# Is time travel impossible? A financial proof

*"It never has existed and furthermore never will exist."*

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Ideas born out of science fiction often presage concrete scientific discoveries — space flight and robotics are but two examples. One of the most intriguing concepts to come forth from the science fiction literature is that of time travel, the ability to move backward or forward in time. Since H.G. Wells authored *The Time Machine* in 1895, the imaginations of numerous authors have been touched by the possibilities, and potential inconsistencies, associated with time travel.

The purpose of this note is to demonstrate that, unlike space flight, time travel is pure fantasy — it never has existed and furthermore never will exist; inexpensive time machines will never be invented. The proof for the impossibility of time travel is not based upon our current understanding of the laws of nature and the physical sciences. Rather, the impossibility of time travel can be proved by invoking simple economic reasoning. The proof will show that current economic conditions rule out the possibility of past, present, or future time machines.

Time travel assumes that individuals can transport themselves costlessly and instantaneously (with respect to their own biological clocks) to any point in time. Science fiction authors speculate over the historical consequences of such travel, but let us focus here on the economic behavior of such time travelers.

One of the most fundamental postulates of economics is that individuals prefer more wealth to less. Now let us assume our time traveler deposits $100 in a savings account in 1985 at an annual rate of interest of 10%. In seven calendar years, the value of the savings account would be $194.87. Under the 10% assumption, therefore, the time traveler could instantaneously travel to 1992, withdraw the $194.87, redeposit it, and begin the process again. As long as the interest rate remained at 10%, the time machine actually would be a money machine; the wealth of the time traveler would be unbounded.

The time traveler could engage in arbitrage across calendar time. These arbitrage profits could be consumed at any point in time. Thus, the time traveler could consume his wealth during any period, which means that the calendar periods in which the arbitrage profits are earned are independent of the periods in which they are consumed.

The only condition under which the time machine would no longer be a money machine should be apparent. This would occur when $100 deposited in a 1985 savings account is withdrawn in 1992 and $100 dollars is returned to the investor. Stated differently, arbitrage profit opportunities will be eliminated only if the nominal rate of interest is zero at all times.

As long as time travel is costless, and as long as the cost of transacting is nil,[1] time travelers will drive the nominal rate of interest to zero by engaging

1. Footnotes appear at the end of the article.

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in arbitrage transactions. Conversely, *the existence of positive nominal rates of interest suggests that time travelers do not exist.*

Negative rates of return also are inconsistent with costless time travel. Suppose that a financial asset in Period 1 costs $100 and drops in value to $50 in Period 2. This scenario allows time travelers to construct a money machine; they can either short-sell this financial asset in Period 1 and instantaneously transport to Period 2 and repurchase it, or they can buy it in Period 2, take it back to Period 1, and sell it. In either case, time travelers earn an arbitrage profit of $50. Again, these arbitrage profits disappear only when the interest rate is zero.

A zero nominal rate of interest is the only interest rate that is consistent with the no-arbitrage condition and costless time travel. Positive or negative interest rates create arbitrage profit opportunities. The sign of the interest rate does not matter, because time travelers can move either forward or backward in time with equal ease. Because the rate of return on financial assets must always be zero, these assets cannot be risky. Of course, given the ability of time travelers to "see" the future, the absence of risk is hardly surprising.

The ability of governments to inflate or deflate a fiat money economy is apt to be absent in a universe with costless time travel. Suppose that inflation merely changes the rate of exchange between goods and dollars. Assume that the government tries to induce inflation by increasing the money supply — the classic case of more dollars chasing the same goods. To time travelers, such schemes present wealth-increasing opportunities. In the period of increased money, time travelers would sell goods in exchange for money. They would then take this money to the prior period and would be able to purchase more goods than they sold. These goods would then be transported to the future period and resold for more dollars than they cost. This cycle of wealth-increasing transactions would cease when the rate of exchange between goods and dollars was equal in the two periods, or, in other words, when inflation was eliminated. In effect, time travelers are able to shift future inflationary increases in the money supply back to the present.

The zero interest rate condition implied by costless time travel would hold even if we were unable to build time machines using current technology. Stated differently, even if the technology to build time machines were to be invented 500 years from now, we would still observe interest rates of zero in capital markets today.

The reasoning is straightforward. Time travelers would realize that whenever the rate of exchange for assets between two different time periods is anything but one-for-one, arbitrage profits can be earned. The time period in which time travelers earn the arbitrage profits is a matter of indifference; they can use their time machines to consume their new-found wealth in any period they choose. Thus, time travelers from the 25th century would willingly travel to the 1980s to take advantage of non-zero rates of interest, because their investment and consumption activities can take place at different calendar dates.

Zero interest rates do not imply that there would be no savings or investment. Capital markets allow individuals to consume their income in periods other than the period in which they earn their income. To the extent that the desired (and feasible) consumption path for individuals differed from their income stream, some individuals would save and some would borrow. A zero rate of interest only implies that the rate of exchange between today's and tomorrow's consumption goods is one-for-one; individuals may still decide to forgo current consumption and save for future consumption.

Capital markets also aid investors in planning their outlays in productive assets. As in the simple, two-period Fisherian model with both productive and financial assets, the optimal strategy is to invest in productive assets up until the point where the marginal return on the productive assets equals the rate of interest. In an environment with a zero interest rate, more — not fewer — resources would be invested in productive assets than in an environment with positive interest rates, ceteris paribus. With costless time travel, the required rate of return, or hurdle rate, on investment projects would be zero. Thus, any project with a positive rate of return would be undertaken.

The arguments outlined above assume that time travel is costless and that time machines are competitively supplied. These assumptions may be reasonable. For example, the costs of time travel may be all development costs. The research and development activities could be funded by government agencies or conducted under the auspices of university laboratories. In either case, once the technology was developed, the marginal cost of time travel might be zero. Time machines might be inexpensive to produce and almost costless to use.

The zero interest rate condition, however, may prevail under conditions other than just costless time travel. In a universe with infinite time, the zero interest rate condition, at least in an asymptotic sense, will hold as long as the costs of time travel are bounded.

Suppose that the interest rate is always positive and strictly bounded away from zero. Given a long enough time horizon, the arbitrage profits that a time traveler could earn become arbitrarily large. This situation is akin to diners in John Adams's *Restaurant at the End of the Universe,* who can afford meals at this most expensive entertainment spot in the distant future as long as they deposit a penny in a special account in the time period from which they come. In an infinite time horizon model, arbitrage profits can become infinite with non-zero interest rates. Thus, as long as the cost of time travel is bounded, interest rates will be zero, or at least they will be non-zero in only a sufficiently small number of periods.[2]

Suppose again that time travel is costless, but now assume that time machines are owned by only one individual rather than being competitively supplied. The sole time traveler may decide not to force the interest rate to zero in order to maximize his or her wealth. In this scenario, the time traveler realizes that he or she can affect the interest rate by controlling the number of arbitrage transactions. If, as in the case of the standard monopoly analysis, the time traveler must trade at a single price (interest rate) with all market participants, then he or she will stop engaging in arbitrage transactions when the marginal revenues are zero with costless time travel. A zero marginal revenue condition does not imply zero rates of interest. On the other hand, if this time traveler can perfectly discriminate, then the interest rate on the last transaction will be driven to zero. A positive interest rate would mean that the time traveler could engage in at least one more transaction profitably. With perfect discrimination and costless time travel, this argument could be repeated until the interest rate on the last transaction is zero.

The speculations of science fiction authors often become reality. Yet the fantasy of time travel is apt to be just that. Based on standard and simple economic reasoning, the constellation of non-zero interest rates observed in financial markets strongly suggests that the technology for inexpensive time travel never has existed and, furthermore, never will exist.

This particular problem shows that the tools of economics, a social science, can make a strong prediction about the physical universe. Furthermore, it highlights the critical role that time plays in models of financial economics.

[1] The assumption of zero transaction costs is common in models of capital market equilibrium.

[2] In exceptional cases, even if the cost of time travel is bounded and the time horizon is infinite, interest rates could be different from zero. For example, if interest rates are not bounded away from zero, they could fluctuate around zero — sometimes assuming positive values and sometimes negative ones. On average, they might be near zero; however, if time travel is costly, it might be too expensive to smooth out the fluctuations. Even if interest rates are always positive and large in one period, they might be approaching zero so rapidly that it would never pay for an individual to engage in an arbitrage transaction.