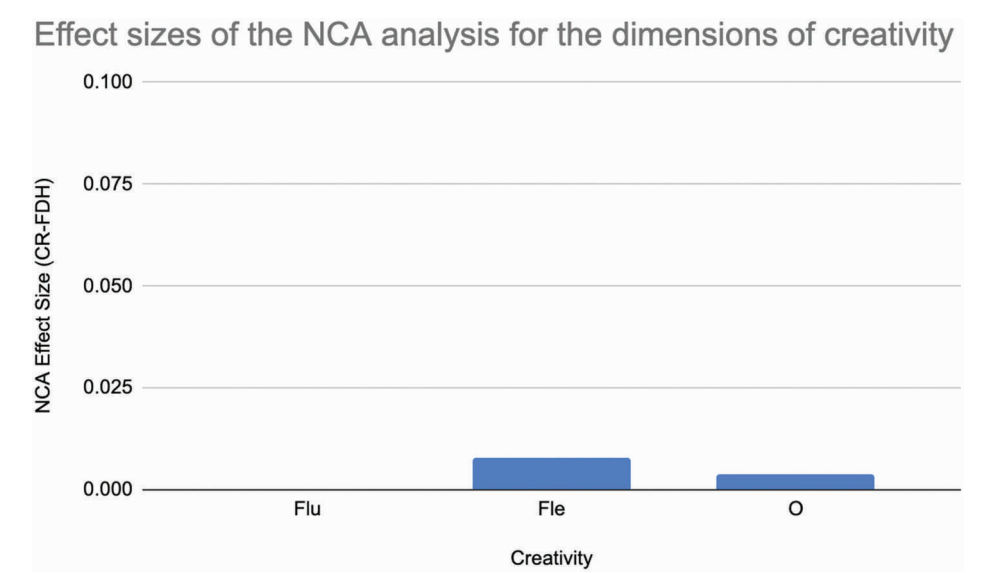


Figure 2. Effect sizes (CR-FDH) of the NCA analysis for the dimensions of creativity. Note: flu = fluency; fle = flexibility; O = originality.

Effect sizes of the NCA analysis for the flow state dimensions

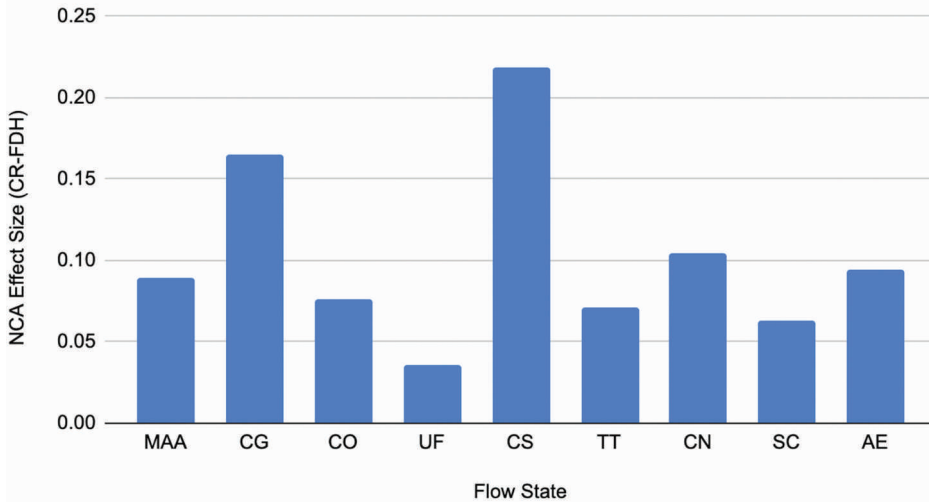


Figure 3. Effect sizes (CR-FDH) of the NCA analysis for the flow state dimensions. Note: MAA = merging actions and awareness; CG = clear goals; CO = concentration on task at hand; UF = unambiguous feedback; CS = challenge skill balance; TT = transformation of time; CN = sense of control; SC = loss of self-consciousness; AE = autotelic experience.

sizes of the dimensions of creativity (see Figure 2), they are all below the 0.1 threshold for being considered necessary conditions.

Regarding *flow state*, the NCA analysis indicates it as a necessary condition for mathematical talent with a medium effect size of $d = 0.258$ (CR-FDH), requiring an average score in the test applied. The accuracy of the ceiling line is 96.1%. Observing the effect sizes of the dimensions of flow state (see Figure 3), three dimensions are necessary conditions ($d \geq 0.1$) for mathematical talent with a medium effect size: sense of control with $d = 0.104$ (CR-FDH) and 96.1% accuracy, clear goals with $d = 0.165$ (CR-FDH) and 97.4% accuracy and challenge-skill balance with $d = 0.219$ (CR-FDH) and 96.1% accuracy.

Discussion and conclusions

This study makes a significant contribution to the field by examining three key components of gifted behavior – ability, creativity, and task commitment – as necessary conditions for the manifestation of mathematical talent, based on Renzulli's (1978) Three-Ring Conception of Giftedness. By applying a novel methodological approach, NCA, we shift the focus from correlates or predictors to the identification of indispensable cognitive requirements that must be present for talent to develop. Unlike traditional regression methods, NCA reveals the minimal conditions without which mathematical talent cannot manifest, while also recognizing that such necessary conditions are not by themselves sufficient to guarantee its emergence.

Our findings are based on a highly specific and underexplored population: gifted students with exceptional mathematical talent identified through Mathematical

Olympiad problems. In this way, we build upon prior research (e.g. Leikin et al., 2017) by offering empirical evidence that informs about this distinctive cognitive profile. While earlier studies, including those discussed in the theoretical framework, have associated creativity, task commitment, and cognitive aptitudes with mathematical giftedness, this study is the first attempt to empirically establish which of these factors are structurally necessary for talent to emerge.

Building on Krutetskii's (1976) work on problem-solving abilities, we identified numerical aptitude, abstract reasoning, and spatial ability as essential cognitive prerequisites within the ability ring. Specifically, our results indicate that moderate levels of numerical aptitude and abstract reasoning, along with high levels of spatial ability, are necessary for the manifestation of mathematical talent. This finding refines existing literature (e.g. Leikin et al., 2017; Lubinski & Benbow, 2006) by demonstrating that these abilities are not merely correlated with talent but are foundational to its emergence and development.

Task commitment, the second component of Renzulli's model, was assessed through the experience of flow during problem solving. Our findings show that a moderate level of flow state is a necessary condition, particularly the dimensions of sense of control, clear goals, and challenge-skill balance. Interestingly, enjoyment, often considered central to flow, did not emerge as a required factor. This aligns with observations by Csikszentmihalyi et al. (1993), who noted that mathematically gifted students often remain committed due to extrinsic motivators – such as the perceived utility or social value of mathematics – rather than intrinsic rewards.

In contrast, our results indicate that general creativity and its dimensions, as assessed in this study, do not constitute the necessary conditions for mathematical talent. This raises important questions about the tools commonly used for identification. Our study employed a general creativity test, aligning with institutional identification protocols used to identify gifted students across multiple areas as well as with research emphasizing the influence of general creativity on domain-specific creativity (Hong & Milgram, 2010) and its role in creative mathematical thinking (Schoevers et al., 2020). While general creativity tests offer advantages such as standardization, broad applicability, and the ability to assess transferable skills (e.g. fluency, originality, and flexibility) across various domains of knowledge, domain-specific tools, such as those developed by Leikin et al. (2017), may offer greater precision for evaluating creativity in problem-solving contexts.

These findings enhance our understanding of mathematical talent by identifying its fundamental cognitive requirements, thereby paving the way for the development of more comprehensive theoretical and practical models. Additionally, the findings provide actionable insights for enhancing the identification, support, and development of mathematically talented individuals in educational contexts, with the broader aim of fostering long-term engagement in STEM fields.

Educational and policy implications

The findings of this study provide valuable insights for educators, policymakers, and stakeholders, to improve the identification and development of mathematical talent in schools. By establishing numerical aptitude, abstract reasoning, and spatial ability as bottleneck factors – necessary conditions that must be met for mathematical talent to

emerge – educators can refine assessment tools to focus not only on general academic performance but also on these critical cognitive prerequisites. This addresses the identification aspect by ensuring that students who meet the minimum requirements are accurately identified. For example, diagnostic tests should incorporate tasks that assess pattern recognition (linked to abstract reasoning) and mental manipulation of two- and three-dimensional objects (e.g. rotation, folding, or unfolding) based on visuospatial stimuli, rather than relying solely on numerical exercises. This bottleneck-oriented approach can improve early identification of mathematically talented students, particularly those whose abilities are not reflected in traditional academic assessments.

Moreover, this study underscores the importance of interventions that cultivate these bottleneck factors, offering a practical approach for fostering sustained engagement in STEM careers. Educational programs should integrate challenging activities such as spatial visualization exercises, logic puzzles, and advanced problem-solving tasks to strengthen spatial, numerical, and abstract reasoning skills. These activities can be incorporated into regular curricula or offered through extracurricular programs, such as mathematics clubs or Olympiad training. Mathematical Olympiad problems serve as powerful tools for identifying and fostering mathematical talent. Beyond enhancing problem-solving skills, participation in Olympiads also plays a crucial role in shaping students' academic and professional trajectories. As Subotnik et al. (1996) argue, such competition immerses mathematically talented students in a social environment that reinforces their intellectual identity and achievement orientation. Therefore, the Olympiad model offers a robust framework for designing interventions that guide mathematically talented students toward their STEM careers.

However, as highlighted by Assouline and Lupkowski-Shoplik (2021), many educators lack formal training to effectively support mathematically talented students, and traditional curricula often fail to provide the necessary rigor to challenge them. Bridging this gap requires pedagogical strategies that extend beyond the standard instruction. Enrichment programs expose students to advanced mathematical concepts, encouraging exploration beyond the curriculum, whereas acceleration strategies allow them to progress at a pace aligned with their abilities. Open-ended problem solving promotes critical and creative thinking by requiring students to deeply engage in complex mathematical ideas rather than applying rote procedures.

Teachers play a fundamental role in fostering student engagement in mathematics. Teachers who convey enthusiasm for a subject and cultivate intellectual curiosity can significantly influence students' motivation and persistence (Csikszentmihalyi et al., 1993). Just as important as developing their cognitive abilities is encouraging them to experience flow while pursuing their talents, as this state is crucial for sustaining long-term engagement in STEM. In fact, Csikszentmihalyi et al. (1993) point out that experiencing flow is a better predictor of engagement than academic ability, family support, or personality. As a bottleneck factor, its absence can hinder the emergence of mathematical talent. Teachers can facilitate flow by offering autonomy, allowing students to choose problems whose level of challenge is balanced with their abilities, and providing freedom in the methods of resolution. As Csikszentmihalyi et al. (1993) point out, adolescents tend to oscillate between anxiety when challenges are excessive and boredom when they are too easy; therefore, it is essential to adjust tasks to each student's level and progress, maintaining positive tension. Strategies such as providing continuous feedback

during the process, clarifying expectations, subdividing objectives, and progressively increasing the complexity of challenges help students feel in control of their progress and enter a state of flow. In addition to these practices, teachers should encourage inquiry and discovery to inspire students and show them that mathematics is a field rich in challenges and possibilities (Assouline & Lupkowski-Shoplík, 2021). Ensuring access to advanced content, mentoring, and opportunities for exploration – both independent and collaborative – helps mathematically talented students stay motivated and develop the skills necessary for their future careers in STEM fields.

At the policy level, this study highlights the need for systemic strategies to support mathematically talented students, particularly in countries such as Uruguay, where recognition of giftedness remains in its early stages. Uruguay faces a pressing challenge: fewer than 20% of its university students pursue STEM-related fields (Instituto Nacional de Estadística, 2024), underscoring the urgency of addressing this issue. Policymakers should prioritize professional development initiatives to equip teachers with the skills to identify and nurture mathematical talent, with a focus on bottleneck factors such as spatial ability, abstract reasoning, and numerical aptitude, alongside motivational dimensions such as task commitment. Inclusive educational policies should ensure differentiated instruction and enrichment opportunities for gifted students regardless of their socioeconomic or cultural backgrounds, thereby broadening access to talent development resources and addressing the critical shortage of STEM professionals.

Limitations and future research

The findings of this study should be interpreted with caution because of the sample limitations. The sample came from one of the few institutions in Uruguay with a structured approach to gifted education, providing a valuable context for studying mathematical talent. Since few individuals in Uruguay have been formally identified as gifted, this study relied on students who had already been recognized for their high abilities or creativity within this pioneering institution to ensure research feasibility. Future studies should validate these findings by using broader and more heterogeneous samples. However, our research represents an important first step toward identifying the necessary cognitive conditions for mathematical talent in a small country with underrepresented educational context.

The sample was balanced by biological sex and homogeneity in terms of socioeconomic status and formal education, which mitigated potential biases. However, no statistical comparisons were conducted between girls and boys, as gender differences were beyond the scope of this initial study. Future research should examine potential differences to better understand how mathematical talent manifests across genders. All the participants received bicultural education and were learning mathematics in two languages, which may have influenced their spatial ability levels, potentially skewing them higher than those of a representative sample. Given these factors, exploring how mathematical talent manifests in diverse educational and cultural contexts is important.

Another limitation is the lack of representation of students with the highest levels of talent, which restricts the complete exploration of the conditions required for those at the extreme end of the spectrum. While highlighting an area for future research, this gap may partially reflect the strict evaluation criteria set by the jury handling the corrections to avoid ceiling effects. Notably, some students

in the preceding talent category achieved the highest scores on national selection tests and represented Uruguay in the International Olympiads, suggesting that their performance approached the highest levels of mathematical talent. As this was an initial study and given the small population of mathematically gifted students in Uruguay, future studies should replicate this research with larger, more diverse, or international samples to better capture the variability across different levels of talent and enhance the generalizability of the findings.

Therefore, the use of the Mathematical Olympiad problems to identify mathematically talented students warrants further consideration. Olympiad problems offer unique advantages, particularly when combined with techniques that assess cognitive aptitude, as implemented in our study. Unlike traditional assessments, Olympiad problems go beyond standard mathematical content, offering multiple solution paths that require creativity, advanced reasoning, and the ability to address unfamiliar challenges. Their high level of difficulty minimizes ceiling effects, making them particularly effective in identifying a broad spectrum of talent. Additionally, these problems often engage students who are passionate about mathematics but may feel unchallenged or unmotivated by routine classroom exercises, thereby identifying individuals who might otherwise remain unnoticed. Nonetheless, it is important to acknowledge that the Olympiad problems may not fully capture other forms of mathematical talent. Future research can explore the necessary conditions by incorporating alternative assessments, such as academic aptitude tests (e.g. SAT-M), to examine additional facets of mathematical ability and complement the Olympiad-based evaluations. A multimethod approach integrating diverse assessments can provide a more comprehensive understanding of the various expressions of mathematical talent.

In this regard, future research should integrate domain-specific and general creativity tests using the NCA methodology to deepen our understanding of the role creativity plays in mathematical talent. Moreover, in contexts such as Mathematical Olympiads, where originality is critical but generating multiple solution approaches is challenging for a single individual because of the complexity of the problems, refined instruments are required to evaluate creativity at this advanced level, which would contribute to the ongoing debate about the domain-specific nature of creativity in mathematics.

Another promising avenue for future research is to study the effects of training on mathematical talent. Many participants in this study attended weekly training workshops for Mathematical Olympiads; however, the influence of this training was not assessed. Investigating how structured training affects the development of mathematical talent, can offer valuable insights for effective educational interventions.

Note

1. It is important to emphasize that this research is conducted within an educational institution, and for this reason, the evaluation is intended for educational purposes and is not exhaustive. The number of students identified as gifted represents potential candidates for further clinical assessment to refine their giftedness.

Acknowledgments

We thank the students, parents and colleagues who took part in this study.

Disclosure statement

No potential conflict of interest was reported by the author(s).

Funding

This work was supported by the Agencia Nacional de Investigación e Innovación [POS_NAC_2019_1_158276]; Comisión Académica de Posgrado de la Universidad de la República [BFPD_2024_1#42368135].

ORCID

Natalia Colino  <http://orcid.org/0000-0001-9308-6293>

Alejandro Maiche  <http://orcid.org/0000-0002-5006-1544>

Mirko Antino  <http://orcid.org/0000-0001-7578-4899>

References

- Acar, S., Cevik, E., Fesli, E., Bozkurt, R. N., & Kaufman, J. C. (2024). Testing the domain specificity of creativity with Kaufman domains of creativity scale: A meta-analytic confirmatory factor analysis. *Journal of Creative Behavior*, 58(1), 171–189. <https://doi.org/10.1002/jocb.641>
- Andreescu, T., & Gelca, R. (2008). *Mathematical olympiad challenges*. Springer Science & Business Media.
- Arribas, D., Santamaría, P., Sanchez-Sanchez, F., & Fernandez-Pinto, I. (2013). *BAT-7. Batería de Aptitudes de TEA* [TEA Aptitude Battery]. TEA Ediciones.
- Artola, T., Barraca, J., Martín, C., Mosteiro, P., Ancillo, I., & Poveda, B. (2008). *PIC-J, Prueba de Imagination Creativa-Jóvenes* [PIC-J, Creative Imagination Test-Youth]. TEA Ediciones.
- Assouline, S. G., & Lupkowski-Shoplik, A. (2021). *Developing math talent: A comprehensive guide to math education for gifted students in elementary and middle school*. Routledge.
- Benbow, C. P. (2012). Identifying and nurturing future innovators in science, technology, engineering, and mathematics: A review of findings from the study of mathematically precocious youth. *Peabody Journal of Education*, 87(1), 16–25. <https://doi.org/10.1080/0161956X.2012.642236>
- Boran, A. İ., Açıkgül, K., & Köksal, M. S. (2015). Relationship of mathematics Olympiad performance of gifted students with IQ and Mathematics achievement. *Journal of Theoretical Educational Science*, 8(2), 185–203.
- Colino, N., & Maiche, A. (2022). Las altas habilidades en el dominio específico de la matemática: Una revisión sistemática de los hallazgos empíricos en neurocognición [Domain-specific high abilities in mathematics: A systematic review of empirical neurocognitive evidence]. *Cuadernos de Neuropsicología*, 16(3), 38–54.
- Csikszentmihalyi, M. (1996). *Flow and the psychology of discovery and invention*. Harper Collins.
- Csikszentmihalyi, M. (2014). *Flow and the foundations of positive psychology*. Springer.
- Csikszentmihalyi, M., Rathunde, K., & Whalen, S. (1993). *Talented teenagers: The roots of success and failure*. Cambridge University Press.
- Dai, D. Y. (2018). A history of giftedness: A century of quest for identity. In S. I. Pfeiffer, E. Shaunessy-Dedrick, & M. Foley-Nicpon (Eds.), *APA handbook of giftedness and talent* (pp. 3–23). American Psychological Association.

- Deci, E. L., & Ryan, R. M. (1985). *Intrinsic motivation and self-determination in human behaviour*. Plenum Press.
- Dul, J. (2016). Necessary condition analysis (NCA): Logic and methodology of “necessary but not sufficient” causality. *Organisational Research Methods*, 19(1), 10–52. <https://doi.org/10.1177/1094428115584005>
- Elgrably, H., & Leikin, R. (2021). Creativity as a function of problem-solving expertise: Posing new problems through investigations. *zdm-Mathematics Education*, 53(4), 891–904. <https://doi.org/10.1007/s11858-021-01228-3>
- Falk, M. (2001). Olimpiadas de Matemáticas: retos, logros (y frustraciones) [Mathematical Olympiads: Challenges, achievements (and frustrations)]. *Boletín de la Asociación Matemática venezolana*, 8(1), 15–26.
- Freeman, B., Marginson, S., & Tytler, R. (2019). An international view of STEM education. In A. Sahin & M. J. Mohr-Schroeder (Eds.), *STEM education 2.0* (pp. 350–363). Brill.
- Gagné, F. (1985). Giftedness and talent: Reexamining a reexamination of the definitions. *The Gifted Child Quarterly*, 29(3), 103–112. <https://doi.org/10.1177/001698628502900302>
- Gagné, F. (2019). Implementing the DMGT’s constructs of giftedness and talent: What, why, and how? In S. R. Smith (Ed.), *International handbook of giftedness and talent development in the Asia-Pacific* (pp. 1–32). Springer International Handbooks of Education.
- Gagné, F. (2020). *Differentiating giftedness from talent: The DMGT perspective on talent development*. Routledge.
- Gardner, H. (2003). *Frames of mind: The theory of multiple intelligences*. Basic Books.
- Green, C. T., Bunge, S. A., Chiongbian, V. B., Barrow, M., & Ferrer, E. (2017). Fluid reasoning predicts future mathematical performance among children and adolescents. *Journal of Experimental Child Psychology*, 157, 125–143. <https://doi.org/10.1016/j.jecp.2016.12.005>
- Guilford, J. P. (1950). Creativity. *The American Psychologist*, 5(9), 444–454. <https://doi.org/10.1037/h0063487>
- Guilford, J. P. (1967). Creativity: Yesterday, today and tomorrow. *Journal of Creative Behavior*, 1(1), 3–14. <https://doi.org/10.1002/j.2162-6057.1967.tb00002.x>
- Haavold, P., Sriraman, B., & Lee, K. H. (2020). Creativity in mathematics education. In S. Lerman (Ed.), *Encyclopaedia of mathematics education* (pp. 145–154). Springer.
- Hansen, K., Johannsen, M., Langemeyer, L., & Krüger, N. (2022). What is mathematical giftedness? Associations with intelligence, openness, and need for cognition. *Journal of Intelligence*, 10(4), 94. <https://doi.org/10.3390/jintelligence10040094>
- Hernández-Torrano, D., & Kuzhabekova, A. (2020). The state and development of research in the field of gifted education over 60 years: A bibliometric study of four gifted education journals (1957–2017). *High Ability Studies*, 31(2), 133–155. <https://doi.org/10.1080/13598139.2019.1601071>
- Hong, E., & Milgram, R. M. (2010). Creative thinking ability: Domain generality and specificity. *Creativity Research Journal*, 22(3), 272–287. <https://doi.org/10.1080/10400419.2010.503535>
- Instituto Nacional de Estadística. (2024). *Anuario Estadístico Nacional* (101 ed.). INE.
- Kang, W. (2015). Implications from Polya and Krutetskii. In S. J. Cho (Ed.), *Selected regular lectures from the 12th International Congress on mathematical Education* (pp. 405–416). Springer International Publishing.
- Knopik, T., & Oszwa, U. (2023). Mathematical resilience and motivation to study in mathematically gifted students-self-determination approach. *Przegląd Badań Edukacyjnych*, 1(41), 5–24. <https://doi.org/10.12775/PBE.2023.001>
- Krutetskii, V. A. (1976). *The Psychology of mathematical abilities in schoolchildren*. University of Chicago Press.
- Kurnaz, A. (2018). Examining effects of mathematical problem-solving, mathematical reasoning and spatial abilities on gifted students’ mathematics achievement. *World Scientific Research*, 5(1), 37–43. <https://doi.org/10.20448/journal.510.2018.51.37.43>
- Leikin, R. (2019). Developing mathematical talent in schoolchildren: Who, what, and how? In R. F. Subotnik, P. Olszewski-Kubilius, & F. C. Worrell (Eds.), *The psychology of high*

- performance: Developing human potential into domain-specific talent (pp. 173–199). American Psychological Association.
- Leikin, R. (2021). When practice needs more research: The nature and nurture of mathematical giftedness. *ZDM – Mathematics Education*, 53(7), 1579–1589. <https://doi.org/10.1007/s11858-021-01276-9>
- Leikin, R., Leikin, M., Paz-Baruch, N., Waisman, I., & Lev, M. (2017). On the four types of characteristics of super mathematically gifted students. *High Ability Studies*, 28(1), 107–125. <https://doi.org/10.1080/13598139.2017.1305330>
- Liashenko, A. K., Khalezov, E. A., & Arsalidou, M. (2017). Methods for identifying cognitively gifted children. *Psychology: Journal of Higher School of Economics*, 14(2), 207–218.
- Lubinski, D., & Benbow, C. P. (2006). Study of mathematically precocious youth after 35 years: Uncovering antecedents for the development of math-science expertise. *Perspectives on Psychological Science*, 1(4), 316–345. <https://doi.org/10.1111/j.1745-6916.2006.00019.x>
- Marginson, S., Tytler, R., Freeman, B., & Roberts, K. (2013). *STEM: Country comparisons: International comparisons of science, technology, engineering and mathematics (STEM) education. Final report*. Australian Council of Learned Academies.
- Mix, K. S., & Cheng, Y.-L. (2012). The relation between space and math: Developmental and educational implications. In J. B. Benson (Ed.), *Advances in child development and behavior* (pp. 197–243). Elsevier Academic Press.
- Payne, B. R., Jackson, J. J., Noh, S. R., & Stine-Morrow, E. A. L. (2011). *Activity flow state scale*. *Activity flow state scale* [Database record]. PsycTESTS.
- Paz-Baruch, N., & Hazema, H. (2023). Self-regulated learning and motivation among gifted and high-achieving students in science, technology, engineering, and mathematics disciplines: Examining differences between students from diverse socioeconomic levels. *Journal for the Education of the Gifted*, 46(1), 34–76. <https://doi.org/10.1177/01623532221143825>
- Pérez, S. G., & Bendelman, K. (2018). Lista de verificación de indicadores de Altas Habilidades/ Superdotación [Checklist of indicators for High Abilities/Giftedness]. En *Manual para la identificación de Altas Habilidades/Superdotación* [Manual for the Identification of High Abilities/Giftedness] (p. 25). Isadora Ediciones.
- Ramírez Uclés, R., Del Río Cabeza, A., & Flores Martínez, P. (2018). Mathematical talent in braille code pattern finding and invention. *Roeper Review*, 40(4), 255–267. <https://doi.org/10.1080/02783193.2018.1501782>
- Raven, J. C. (2006). *Test de matrices progresivas. Escala avanzada. Cuaderno de matrices/Serie II*. [Raven's advanced progressive matrices test series II]. Paidós. <https://www.amazon.com/-/es/Matrices-Progresivas-Escala-Avanzada-Spanish/dp/9501251608>
- Renzulli, J. S. (1978). What makes giftedness? Reexamining a definition. *Phi Delta Kappa*, 60(3), 180–184, 261. <https://doi.org/10.1177/003172171109200821>
- Renzulli, J. S. (2020). Reflections on my work: The identification and development of creative/productive giftedness. In D. Y. Dai & R. J. Sternberg (Eds.), *Scientific inquiry into human potential* (pp. 197–211). Routledge.
- Renzulli, J. S., & Gaesser, A. H. (2015). Un sistema multicriterial para la identificación del alumnado de alto rendimiento y de alta capacidad creativo-productiva [A multi-criteria system for the identification of high-performing students with high creative-productive capacity]. *Revista de Educación*, 368(4), 96–131. <https://doi.org/10.4438/1988-592X-RE-2015-368-290>
- Renzulli, J. S., & Reis, S. M. (2021). The three ring conception of giftedness: A change in direction from being gifted to the development of gifted behaviours. In R. J. Sternberg & D. Ambrose (Eds.), *Conceptions of giftedness and talent* (pp. 335–356). Palgrave Macmillan.
- Ridwan, M. R., Hadi, S., & Jailani, J. (2023). A meta-analysis of numerical aptitude's effect on learning outcomes and mathematical ability. *TEM Journal*, 12(1), 434. <https://doi.org/10.18421/TEM121-53>
- Roldán, O., & Ferrando, I. (2021). Identificación de indicadores propios de estudiantes de talento matemático: fluidez, flexibilidad, originalidad, elaboración y creatividad [Identification of specific indicators of mathematical talent: fluency, flexibility, originality, elaboration and creativity]. *Contextos educativos*, (28), 9–28. <https://doi.org/10.18172/con.4989>

- Ruthsatz, J., Ruthsatz-Stephens, K., & Ruthsatz, K. (2014). The cognitive bases of exceptional abilities in child prodigies by domain: Similarities and differences. *Intelligence*, 44, 11–14. <https://doi.org/10.1016/j.intell.2014.01.010>
- Schoevers, E. M., Kroesbergen, E. H., & Kattou, M. (2020). Mathematical creativity: A combination of domain-general creative and domain-specific mathematical skills. *Journal of Creative Behavior*, 54(2), 242–252. <https://doi.org/10.1002/jocb.361>
- Sipahi, Y., & Bahar, A. K. (2024). Who are the mathematically gifted? A systematic review of the research on cognitive characteristics. *Journal of Educational Studies in Science and Mathematics*, 3(2), 45–76. <https://doi.org/10.29329/jessm.2024.1110.1>
- Sriraman, B. (2005). Are giftedness and creativity synonyms in mathematics? *Journal of Secondary Gifted Education*, 17(1), 20–36. <https://doi.org/10.4219/jsge-2005-389>
- Stanley, J. C., & Brody, L. E. (2001). History and philosophy of the talent search model. *Gifted and Talented International*, 16(2), 94–96. <https://doi.org/10.1080/15332276.2001.11672967>
- Sternberg, R. J. (1995). *A triarchic approach to giftedness (research monograph 95126)*. University of Connecticut, The National Research Center on the Gifted and Talented.
- Sternberg, R. J. (2024). A duplex model for giftedness. *The Gifted Child Quarterly*, 68(2), 91–106. <https://doi.org/10.1177/00169862231217730>
- Subotnik, R. F., Miserandino, A. D., & Olszewski-Kubilius, P. (1996). Implications of the Olympiad studies for the development of mathematical talent in schools. *International Journal of Educational Research*, 25(6), 563–573. [https://doi.org/10.1016/S0883-0355\(97\)86733-X](https://doi.org/10.1016/S0883-0355(97)86733-X)
- Subotnik, R. F., Olszewski-Kubilius, P., & Worrell, F. C. (2011). Rethinking giftedness and gifted education. *Psychological Science in the Public Interest*, 12(1), 3–54. <https://doi.org/10.1177/1529100611418056>
- Tourón, J. (2020). Las altas capacidades en el sistema educativo español: reflexiones sobre el concepto y la identificación [High abilities in the spanish educational system: reflections on the concept and identification]. *Revista de Investigación Educativa*, 38(1), 15–32. <https://doi.org/10.6018/rie.396781>
- Uttal, D. H., Meadow, N. G., Tipton, E., Hand, L. L., Alden, A. R., Warren, C., & Newcombe, N. S. (2013). The malleability of spatial skills: A meta-analysis of training studies. *Psychological Bulletin*, 139(2), 352–402. <https://doi.org/10.1037/a0028446>
- Wagner, H., & Zimmermann, B. (1986). Identification and fostering of mathematically gifted students. *Educational Studies in Mathematics*, 17(3), 243–260. <https://doi.org/10.1007/BF00305072>
- Wai, J., Lubinski, D., & Benbow, C. P. (2009). Spatial ability for STEM domains: Aligning over 50 years of cumulative psychological knowledge solidifies its importance. *Journal of Educational Psychology*, 101(4), 817–835. <https://doi.org/10.1037/a0016127>
- Yan Kong, F. Z., & Benny, Y. C. (2018). Pattern recognition for identifying mathematically gifted children. In C. Speelman & D. Chhabra (Eds.), *7th annual international conference on cognitive and behavioral psychology* (pp. 18–21). Global Science & Technology Forum.
- Zhang, L., Gan, J. Q., & Wang, H. (2014). Optimized gamma synchronization enhances functional binding of fronto-parietal cortices in mathematically gifted adolescents during deductive reasoning. *Frontiers in Human Neuroscience*, 8, 430. <https://doi.org/10.3389/fnhum.2014.00430>
- Zubova, S. P., Lysogorova, L. V., Kochetova, N. G., & Fedorova, T. V. (2021). Olympiad potential for identifying mathematical giftedness in elementary schoolers In A. I. Savenkov (Ed.), *SHS web of conferences* (Vol. 117, p. 02005). EDP Sciences.