Bargaining under Incomplete Information

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This paper presents and analyzes a bargaining model of bilateral monopoly under uncertainty. Under the bargaining rule proposed, the buyer and the seller each submit sealed offers that determine whether the good in question is sold and the transfer price. The Nash equilibrium solution of this bargaining game implies an offer strategy of each party that is monotonic in its own reservation price and depends on its assessment of the opponent's reservation price. Issues of relative bargaining advantage and efficiency are examined for a number of special cases. Finally, we discuss the appropriateness of the Nash solution concept.

THIS PAPER presents a simple model of two-person bargaining under incomplete information. Applications of the model range from the settlement of a claim out of court, to union-management negotiations, to the purchase and sale of a used automobile. The common feature shared by these examples is that each bargainer, while certain of the potential value it places on the transaction, has only probabilistic information concerning the potential value of the other. For instance, in haggling over the price of a used car, neither buyer nor seller knows the other's walk-away price.

Our principal aim is to investigate equilibrium bargaining behavior under uncertainty and to study how the parties fare, individually and collectively, under a simple class of bargaining procedures. A main result is that, given incomplete information, not all mutually beneficial agreements can be attained via bargaining. Even when the buyer values the good more highly than the seller, a successful sale may be impossible. Additionally, we present a number of results characterizing the players' optimal bargaining strategies and give explicit solutions for a number of examples. These examples provide useful comparative statics results—indicating the effect on bargaining behavior of changes in the negotiation

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ground rules, the information available to the players, and the degree of the players' risk aversion.

Complete vs. Incomplete Information. Most game-theoretic literature on bargaining has assumed that the participants possess complete information about the negotiation situation. Important contributions are provided by Nash [1950], Harsanyi [1956], Schelling [1960], Cross [1969], and Roth [1979]. Young [1975] provides an excellent summary and critique of the most important literature on the bargaining problem. In two-person bargains (to which our discussion will be limited), it is customary to stipulate that the set of possible actions of the parties and the player payoffs from any combination of actions are common knowledge. In the model of bilateral monopoly considered here, with the presumption of complete information, each bargainer, seller and buyer, knows the other's walk-away price. Then the bargainers negotiate a final price in the known range of mutually acceptable prices (i.e. between the seller's minimum and the buyer's maximum acceptable prices). Since any price in this range can be supported as an equilibrium outcome, bargaining solutions are usually determined either by specifying a concession mechanism leading to a distinct outcome, or a set of axioms that a "reasonable" outcome should satisfy.

Despite its many insights, the complete information approach fails to mirror key features of actual negotiations: 1) the fact that each bargainer is uncertain about its adversary's payoff, and 2) the occurrence of "unreasonable" bargaining outcomes—breakdowns in negotiations, strikes, and work stoppages—when mutually beneficial agreements are possible. A bargaining model that embraces these features carries a number of noteworthy implications. First, each party's bargaining strategy will depend directly on the information it possesses about his adversary's possible payoffs. For instance, as shown in later examples, the more optimistic a player's assessment of his opponent's stake in the negotiation, the more aggressive it can afford to be when bargaining. Second, as has been recently noted by Samuelson [1980] and Crawford [1982], the employment of optimal bargaining strategies may result in the breakdown of negotiations altogether—this despite the fact that a mutually beneficial agreement may exist ex post. Bargaining under uncertainty will, in general, fail to be Pareto efficient.

The present paper examines these issues by modeling bargaining as a game of incomplete information following the pioneering approach of Harsanyi [1967–1968]. The application of these ideas to the bargaining problem is found in Harsanyi and Selten [1972]. Like Harsanyi and Selten, we frame the bargaining situation as a noncooperative game and focus on the resulting set of equilibrium outcomes. In other respects, however, our methods differ. First, we substitute a single stage bargaining

procedure for the multistage representation of Harsanyi and Selten. Acting independently and without prior communication, the bargainers submit price offers. If these offers are compatible (in a sense to be defined later), a transaction is concluded at a price that depends on the offers; if they are not, then no transaction takes place. Though abstracting from the dynamics of the negotiation process, the single stage bargaining procedure emphasizes the basic strategy trade-off faced by each player. By making a more aggressive price offer, a player earns a greater profit in the event of an agreement but, at the same time, increases the risk of a disagreement (i.e. that his offer is unacceptable to the other).

Second, our approach employs continuous distributions to summarize the probability beliefs of the bargainers whereas Hansanyi and Selten focus on discrete distributions. Our aim is to investigate player bargaining strategies—that is, the mapping between the continuum of possible values that a player might hold and its price offers. The use of continuous distributions allows us to characterize a class of equilibria in which these bargaining strategies are well-behaved (i.e. differentiable). Additionally, the assumption facilitates the presentation of comparative statics results for specific examples.

Recently, a number of authors (Rosenthal [1978], Myerson [1979], and Myerson and Satterthwaite [1981] have examined the performance of arbitration procedures in the bargaining context. While our model offers predictions about the frequency of negotiation breakdown, bargaining efficiency, and relative bargaining advantage, our emphasis is on individual bargaining strategy and not on arbitration performance.

1. THE BASIC MODEL

Our analysis investigates bargaining behavior in the well-known case of bilateral monopoly. Suppose that a single seller of an indivisible good faces a single potential buyer. A successful bargain is concluded if and only if the good is transferred at a mutually acceptable price. Let v_s denote the seller's reservation price—the smallest monetary sum he will accept in exchange for the good (independent of his level of income). Similarly, let v_b denote the buyer's reservation price (the greatest sum he is willing to pay for the good). Since a bargain is struck only when it is agreeable to both parties, the sale price P must satisfy $v_s \leq P \leq v_b$ if such an interval exists.

Incomplete information of the bargainers is modeled by the following assumption: Each party knows his own reservation price, but is uncertain about his adversary's, assessing a subjective probability distribution over the range of possible values that his opponent might hold. Specifically, the buyer regards v_s as a random variable possessing a cumulative

distribution function $F_b(v_s)$ satisfying $F_b(\underline{v}_s) = 0$ and $F_b(\bar{v}_s) = 1$, and which is strictly increasing and differentiable on $[\underline{v}_s, \bar{v}_s]$. The seller's knowledge of v_b is summarized by $F_s(v_b)$ which satisfies $F_s(\underline{v}_b) = 0$ and $F_s(\bar{v}_b) = 1$, and which is strictly increasing and differentiable on $[\underline{v}_b, \bar{v}_b]$. These distribution functions are common knowledge in the sense of Aumann [1976]—that is, each side knows these distributions, knows that they are known by the other side, knows that the latter knowledge is known, and so forth.

In this framework, bargaining behavior depends on a player's reservation price v_i, his assessment of the opponent's reservation price, $F_i(v_j)$, and the knowledge of the opponent's assessment, $F_j(v_i)$, for $i = s, b$.

We will consider the following simple bargaining procedure:

Bargaining Rule. Seller and buyer submit sealed offers, s and b respectively. If $b \geq$ s, a bargain is enacted and the good is sold at price, $P = kb + (1 - k)s$, where $0 \leq k \leq 1$. If $b < s$, there is no sale and no money trades hands.

Special cases of this rule are of some interest. When k equals 1, the rule is equivalent to granting the buyer the right to make a first and final offer that the seller can accept or reject. In this instance, the sale price is determined solely by the buyer's offer, while the seller's offer serves only to determine whether there is an agreement or not. The seller's optimal strategy is to submit offers $s = v_s$ for all v_s (i.e. his reservation price), and an agreement is reached if and only if $b \geq s$. Similarly, setting $k = 0$ effectively grants the seller the first-offer right. Finally, when $k = \frac{1}{2}$, the rule determines a final sale price by splitting the difference between the player offers. Both offers carry equal weight in determining the sale price.

Framing the single stage bargain as a noncooperative game, we will characterize the resulting Nash or Bayesian equilibrium solutions. In the event of an agreement, each player earns a profit measured by the difference between the agreed price and his reservation price ($P - v_s$ for the seller and $v_b - P$ for the buyer); in the event of no agreement, each earns a zero profit. Additionally, we assume that each bargainer makes offers to maximize his expected profit. The notation $b = B(v_b)$ and $s = S(v_s)$ indicates that the players' price offers depend on their respective reservation prices. The functions $B(\,)$ and $S(\,)$ will be referred to as the player offer strategies. Then, the expected profit of the buyer is given by

$$\begin{aligned} \pi_b(b, v_b) &= \int_{\underline{s}}^{b} (v_b - P) g_b(s) ds \quad \text{if} \quad b \geq \underline{s} \\ &= 0 \quad \text{if} \quad b < \underline{s}, \end{aligned} \tag{1}$$

where $g_b(s)$ is the density function of seller offers induced by the offer strategy $S(\,)$ and the underlying distribution of values $F_b(v_s)$. The upper

and lower supports of the offer distribution are $\bar{s}$ and $\underline{s}$, respectively. Similarly, the expected profit of a seller with value v_s who makes offer s and faces a potential buyer using offer strategy $B(\,)$ is

$$\pi_s(s, v_s) = \int_s^{\bar{b}} (P - v_s)g_s(b)db \quad \text{if} \quad s \leq \bar{b},$$

$$= 0 \quad \text{if} \quad s > \bar{b}.$$

Conditional on v_s, the seller's offer s^* is a best response against $B(\,)$ if $\pi_s(s^*, v_s) \geq \pi_s(s, v_s)$, for all s. Similarly, a buyer holding v_b makes offer b^* that is a best response against $S(\,)$ if $\pi_b(b^*, v_b) \geq \pi_b(b, v_b)$, for all b. Then player i employs a best response strategy if for each v_i his offer is a best response against his opponent's strategy. A pair of best response offer strategies constitute a Nash (or Bayesian) equilibrium. By definition, neither player can increase his expected profit by unilaterally altering his chosen strategy.

2. PROPERTIES OF EQUILIBRIUM STRATEGIES

This section presents a number of results characterizing the equilibrium offer strategies of the bargaining game. Offer strategies satisfying $S(v_s) > \bar{v}_b$ and $B(v_b) < \underline{v}_s$ for all v_s and v_b constitute a no trade equilibrium. Obviously, these offer strategies are not very interesting. The results in this section describe equilibria in which an agreement occurs with a positive probability.

A fundamental property of equilibrium offer strategies for which trades occur is that they are increasing in the individual reservation prices. The higher the value placed on the good by the seller (buyer), the higher the price he demands (offers). The proof of this result requires no special assumptions about the offer strategies (e.g. continuity or differentiability).

THEOREM 1. *Under the sealed offer bargaining rule, the equilibrium bargaining strategies of the buyer and seller are increasing in the respective reservation prices.*

Proof. We present the proof for only the buyer offer function $B(\,)$; the seller proof is analogous. Consider v_b and v_b' such that $v_b \neq v_b'$ and let b and b' be the buyer's optimal offers for these respective values. Then by assumption, $\pi_b(b, v_b) \geq \pi_b(b', v_b)$, and $\pi_b(b', v_b') \geq \pi_b(b, v_b')$. Combining these inequalities, we find

$$\pi_b(b, v_b) - \pi_b(b', v_b) + \pi_b(b', v_b') - \pi_b(b, v_b') \geq 0. \qquad (2)$$

The expression for the buyer's expected profit in (1) implies that $\pi_b(b, v_b') - \pi_b(b, v_b) = G_b(b)(v_b' - v_b)$ where $G_b(b)$ is the cumulative distri-

bution function of s. In turn, it follows that $\pi_b(b', v_b') - \pi_b(b', v_b) = G_b(b')(v_b' - v_b)$. Substituting these expressions into (2) yields $[G_b(b') - G_b(b)][v_b' - v_b] \geq 0$. For $v_b' > v_b$, then $G_b(b') \geq G_b(b)$. Since $G_b(\,)$ is a distribution function, $b' \geq b$ when the strict inequality holds. If $G_b(b') = G_b(b)$, the buyer's profit maximization behavior ensures that $b' \geq b$. If b' were smaller than b, a buyer holding v_b could lower his offer to b' and increase his profit in the event of a bargain without affecting the probability of a bargain. This would imply that b' and not b was optimal for v_b—a contradiction. This illustrates the basic principle that a player should never place an offer in an interval where the other player makes no offer.

Our principal objective is to characterize the class of equilibria for which player offer strategies are well-behaved. We shall consider the family $\underline{A}$ of equilibria for which the following assumption is met: Each offer strategy is bounded above and below and is strictly increasing and differentiable except possibly at these offer bounds. Theorems 2 and 3 that follow characterize class $\underline{A}$ equilibria.

THEOREM 2. *In a class $\underline{A}$ equilibrium, over intervals of reservation prices for which the offer strategies are strictly increasing, $S(\,)$ and $B(\,)$ must satisfy the linked differential equations*

$$kF_b(y)S'(y) + f_b(y)S(y) = B^{-1}(S(y))f_b(y) \qquad (3a)$$

$$(1 - k)(1 - F_s(x))B'(x) - f_s(x)B(x) = -S^{-1}(B(x))f_s(x). \qquad (3b)$$

Proof. Start with the buyer's equilibrium strategy. His expected profit is

$$\pi_b(b, v_b) = \int_s^b [v_b - kb - (1 - k)s]\, g_b(s)ds.$$

The first order condition for a maximum is

$$\partial\pi_b/\partial b = (v_b - b)g_b(b) - kG_b(b) = 0.$$

Note that $G_b(b) = F_b(S^{-1}(b))$, where S^{-1} is the inverse offer function, i.e. $v_s = S^{-1}(b)$. Equivalently, $G_b(b) = F_b(y)$, where y is a dummy variable defined so that $b = S(y)$. After noting that $g_b(b) = f_b(y)/S'(y)$ and $v_b = B^{-1}(Sy))$, we can rewrite the first order condition as

$$[B^{-1}(S(y)) - S(y)]f_b(b) - kS'(y)F_b(y) = 0,$$

which, after some rearrangement, is identical to (3a).

Similarly, the seller's first order condition is

$$\partial\pi_s/\partial s = (v_s - s)g_s(s) + (1 - k)(1 - G_s(s)) = 0.$$

Then by employing the dummy variable $x = B^{-1}(s)$ and making the appropriate substitutions, we arrive at (3b).

Equations 3a and 3b are "linked" differential equations indicating that the player strategies are interdependent. The equations are a precise expression of the fact that a player's optimal price offer depends not only on v_i and $F_i(v_j)$, but also indirectly on $F_j(v_i)$ since the latter influences the opponent's strategy.

We can now characterize the equilibrium strategies at the offer bounds. For reference purposes, we denote the solutions to (3a) and (3b) over an unrestricted range of reservation prices by $\tilde{S}(\ )$ and $\tilde{B}(\ )$ respectively. We denote the corresponding inverse functions similarly. Since these functions are strictly increasing, the maximum and minimum price offers are $\tilde{S}(\bar{v}_s)$ and $\tilde{S}(\underline{v}_s)$ for the seller and $\tilde{B}(\bar{v}_b)$ and $\tilde{B}(\underline{v}_b)$ for the buyer.

To illustrate the nature of the boundary solutions consider the case when $\tilde{B}(\bar{v}_b) > \tilde{S}(\bar{v}_s)$ and $\tilde{B}(\underline{v}_b) < \tilde{S}(\underline{v}_s)$. In this case, $\tilde{B}(\ )$ describes the buyer's optimal strategy only for the interval of reservation prices where the probability of a successful bargain is positive but smaller than unity. When the buyer employs $\tilde{B}(\ )$, a bargain first becomes a certainty when $\tilde{B}(v_b) = \tilde{S}(\bar{v}_s)$—when the buyer's offer just matches the greatest possible seller offer. Equivalently, this occurs at value $v_b = \tilde{B}^{-1}(\tilde{S}(\bar{v}_s))$. Clearly, the buyer improves upon $\tilde{B}(\ )$ by employing the constant offer strategy $B(v_b) = \tilde{S}(\bar{v}_s)$, for $v_b > \tilde{B}^{-1}(\tilde{S}(\bar{v}_s))$, since he increases his profit in instances when he knows a successful bargain is certain. In this case, $\tilde{S}(\bar{v}_s)$ establishes the upper bound for both the buyer and seller offers.

It is straightforward to check that the seller's best response strategy is unchanged with this modification in the buyer's optimal strategy. Against the buyer's modified strategy, the seller's expected profit is

$$\pi_s(s, v_s) = k \int_s^{\bar{s}} b g_s(b)db + k \int_{\bar{s}}^{\bar{b}} \tilde{S}(\bar{v}_s) g_s(b)db + \int_s^{\bar{b}} [(1 - k)s - v_s]\, g_s(b)db.$$

Differentiating this expression with respect to s yields precisely the first order condition of Theorem 2.

Similarly, in the case that $\tilde{B}(\underline{v}_b) < \tilde{S}(\underline{v}_s)$, a bargain is impossible for sufficiently low buyer reservation prices. For lower and lower buyer values, the probability of a bargain first goes to zero at $v_b = \tilde{S}(\underline{v}_s)$. (At a reservation price greater than this, it is suboptimal for the buyer to match the lowest seller offer since by bidding instead in the interval $(\tilde{S}(\underline{v}_s), v_b)$, he can earn a positive profit. Thus, the buyer makes the truthful offer $B(v_b) = v_b$ only at $v_b = \tilde{S}(\underline{v}_s)$.) For $v_b < \tilde{S}(\underline{v}_s)$, the buyer's precise offer strategy is largely irrelevant since in equilibrium no bargain will ever be concluded. A buyer with v_b in this range faces only one restriction—that his offer be smaller than $\tilde{B}(v_b)$ in order to preserve the

zero profit equilibrium. If this restriction failed to hold, the buyer would earn negative expected profits against the seller's best response.

In sum, the pair $\tilde{S}(\ )$ and $\tilde{B}(\ )$ of Theorem 2 describe class $\underline{A}$ equilibria except at boundary conditions that occur as follows:

THEOREM 3. *In a class $\underline{A}$ equilibrium, boundary conditions are*:

1) *If* $\tilde{B}(\bar{v}_b) > \tilde{S}(\bar{v}_s)$, *then* $B(v_b) = \tilde{S}(\bar{v}_s)$ *for* $v_b > \tilde{B}^{-1}(\tilde{S}(\bar{v}_s))$.
2) *If* $\tilde{B}(\bar{v}_b) < \tilde{S}(\bar{v}_s)$, *then* $S(v_s) \geq \tilde{S}(v_s)$ *for* $v_s > \tilde{B}(\bar{v}_b)$.
3) *If* $\tilde{B}(\underline{v}_b) > \tilde{S}(\underline{v}_s)$, *then* $S(v_s) = \tilde{B}(\underline{v}_b)$ *for* $v_s < \tilde{S}^{-1}(\tilde{B}(\underline{v}_b))$.
4) *If* $\tilde{B}(\underline{v}_b) < \tilde{S}(\underline{v}_s)$, *then* $B(v_b) \leq \tilde{B}(\underline{v}_b)$ *for* $v_b < \tilde{S}(\underline{v}_s)$.

Thus, the range of "serious" offers is bounded from below by $\max[\tilde{S}(\underline{v}_s), \tilde{B}(\underline{v}_b)]$ and from above by $\min[\tilde{S}(\bar{v}_s), \tilde{B}(\bar{v}_b)]$.

3. ANALYSIS OF SPECIAL CASES

While Theorems 2 and 3 characterize equilibrium strategies that are well-behaved, this by no means exhausts the set of all equilibrium solutions. For instance, it is possible to construct other equilibria involving discontinuous offer strategies that place probability masses on specific offer values. Furthermore, even the linked differential equations of Theorem 2 resist analytic solutions for all but the most elementary distribution functions. The examples that follow are limited to distributions belonging to the uniform family. We begin with an examination of "identical" bargains—cases in which $F_s(\ ) = F_b(\ )$.

Example 1. Suppose the parties bargain under the sealed offer rule with $F_s(v) = F_b(v) = v/\bar{v}$. We have the following results:

a) The equilibrium offer strategies, $S(\ )$ and $B(\ )$, are given by:

$$S(v_s) = v_s/(2 - k) + ((1 - k)/2)\bar{v} \quad \text{for} \quad 0 \leq v_s \leq ((2 - k)/2)\bar{v}$$

$$S(v_s) \geq v_s/(2 - k) + ((1 - k)/2)\bar{v} \quad \text{for} \quad ((2 - k)/2)\ \bar{v} < v_s \leq \bar{v}$$

$$B(v_b) \leq v_b/(1 + k) + (k(1 - k)/2(1 + k))\bar{v} \quad \text{for} \quad 0 \leq v_b < ((1 - k)/2)\bar{v}$$

$$B(v_b) = v_b/(1 + k) + (k(1 - k)/2(1 + k))\bar{v} \quad \text{for} \quad ((1 - k)/2)\ \bar{v} \leq v_b \leq \bar{v}.$$

b) The probability that a bargain is reached equals $(-k^2 + k + 2)/8$ and achieves it maximum value, 9/32, at $k = \frac{1}{2}$.

c) i) The seller's expected profit is $\pi_s(k) = (\bar{v}/48)(2 - k)^2(1 + k)$ which is strictly decreasing in k.

ii) The buyer's expected profit is $\pi_b(k) = (\bar{v}/48)(1 + k)^2(2 - k)$ which is strictly increasing in k.

iii) The sum of the parties' profits is $\pi_s + \pi_b = (\bar{v}/16)(1 + k)(2 - k)$, which has a maximum of $(9/64)\bar{v}$, at $k = \frac{1}{2}$.

By simple inspection one can check that linear offer strategies above constitute solutions of Equations 3a and 3b when both distribution

functions are linear. Once the equilibrium strategies have been established, the results in parts b and c follow from straightforward computations. Note that part c describes each player's ex ante profit prior to the "draw" of either reservation price.

For easy reference, Figure 1 plots the equilibrium strategies for the

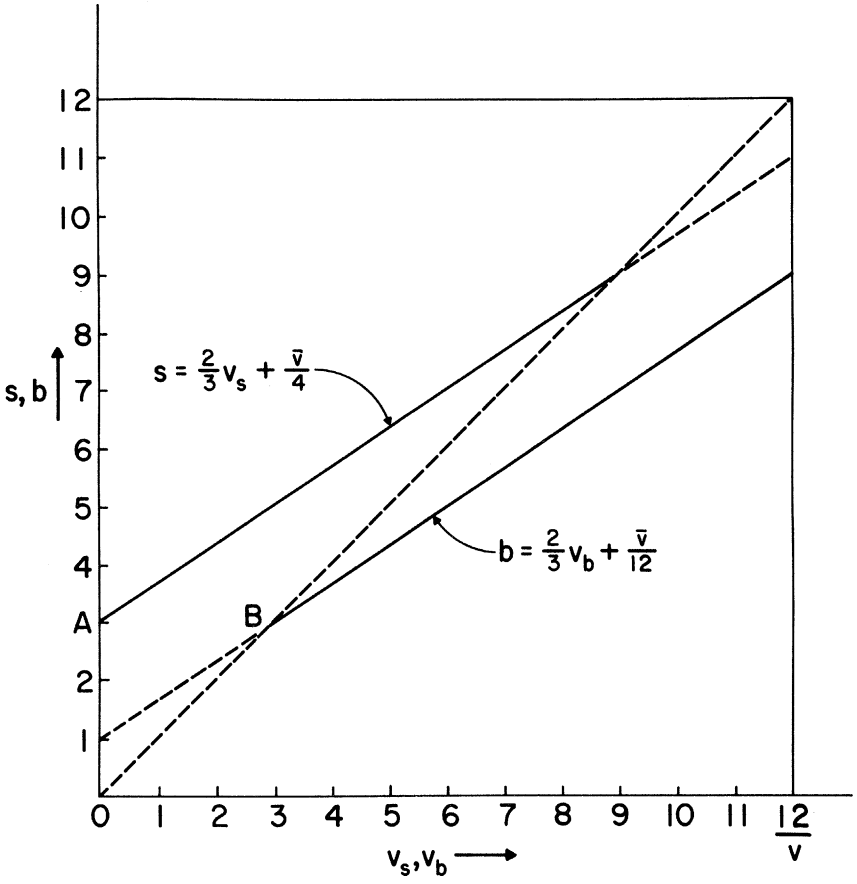


Figure 1. An identical bargain ($k = \frac{1}{2}$).

split-the-difference rule, $k = \frac{1}{2}$. Regardless of the specific rule, however, the equilibria of Example 1 satisfy a number of general properties. The buyer "shades" his offer below his true reservation price except when the probability of a bargain just becomes zero. (In Figure 1, the buyer strategy intersects the 45° line precisely at the seller's lowest offer; point B is level with point A.) Similarly, the optimal seller strategy calls for a "mark up." The combination of linear distributions, offer strategies, and bar-

gaining rule imply that the buyer's (seller's) conditional expected profit increases (decreases) as the square of his reservation price.

The employment of shading and mark up strategies in equilibrium precludes sales in many circumstances when mutually beneficial bargains exist. A bargain is possible provided $v_s \leq v_b$ but will only be concluded when $s \leq b$, or when $v_s \leq ((2 - k)/(1 + k))v_b - ((1 - k)(2 - k)/2(1 + k))\bar{v}$, using the results in part a. To put this another way, the probability of a successful bargain would be ½ if both parties made truthful offers (since $F_s = F_b$). The maximum attainable probability when optimal offer strategies are used is 9/32. Similarly, expected group profit under truthful offers is 1/6; under the equilibrium strategies it is at most 9/64.

One conclusion of Example 1 is that one's intuitive notion of who gains from different kinds of compromises can be mistaken. Suppose that the bargaining rule is $k = ½$. A suggestion is made to change the rule to $k = ¾$, so that the sale price is determined according to $P = (¾)b + (¼)s$. In whose interest, buyer or seller, is such a change? It would appear at first blush that this change benefits the seller and harms the buyer. Since a sale is made only when there is some "bargaining space" between the two offers (i.e., when $b - s \geq 0$), the move would seem to signify a surrender of this space by the buyer. Should not the switch signify an increase in profit for the seller and a reduction for the buyer?

The answer is no. The fallacy of this line of reasoning is that it implicitly assumes that the bargaining strategies are unaffected by the rule change. It is true that the seller would gain and the buyer would lose if the strategies employed for $k = ½$ were continued to be used for $k = ¾$. This is not the case, however, as the formulas of Example 1 demonstrate. When k increases, both the buyer and the seller shade their offers, the seller moving closer to his reservation price, the buyer toward a greater understatement. The net result is to confer an advantage on the buyer.

In Example 1, the bargain is identical ($F_s = F_b$) and, furthermore, the probability distribution is symmetric. In this instance, the bargaining rule that calls for splitting the difference ($k = ½$) has a natural appeal. It is efficient (i.e., it maximizes expected group profit), and it treats the parties symmetrically. In fact, Myerson and Satterthwaite show that the split-the-difference bargaining rule is an *optimal* mechanism—that is, it attains the greatest group expected profit of all possible bargaining procedures. For these reasons splitting the difference between offers is attractive for bargains that are identical and symmetric.

The next example characterizes the equilibrium strategies for nonidentical bargains when the buyer and seller distributions are uniform.

Example 2. Suppose that v_s is uniformly distributed on the interval [0,

$\bar{v}_s]$ and that v_b is uniformly distributed on $[\underline{v}_b, \bar{v}_b]$, where $0 \leq \underline{v}_b \leq \bar{v}_s \leq \bar{v}_b$. Then we have the following results:

a) The equilibrium offer strategy of the seller is specified by:

$$S(v_s) = \underline{v}_b/(1 + k) + (k(1 - k)/2(1 + k))\bar{v}_b$$
$$\text{for} \quad 0 \leq v_s < ((2 - k)/(1 + k))\bar{v}_b - ((1 - k)(2 - k)/2(1 + k))\bar{v}_b$$

$$S(v_s) = v_s/(2 - k) + ((1 - k)/2)\bar{v}_b$$
$$\text{for} \quad ((2 - k)/(1 + k))\bar{v}_b - ((1 - k)(2 - k)/2(1 + k))\bar{v}_b \leq v_s \leq ((2 - k)/2)\bar{v}_b$$

$$S(v_s) \geq v_s/(2 - k) + ((1 - k)/2)\bar{v}_b \quad \text{for} \quad ((2 - k)/2)\ \bar{v}_b < v_s \leq \bar{v}_s.$$

b) The equilibrium offer strategy of the buyer is specified by:

$$B(v_b) \leq v_b/(1 + k) + (k(1 - k)/2(1 + k))\bar{v}_b$$
$$\text{for} \quad 0 \leq v_b < ((1 - k)/2)\bar{v}_b$$

$$B(v_b) = v_b/(1 + k) + (k(1 - k)/2(1 + k))\bar{v}_b$$
$$\text{for} \quad ((1 - k)/2)\bar{v}_b \leq v_b \leq ((1 + k)/(2 - k))\bar{v}_s + ((1 - k)/2)\bar{v}_b$$

$$B(v_b) = \bar{v}_s/(1 - k) + ((1 - k)/2)\bar{v}_b$$
$$\text{for} \quad ((1 + k)/(2 - k))\bar{v}_s + ((1 - k)/2)\bar{v}_b < v_b \leq \bar{v}_b.$$

The offer strategies are sensitive to changes in the underlying distributions in the natural way. Other things equal, increases in $\underline{v}_b$, $\bar{v}_s$, and $\bar{v}_b$ imply larger (or no smaller) buyer and seller offers—that is, a shift to uniformly higher possible buyer or seller reservation prices (in the sense of stochastic dominance) results in higher offers. As a second example consider a player's offer response to a mean preserving spread in the distribution of his opponent's reservation price. For instance, if the seller becomes more uncertain about the buyer's value (such that $\Delta\bar{v}_b = -\Delta\underline{v}_b > 0$), the range of his offers increases in equilibrium (i.e. he makes higher (lower) offers than before when he holds sufficiently large (small) reservation prices). Furthermore, a simple calculation shows that, with the increase in uncertainty, the seller on average is worse off than before. An analogous result holds for an uncertain buyer. Figures 2 and 3 illustrate a pair of typical equilibria.

A number of points concerning the efficiency of the sealed offer bargaining rule can be summarized. First, in the nonidentical setting the rule of thumb of splitting the difference between offers will no longer maximize expected group profit (except coincidentally). When the dis-

tributions $F_s(\,)$ and $F_b(\,)$ differ, the seller and buyer obviously play asymmetric roles. Clearly then, the expected group profit as a function of the bargaining rule k will vary with $\underline{v}_s$, $\bar{v}_s$, $\underline{v}_b$, and $\bar{v}_b$. To take an extreme case, suppose there is no uncertainty surrounding the buyer's reservation price. The bargaining rule that calls for the seller to make the first and final offer ($k = 0$) extracts 100% of the potential group

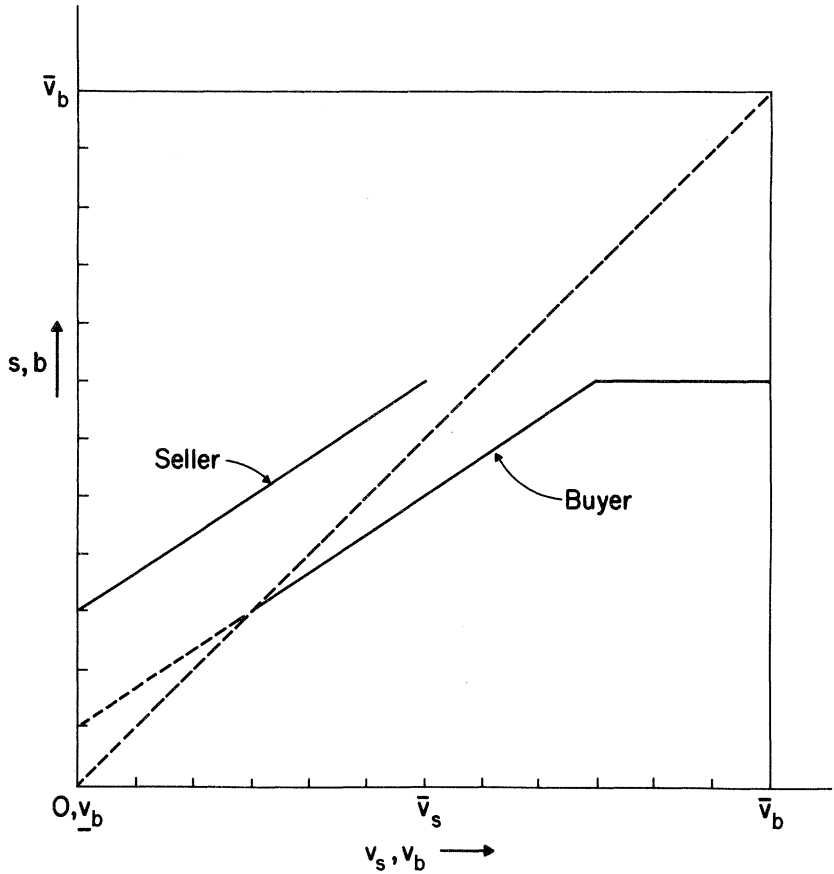


Figure 2. Nonidentical bargain, $k = \frac{1}{2}$.

profit (since the seller offers $s = v_b$ when $v_s \leq v_b$). But a buyer who makes a first and final offer ($k = 1$) will shade his offer below his true value; consequently, a number of mutually beneficial bargains will be missed. Here, a seller's first offer is superior to any other bargaining rule. In short, the bargaining rule that maximizes group profit must be determined on a case-by-case basis.

Second, a tedious but straightforward calculation can confirm that the

expected profit of the seller (buyer) is decreasing (increasing) in k. This result, which held in the identical bargain example, continues to hold for nonidentical uniformly distributed bargains.

Player Risk Aversion. Additional insight into the nature of the bargaining equilibrium can be gained by relaxing the assumption that the bargainers are risk neutral. Suppose that the seller and the buyer,

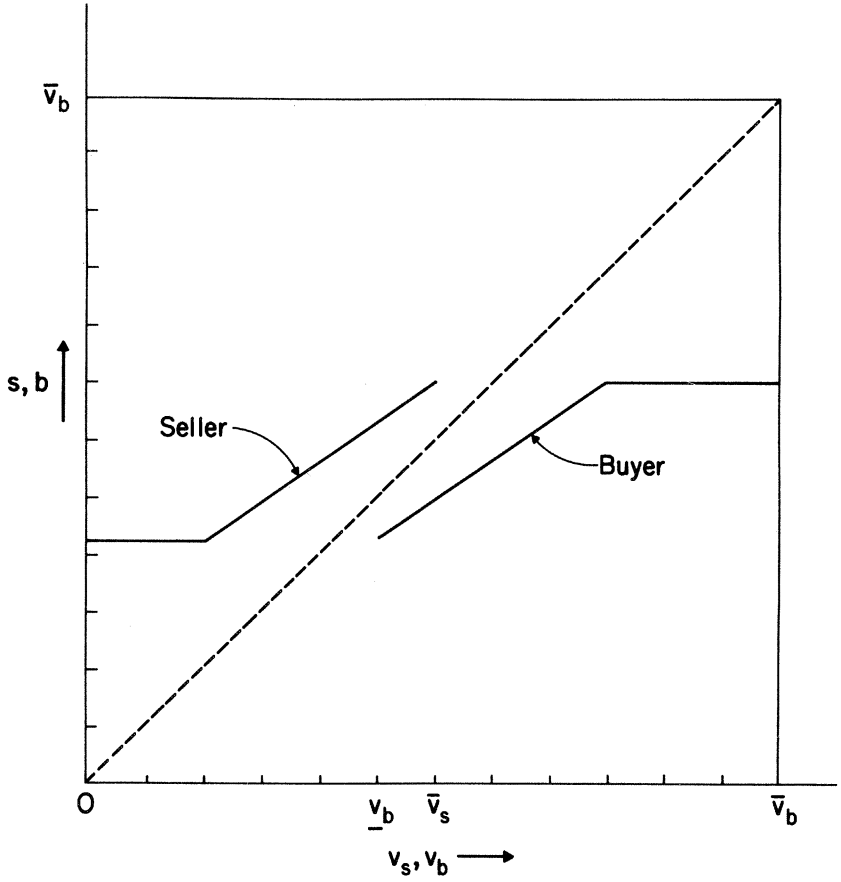


Figure 3. Nonidentical bargain, $k = \frac{1}{2}$.

respectively, earn von Neumann-Morgenstern utilities $u_s(P - v_s)$ and $u_b(v_b - P)$ in the event of an agreement and zero utilities otherwise. Then it is easy to check that the new first order conditions are

$$u_b(v_b - b) - k \int_{\underline{s}}^{b} u_b(v_b - kb - (1 - k)s)g_b(s)ds = 0,$$

and

$$u_s(s - v_s) - (1 - k) \int_s^{\bar{b}} u_s(kb + (1 - k)s - v_s)g_s(b)db = 0.$$

With these expressions, we can present equilibria in the case that: 1) the players' utility functions display constant relative risk aversion, $u(y) = y^\alpha$, where $0 < \alpha < 1$, and 2) the underlying distributions are uniform. As a simple illustration, suppose the players have the same degree of risk aversion, the underlying distributions are both uniform on [0, 1], and the bargaining rule is $k = \frac{1}{2}$. Then the equilibrium strategies are linear:

$$s = (\beta - \tfrac{1}{2})/(2\beta^2 - \tfrac{1}{2}) + [1 - (1/2\beta)]v_s$$

$$b = (1 - \tfrac{1}{2}\beta)/(4\beta^2 - 1) + [1 - (1/2\beta)]v_b$$

where $\beta = 2^{1/\alpha} - \frac{1}{2}$. In this symmetric example, the offer strategies have the same slope, and the buyer's intercept is smaller than the seller's. Consider the effect on the bargaining strategies as the players become increasingly risk averse: As α decreases, β increases causing the slope of each offer strategy to increase (and the intercept to fall). The result is generally lower seller offers and higher buyer offers. The intuition underlying this result is that, with risk aversion, the marginal increment in profit associated with a slighty more agressive offer (a higher seller offer or a lower buyer offer) is weighted less heavily than the possible loss, if as a result of the change, an agreement is precluded ($b < s$). This fact leads risk-averse bargainers to make offers closer to their true values than their risk-neutral counterparts. In the limit as α approaches 0 (i.e. the players become infinitely risk averse), the equilibrium strategies approach the strategies $s = v_s$ and $b = v_b$ for all v_s and v_b. Consequently, $b \geq s$ if and only if $v_b \geq v_s$, and so all mutually beneficial agreements are attained. Short of this extreme case, however, potentially beneficial bargains will be lost.

In the case of differing degrees of risk aversion, one can show that, other things equal, an increase in the risk aversion of the seller (buyer) implies uniformly lower (higher) offers by *both* parties in equilibrium. Not surprisingly, the opponent's best response to more truthful bids by the player who has become more risk averse is to make more aggressive offers himself.

4. CONCLUDING REMARKS

In what respects do the equilibria described in Theorems 2 and 3 provide an accurate representation of bargaining under uncertainty? Since the conclusions of any model depend on its premises, it is well to examine the extent to which the model's assumptions capture or approx-

imate the actual bargaining conditions. First, uncertainty concerning the opponent's walk away price is frequently encountered in bargaining situations. It is the task of each bargainer to assess the likely reservation price of his opponent. Indeed, the better the bargainer's information about his opponent, the better he can expect to fare in the negotiations. For instance, the man on the street is better equipped to bargain with a car dealer if he possesses the "book" that lists the prices the dealer himself has paid for various models and if he has assessed the current state of demand for automobiles.

Probability assessment becomes more complicated in an environment with stochastic dependence between the player values. For instance, suppose that each bargainer's value for the good depends on future (unknown) market conditions as well as on his personal characteristics. Additionally, suppose that each bargainer possesses differential information bearing on the good's potential value. Then, based on this information, each player must estimate his opponent's value *and* his own value in the event there is an agreement or not. In particular, the fact that the players would conclude an agreement (or not) is informative of the good's ultimate value.

While the single-stage bargaining rule fails to capture the pattern of reciprocal concessions observed in everyday practice, it is a useful idealization and a starting point for other investigations. A more general model would allow the bargainers multiple rounds in which to exchange offers (potentially incurrng "transaction" costs in the process). To the extent that the exchange of offers conveys information about the reservation prices, one would expect agreements to become more frequent. For example, when a bargain is unsuccessful at the initial stage, each party could revise his probability assessment of the other's reservation price and could make a concession in his next offer. These kinds of multistage bargains can be properly analyzed within our framework and deserve further attention. (For an example, see Sobel and Takahashi [1980].)

In our view, there are strong arguments for the monotonic equilibria of Theorems 2 and 3 even when other equilibria may exist. An individual is likely to accept the following proposition as reasonable: "The higher the seller's value the more he demands for the good; the higher the buyer's value the more he is willing to offer." Accepting this principle and believing firmly that any bargaining opponent will accept it as well, the individual then seeks a best monotonic offer strategy.

Monotonic offer strategies may also provide a focal point for the bargaining process. As a simple example, suppose the buyer's reservation price is uniformly distributed on the interval [0.5, 1] and the seller's price is similarly distributed on the interval [0, 0.5]. In this case, the constant offers $S(v_s) = B(v_b) = 0.5$ constitute an equilibrium and guar-

antee that the players always concluded a bargain. Each party chooses not to push further its demand, nor to retreat, expecting his opponent to feel the same way. One must not overlook, however, the existence of a second equilibrium in monotonic strategies. In this case, both parties make offers that are smaller than 0.5 for sufficiently low reservation prices and offers greater than 0.5 for high prices. Thus, it is prudent for a buyer who values the good highly to make a generous offer, thereby increasing the likelihood of an agreement when facing a seller who may not cooperate with an offer of $S(v_s) = 0.5$.

Equilibrium theory can furnish no definitive conclusion indicating which outcome represents the "more logical" bargaining focal point. Observe, however, that the monotonic equilibrium is responsive to changes in the underlying distributions F_s and F_b, while the constant strategy equilibrium is not. If, for instance, the buyer's value is drawn uniformly from the interval [0.5, 5] instead of [0.5, 1], the constant strategy equilibrium $S(v_s) = B(v_b) = 0.5$ becomes much less compelling. Is this a logical resting point—a position from which neither player expects the other to retreat? It would seem that the buyer could be expected to concede (settling for a smaller but still substantial profit) if the seller insisted. The monotonic equilibrium is free from these criticisms since it depends explicitly on the underlying distributions. With the shift in possible buyer values, the offers of the buyer become more generous and those of the seller more demanding. This response is consistent with the expectation that such a shift should benefit both parties (not just the buyer).

Finally, when $\bar{v}_s > \underline{v}_b$, there is no guarantee that a mutually beneficial agreement is always available. In this case of a "close" bargain, there is no constant strategy equilibrium. Indeed, the case of an identical bargain $F_s = F_b$ points out the main difficulty with a constant offer strategy. The employment of an aggressive offer will likely preclude an agreement, while an offer making too great a concession will be unprofitable. In short, players will adopt monotonic strategies because they are more efficient.

We would argue that close bargains occur most frequently in actual practice. In most bargaining situations, the opportunity for the seller or the buyer to cease negotiations and to consider a third-party transaction is available, at least implicitly. It is unusual for the buyer to be the only potential customer for the good or for the seller to have a monopoly position. In this instance, each party's reservation price will reflect, at least partially, the price (net of search transaction costs) that he could expect to obtain elsewhere. To the extent to which beliefs about the prices available from third parties transactions differ, or access to these parties differs, the reservation prices of the buyer and seller will diverge. Nevertheless, one would expect such an opportunity to minimize the gap

between the possible reservation prices of the parties. This suggests that close bargains may be of the greatest practical importance.

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