OPTIMAL RESILIENCE IN MULTITIER SUPPLY CHAINS *

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Forward-looking investments determine the resilience of firms' supply chains. Such investments confer externalities on other firms in the production network. We compare the equilibrium and optimal allocations in a general equilibrium model with an arbitrary number of vertical production tiers. Our model features endogenous investments in protective capabilities, endogenous formation of supply links, and sequential bargaining over quantities and payments between firms in successive tiers. We derive policies that implement the first-best allocation, allowing for subsidies to input purchases, network formation, and investments in protective capabilities. The first-best policies depend only on production function parameters of the pertinent tier. When subsidies to transactions are infeasible, the second-best subsidies for resilience depend on production function parameters throughout the network, and subsidies are larger upstream than downstream whenever the bargaining weights of buyers are nonincreasing along the chain. *JEL codes:* D21, D62.

I. Introduction

A spate of highly publicized supply chain disruptions—owing not only to the COVID-19 pandemic but also to natural disasters, cyberattacks, extreme weather events, logistics bottlenecks, geopolitical tensions, and a host of other causes—have drawn policy makers' attention to the importance of supply chain resilience. International institutions such as the Organisation for Economic Co-operation and Development (OECD 2021) and European Parliament (2021) have issued reports with "resilience"

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or "robustness" in their titles.[1] Government publications, such as the U.K. Department of International Trade (2022) and the U.S. *Economic Report of the President* (Council of Economic Advisors 2022, chapter 6) and international organizations such as the World Bank (2023) have also addressed these issues. Think tanks, such as McKinsey Global Institute (Lund et al. 2020) and the Brookings Institution (Iakovou and White 2020), have offered guidance as well. Yet little formal economic analysis has addressed the topic of optimal government policy in the face of ongoing risks of supply chain disturbances.

In this article, we examine the market failures that may generate suboptimal resilience in complex supply chains. We seek to capture in a stylized but realistic way one of the canonical supply-chain forms described in Lund et al. (2020) and the *Economic Report of the President* (Council of Economic Advisors 2022, fig. 6.1, panel B).[2] In what that report calls "outsourcing with isolated industries," inputs travel downstream through several or many tiers until they are ultimately transformed into a consumer good. Lead firms create the product designs and oversee specifications, at least from their immediate suppliers if not further up the chain, but they typically do not own or control most of these suppliers. Often, sourcing takes place sequentially (Yoo, Choi, and Kim 2021) and lead firms (a.k.a. original equipment manufacturers, OEMs) delegate procurement of components to their upstream partners (Guo, Song, and Wang 2010). These features of sequential and delegated procurement are described more fully in Mena, Humphries, and Choi (2013) and the references therein.

The McKinsey report describes another salient characteristic of modern supply chains, namely, the large numbers of firms

1. Baldwin and Freeman (2022) cite the business literature to distinguish between resilience and robustness. They describe resilience as "the ability of organizations and supply chains to plan for, respond to, and recover from disruptions in a timely and cost-effective manner" (Martins de Sá et al. 2019) and robustness as "the ability to maintain operations during a crisis (Brandon-Jones et al. 2014). In our static framework, we cannot distinguish between these two concepts, and so we use the term "resilience" to refer to both forms of protection from disruptions.

2. Baldwin and Venables (2013) coined the terms "snake" and "spider" to distinguish supply chains in which an input passes through multiple stages with sequencing dictated by engineering considerations from chains that involve the assembly of parts in no particular order. They focus on the effects of a reduction in international frictions on the location of production in these alternative types of global supply chains. Our model is something of a hybrid, with a spider structure at every tier and a snake structure that links the different tiers.

that are typically involved. They examined lists of publicly disclosed suppliers for 668 large manufacturing companies and report that most have hundreds of direct suppliers, who collectively have thousands of suppliers in the tier above. For example, General Motors reports 856 direct suppliers and a total of more than 18,000 suppliers to those direct suppliers. For Apple, those numbers are 638 and more than 7,400, respectively, and for Nestlé they are 717 and more than 5,000. Moreover, as Carvalho and Tahbaz-Salehi (2019) observed, input suppliers often sell to several or many lead firms. Dell and Lenovo share 2,272 direct suppliers among the total of 7,033 serving the former company and the 6,240 serving the latter (Lund et al. 2020, 9).

Guided by these observations, we develop a novel general equilibrium model of network production featuring multitier supply chains, arm's-length transactions between firms in different layers, many input suppliers for each manufacturer, many customers for each intermediate producer, and sequential procurement. The supply chains that we envision do not involve off-the-shelf inputs that might be available on anonymous markets. Rather, inputs are customized and sold to order. In our model, each producer negotiates the terms of a purchase contract with each potential supplier. The contracts specify the quantities that will be delivered by the upstream firms and the payments that will be made in return. Transactions take place only between firms that have borne the prior fixed costs of forming relationships. In this setting, we introduce risks of disruption at every node along the chains.

More specifically, we model an economy with a finite measure of firms that produce differentiated consumer goods and sell them to households in a setting of monopolistic competition. These lead firms, which are active in what we denote by tier S, produce their unique varieties using labor and bundles of differentiated intermediate inputs that they purchase from firms operating in tier $S - 1$. The firms in tier $S - 1$, in turn, fulfill their orders by combining labor and differentiated inputs procured from their partners in tier $S - 2$. Firms in tier $S - 2$ buy inputs from suppliers further upstream, and so on up the chain. The vertical chain ends with tier 0, where companies produce inputs from labor alone and sell them to firms in tier 1.

Since each supplier has many customers and each customer has many suppliers, and since firms have overlapping but different networks, it would be impractical for a grand negotiation to take place among all firms in the economy. Instead, we assume cooperative but simultaneous bargaining among isolated pairs in adjacent tiers. We assume a Nash-in-Nash equilibrium for the bargaining outcomes between all firms in some tier s and those in tier $s - 1$ (Horn and Wolinsky 1988); that is, each member of a pair takes as given the outcomes of its negotiations with all of its other suppliers or buyers, as the case may be. Meanwhile, we impose a sequential structure to the series of negotiations across tiers, in keeping with a prominent strategy described by Yoo, Choi, and Kim (2021).[3] Bargaining begins with negotiations between firms in tier S and their suppliers in tier $S - 1$ and proceeds upstream until firms in tier 1 sign contracts with firms in tier 0. All pairs are forward-looking, recognizing that their agreements have implications for their subsequent purchases and payments both on and off the equilibrium path.

We assume that each firm faces a positive probability of a catastrophic supply disruption. If a firm suffers such a disturbance, it will be unable to produce in the period captured by the model. The risks of disruption depend on actions undertaken by the firms to foster resilience and may vary across tiers of the supply chain. A firm's profits depend on its own fate and that of all of its suppliers and customers.

To capture the private opportunities available to promote supply chain resilience, we grant firms two means to moderate their risks. First, firms may invest in protective capability, which MacDuffie Fujimoto, and Heller (2021, 20) define as "the ability of firms to minimize damage inside facilities, sites and routes of the supply chain." Firms might choose to install equipment and erect buildings that are protected from weather shocks, establish strict health and safety protocols, design facilities that inhibit the spread of disease, and invest in cybersecurity. Under the heading of protective capabilities, we would also include what *The Economic Report of the President* (2022, 212) refers to as investments in agility, by which they mean "workers' ability to solve problems that ... enabl[es] them to pivot quickly to alternative products or processes or react to abnormal situations." In short, we allow firms to devote resources to reducing the probability that their own operations will be disrupted.

3. Yoo, Choi, and Kim (2021) cite the example of Google, which outsources the manufacturing of its built-in streaming technology Chromecast to Flex, while delegating to Flex the sourcing decisions from second-tier suppliers.

Second, we allow firms to invest in network thickness. Each firm chooses the fraction of suppliers in the tier immediately above its own with whom it forms relationships. Having multiple suppliers protects a firm against the event that some of its partners are unable to produce. *The Economic Report of the President* (2022, 211) describes a thick network as providing redundancy, that is, the wherewithal to replace a particular input supplier with another that offers a close substitute. In our model, where firms demand a variety of inputs, none of which are critical to its operation, a thicker network directly boosts productivity in the face of supplier outages. We assume that developing relationships is costly, as potential suppliers must be identified, vetted, instructed about specifications, and have their prototypes tested for quality.

Our analysis focuses on the "wedges" that emerge between private and social incentives at different stages of the supply chain. To identify these wedges, we solve a planner's direct-control problem and then ask what instruments the government would need to implement the first-best allocation as a decentralized equilibrium. We do not interpret these optimal policies literally as a prescription for industrial policy. Rather, the optimal policies help us identify where inefficiencies can arise in arm's-length supply chains, how the extent of these inefficiencies might vary across tiers that differ in their orders in the chain, and how the inefficiencies in a given tier reflect conditions in other parts of its network.

In general, the government would need three types of policy instruments in our setting to achieve the first best: a set of subsidies or taxes on transactions between firms in adjacent tiers, a set of subsidies or taxes to promote or discourage investments in protective capabilities in different tiers, and a set of subsidies or taxes to encourage or impede the formation of supplier relationships. The first-best transaction subsidy for any pair of firms depends only on the bargaining weights and production parameters for that dyad. The optimal policies to promote first-best resilience depend only on the bargaining weight that a firm achieves in its negotiations with its customers and on the size of the optimal subsidy for its sales to those customers.

We find that the outcome of each bargaining game yields an intuitive "markup factor" relating the payment for inputs by firms in some tier to the production cost for the firms in the tier above. The endogenous markup reflects the relative bargaining weights of the upstream and downstream firms and the substitutability between the various inputs used by the latter. The optimal transaction subsidy counteracts the effect of the markup on marginal cost, much as in settings with imperfectly competitive markets (rather than bilateral bargaining) for standardized inputs.

The optimal policy to promote or discourage investments in protective capabilities reflects two offsetting considerations. On the one hand, such investments confer a positive externality to the clients immediately downstream in a firm's network. On the other hand, the subsidy to transactions that is part of the first-best policy package inflates the private profitability of investments in resilience relative to their social value. If bargaining and technology parameters are common across tiers, then the first-best subsidies to resilience do not vary with a good's place in the supply chain, except for those at the extreme ends of the chain.[4] Alternatively, if goods further downstream are more differentiated than those upstream and other production and bargaining parameters are the same, the optimal subsidies for investments in protective capabilities decline as a good proceeds downstream. In any case, the optimal "subsidy" for investments in protective capabilities by firms in any middle tier may be a tax, if the first-best subsidy for input purchases by those firms is large enough. Finally, we show that the optimal subsidies for network formation are the same as those for protective capabilities, even though firms have a private incentive to use these investments to improve their bargaining position vis-à-vis their suppliers and buyers.

It is perhaps surprising that the first-best policies do not depend on parameters that describe a firm's entire production network. After all, when a firm becomes better protected against supply disruptions or creates a larger network, the greater productivity that results from its presence or from its greater number of suppliers confers a positive externality to other companies upstream and downstream in the firm's network, while conferring a negative externality on firms in other networks, including those in its own tier. We show, however, that in the presence of optimal subsidies to counteract the distorting effects of the negotiated

4. Some authors, like Antràs et al. (2012), refer to the place of an industry in the supply chain as the degree of its "upstreamness." Our finding says that with common production parameters and bargaining weights in all tiers, the first-best subsidy for resilience is independent of this characteristic of an industry.

markups, these positive and negative spillovers to firms that are not direct suppliers cancel in the general equilibrium. What remain are only the benefits that accrue to the firm's immediate customers and the wedge between social and private returns to investment that results from the transaction subsidies.

As noted, the first-best policies for investments in protective capabilities and network formation reflect the fact that the government uses subsidies for input purchases to ensure the ideal sizes of tier-to-tier transactions. But such subsidies may be politically sensitive if they are viewed as corporate handouts. Given the public focus on resilience, we feel it is interesting also to examine a second-best setting in which policies to promote protective capabilities and thicker networks are used in the absence of subsidies to transactions. We find that the second-best policies differ from the first-best policies not only in magnitude but also in the information that enters their design. Whereas the first-best subsidies to investments in resilience depend only on technological parameters relevant to the tier being targeted, the second-best policies reflect technological parameters that describe the whole supply chain. Specifically, the second-best subsidies reflect, among other considerations, an input's place in the chain.

Although our main focus is on the policy imperative that arises from the risk of supply disturbances, this article also contributes a new model to the toolkit on supply chains. Our model is distinctive in its combination of vertical chains with multiple tiers, endogenous network formation, endogenous investments in protective capabilities, bilateral and sequential bargaining, and general equilibrium. Models of endogenous networks such as Oberfield (2018), Acemoglu and Azar (2020), and Kopytov et al. (forthcoming), typically assume roundabout production processes, whereas those with vertical chains such as Ostrovsky (2008), Antràs and Chor (2013), and Johnson and Moxnes (2023) often take the network as given. Like us, Dhyne et al. (2023) allows for costly investments in supplier relationships, but in their case the probability of supply failures is completely exogenous and downstream firms subsequently purchase inputs from their suppliers at marginal cost.

Many of the supply chains modeled in the literature are fully efficient, either because a lead firm organizes all the transactions along the chain (Antràs and de Gortari 2020), because the market structure is perfectly competitive (Kopytov et al. forthcoming; Johnson and Moxnes 2023), or because a stability mechanism weeds out inefficient pairings (Oberfield 2018). These models are not suitable for studying the externalities that arise from investments in protective capability and network thickness, which are the main focus of our analysis.[5]

This article shares some of the concerns addressed in Grossman, Helpman, and Lhuillier (2023), although the economic environments in the studies are very different. Grossman, Helpman, and Lhuillier (2023) use a simple production structure in which a single critical input is used in fixed proportion to final output. Each final producer can purchase its sole input at marginal cost from any supplier with whom it has a prior relationship that survives a potential supply disruption. Their focus is on whether firms have adequate incentive to diversify their sourcing across locations and whether they have appropriate incentive to source in a safer, high-cost country relative to a riskier, low-cost country. There are no investments available to reduce the risk of a disruption and no reasons for a firm to invest in a thicker network aside from providing insurance against the loss of its critical input. Here we are primarily interested in how distortions differ upstream versus downstream, which demands a setting with multitier supply chains. We capture the empirical observation that firms in supply chains have many suppliers and customers, and we explicitly model the bargaining that determines quantities and payments. We also endogenize the probabilities of shocks by allowing firms to invest in protective capabilities. To handle this richer environment, we abstract from critical inputs and from shocks that are common to all firms in a given country.

This study also bears some similarity to recent, independent work by Acemoglu and Tahbaz-Salehi (2024). They study supply chains with endogenous networks that result from costly relationship-specific investments. In their model, like ours, transactions reflect negotiations between isolated pairs of firms, although there are some important differences in the details of the bargaining protocols.[6] Their supply chains have neither a

5. Few models allow for negotiated prices and quantities along the chain. An exception is Alviarez et al. (2023), but they allow for only two production tiers and have no investments in resilience or network formation.

6. Acemoglu and Tahbaz-Salehi (2024) assume that firms can negotiate contracts with two-part tariffs that are contingent on the realized production networks. In effect, all bilateral contracts are renegotiated when any negotiation breaks down. By allowing for renegotiation, they eliminate any inefficiencies in the sizes of equilibrium transactions between firms in an equilibrium network and focus instead on inefficiencies in the extensive margin of the equilibrium

vertical nor a sequential structure, and they do not consider ongoing risks of supply disturbances. Instead, they focus on the macroeconomic propagation of a single, unanticipated shock and especially on how small shocks can generate large changes in aggregate output due to the endogenous dissolution of supply relationships. Although they comment on the inefficiency of equilibrium with endogenous networks, they do not consider the optimal policy response at different points along the supply chain.

Like us, Elliot et al. (2022) study supply chain disturbances with idiosyncratic risks of failure. In their decentralized equilibrium, firms source inputs from multiple suppliers and invest resources to strengthen their relationships. However, there are several differences between their setting and ours. In their model, each firm has a finite set of critical inputs (much as in Grossman, Helpman, and Lhuillier 2023). Also, the microfoundations that they provide in their Online Appendix feature roundabout production, not vertical relationships. Their formulation does not allow for bilateral bargaining to determine quantities and prices. Finally, they address the determinants of resilience only in a single supply chain because the complexity of their model precludes a general equilibrium analysis.

There is an interesting parallel between our findings concerning second-best policies to promote resilience and results reported in Liu (2019) on optimal "industrial policies." Liu introduces exogenous wedges into a generic model of production networks. When the networks have a vertical structure, as here, the government's second-best policy is to provide larger production subsidies to sectors that are relatively farther upstream.

Finally, it is worth emphasizing that in this article, we treat only networks that form in a closed economy. In contrast, Antràs and Chor (2013), Antràs and de Gortari (2020), Grossman, Helpman, and Lhuillier (2023), Alviarez et al. (2023), Johnson and Moxnes (2023), and Fontaine, Martin and Mejean (2023), among others, deal with issues of international specialization in global supply chains. We hope to study optimal policy in the open economy in our future research.

network. In contrast, our analysis admits "double marginalization" that affects both the sizes of transactions and the incentives for investments in supplier relationships and in protective capabilities. See Lee, Whinston, and Yurukoglu (2021, sec 4.2) for a discussion of the empirical literature that established the importance of double marginalization in several industries.

To reiterate, our main contribution is to provide a rich yet tractable framework that can be used to study complex investment decisions in supply chains. Our model features an arbitrary number of tiers, bilateral bargaining, costly supplier relationships, and investments in protective capabilities and network formation. It captures several realistic externalities that arise in this setting, and we provide a complete characterization of first-best and second-best policies for a closed economy.

The remainder of the article is organized as follows. In the next section, we develop our model and describe the outcomes of the sequential bargaining and the equilibrium choices of investments in resilience and network formation. In Section III, we study the first-best allocation, outlining first the solution to the planner's direct-control problem and then the policies that a benevolent government can use to implement the optimum as a decentralized equilibrium. We characterize the optimal subsidies for input transactions, for investments in resilience, and for the formation of supplier relationships. Section IV addresses the second-best policy problem that arises when the government cannot subsidize transactions but can only promote (or discourage) investments in resilience and network formation. Section V concludes.

II. A Model of Multitier Supply Chains

In this section, we develop a general equilibrium model of vertical supply chains with an arbitrary number $S+1$ of production tiers and risks of supply disruptions throughout. A firm in the uppermost tier 0 produces a differentiated intermediate input using labor alone. A firm in a middle tier $s \in \{1, 2, \ldots, S-1\}$ produces an intermediate using labor and a bundle of inputs from tier $s-1$. It procures this bundle by bargaining over quantities and payments with the various suppliers in its production network. A firm in tier S produces a differentiated consumer good using labor and a bundle of tier $S-1$ inputs. We take the measure of firms in each tier s as given, and denote this measure by N_s for $s \in \{0, 1, \ldots, S\}$.[7]

7. We could readily allow for free entry at some fixed costs that vary by tier. This would not change any of our results regarding the first best, provided the government can also subsidize or tax entry.

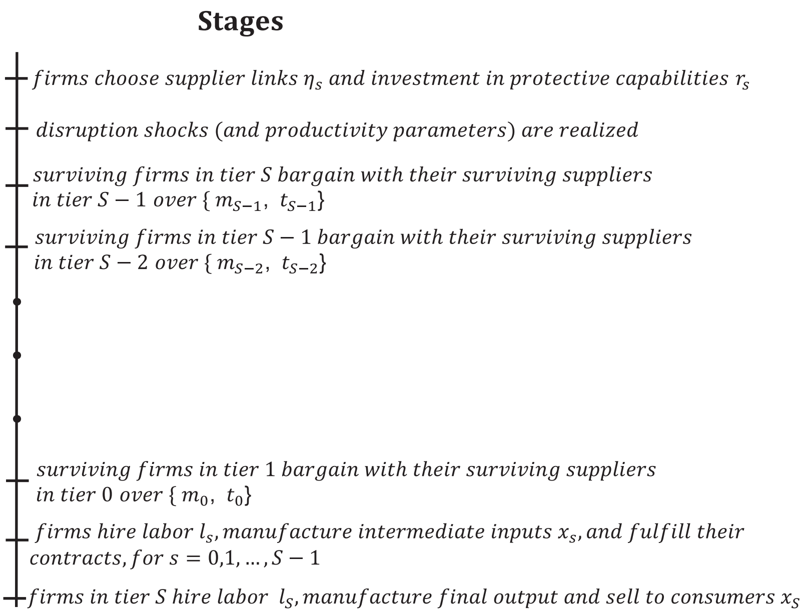

FIGURE I

Sequence of Events and Decisions

II.A. Overview and Notation

As a guide to what follows, we begin with a brief overview of the model and notation. We do so with reference to two figures that describe, respectively, the timing in the model and the transactions between successive tiers.

Figure I portrays the timing. First, firms invest in their protective capabilities and form links with potential suppliers. We let r_s denote the extent of the investments in things like weatherproofing and cybersecurity by firms in tier s. Such investments reduce the probability $1 - \phi_s(r_s)$ that the firm will suffer a catastrophic supply disruption, with $\phi_s'(r_s) > 0$ and $\phi_s''(r_s) < 0$ for all $s \in \{0, 1, \ldots, S\}$. Meanwhile, a typical firm in tier s, $s \in \{1, 2, \ldots, S\}$, elects to form relationships with the fraction η_s of the N_{s-1} suppliers in tier $s-1$ at a cost of k units of labor per relationship.

In the next stage, disruption shocks are realized that disable a fraction $1 - \phi_s$ of firms in tier s, leaving a measure $\phi_s N_s$ of active firms. In the main text, we assume that all surviving

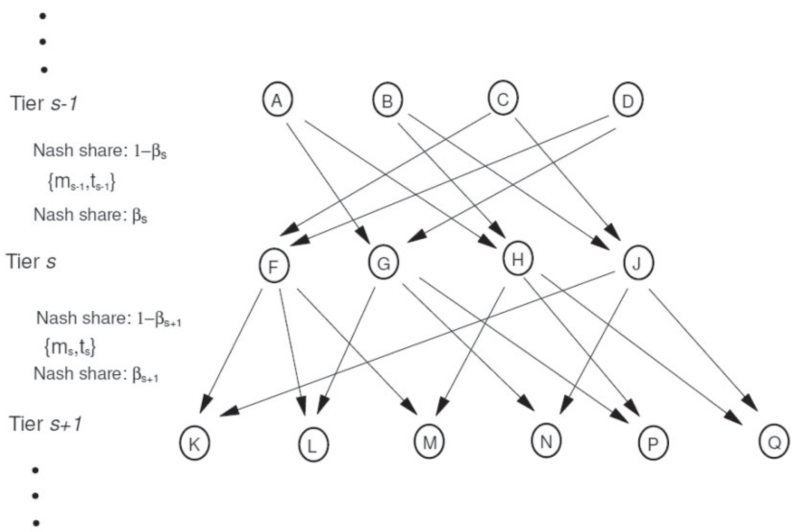

FIGURE II

Supplier Contracts and Relationships

firms in a tier have the same productivity, which we normalize to equal one. In the Online Appendix, we develop a more general version of the model in which surviving firms draw a Hicks-neutral productivity parameter from a known probability distribution with density function $f_s(z)$, as in Melitz (2003). We show in the Online Appendix that the policy conclusions for the model with heterogeneous firms are identical to those in the model with similar firms in a given tier.

Firms that survive the supply disturbances move on to the procurement stage. Procurement takes place sequentially. First, the lead producers negotiate with their surviving suppliers in tier $S-1$. These negotiations take place simultaneously and the negotiants take all other bargaining outcomes as given. After this end-of-chain bargaining has been concluded, firms in tier $S-1$ bargain simultaneously with suppliers in tier $S-2$. Bargaining continues sequentially until finally firms in tier 1 sign contracts with firms in tier 0.

Figure II depicts the sourcing in more detail. First notice that each buyer has multiple suppliers and that each supplier has multiple customers. For example, firm F in tier s supplies inputs to producers K, L, and M in tier $s+1$, while procuring inputs from firms C and D in tier $s-1$. The network for firm F overlaps with that of firm G, but not perfectly so. A firm in tier s negotiates a contract with each of its suppliers in tier $s-1$ that calls for a quantity of inputs, m_{s-1}, and a payment of t_{s-1}.[8] In the extended model with heterogeneous firms outlined in the Online Appendix, the quantities and payments are functions of the productivity of the buyer and the productivity of the supplier. In any case, the Nash bargaining gives weight β_s to the buyer in tier s and the weight $1-\beta_s$ to the supplier in tier $s-1$, as noted in the figure.[9]

After all the contracts have been negotiated, the firms in tier s hire l_s units of labor to combine with their input purchases of m_{s-1} units from each of their $n_s^u \equiv \eta_s \phi_{s-1}(r_{s-1}) N_{s-1}$ suppliers to produce x_s units of output. Again, if firms in tier s are heterogeneous in productivity—as outlined in the Online Appendix—then l_s and x_s will be functions of the productivity of the producer, and m_{s-1} will be a function of both the productivity of the producer and that of the particular supplier. Finally, the lead producers in tier S engage l_S units of labor, produce x_S units of output, and sell their differentiated products in a monopolistically competitive market at price p; these variables also depend on firm productivity in the extended model.

We proceed to analyze the stages of the model in reverse order. We specify the preferences and production technologies and describe the unique equilibrium, beginning with production of final goods, followed by production of inputs, sequential bargaining between suppliers and buyers, and finally investments in protective capabilities and relationship links. In Section II.J we

8. Equivalently, the firms could negotiate a quantity and a per unit price. As in other settings with cooperative bargaining, the firms set the quantity that is jointly optimal, then share the surplus by choice of payment. It follows that we could as well specify that firms negotiate two-part tariffs, as in Acemoglu and Tahbaz-Salehi (2024), with a fixed payment and a price per unit, and then they could allow the buyer to choose the quantity unilaterally.

9. Although the figure depicts a setting with discrete numbers of suppliers and customers, this is for illustrative purposes only. The analysis below treats the case of a continuum of firms. We solve the bargaining problem with "the last firm" by differentiating benefits and costs with respect to the measure of firms and allowing the bargain at the margin to differ from those with the remaining firms. Each firm enjoys a small surplus from the marginal transaction and the Nash bargaining solution applies to these small surpluses, as usual.

spell out the remaining condition for a general equilibrium in an economy with an inelastic labor supply, L. Throughout, we take the wage rate as numeraire.

II.B. Production and Sale of Consumer Goods

Consumers hold preferences defined over differentiated final goods, with a constant elasticity of substitution $\varepsilon > 1$ between every pair of products. Each of the $\phi_S(r_S)N_S$ surviving lead producers faces a demand with constant elasticity $-\varepsilon$ and a "demand shifter" A that is determined in general equilibrium.[10] With a continuum of final producers, each firm takes the demand shifter as given.

The typical firm produces output according to a Cobb-Douglas production function that combines labor and a bundle of intermediate inputs, with cost shares γ_S and $1-\gamma_S$, respectively. The input bundles comprise constant elasticity of substitution (CES) aggregates of the various inputs that firms have contracted to purchase, with elasticity of substitution $\sigma_S > 1$ between every pair. We write

$$x_S = l_S^{\gamma_S}\left[\int_{i\in\Omega_{S-1}^u} m_{S-1}(i)^{\alpha_S}\,di\right]^{\frac{1-\gamma_S}{\alpha_S}}, \tag{1}$$

where $m_{S-1}(i)$ is the agreed quantity that the firm buys from supplier i in tier $S-1$, Ω_{S-1}^u is the firm's set of surviving suppliers in that tier, and $\alpha_S \equiv \frac{\sigma_S-1}{\sigma_S}$.[11]

The market demand implies $p=\left(\frac{x_S}{A}\right)^{-\frac{1}{\varepsilon}}$. The typical firm has n_S^u surviving suppliers in tier $S-1$, where $n_S^u = \eta_S\phi_{S-1}\left(r_{S-1}\right)N_{S-1}$ is the product of the number of relationships it has formed and the survival rate. It has negotiated deals to purchase m_{S-1} units of a differentiated input from each of its suppliers and to pay t_{S-1} to each one. Therefore, the firm chooses l_S

10. The demand shifter $A=\frac{Y}{P^{-\varepsilon}}$, where Y is aggregate real income and P is the aggregate price index of all differentiated consumer goods.

11. In the extended model in the Online Appendix that allows for firm heterogeneity, the right-hand side of equation (1) is preceeded by z, an index of the productivity of the particular lead producer. The same is true for the production functions for goods in middle tiers and in the initial tier, which appear in equations (3) and (5).

at the production stage to maximize

$$\pi_S = A^{\frac{1}{\varepsilon}} l_S^{\frac{\gamma_S(\varepsilon-1)}{\varepsilon}} \left(m_{S-1}\right)^{\frac{(1-\gamma_S)(\varepsilon-1)}{\varepsilon}} \left(n_S^u\right)^{\left(\frac{1-\gamma_S}{\alpha_S}\right)\left(\frac{\varepsilon-1}{\varepsilon}\right)} - l_S - n_{S-1}^u t_{S-1}, \tag{2}$$

the difference between revenues from the sale of x_S units and total production costs.

II.C. Production of Inputs

A firm in a middle tier $s \in \{1, \ldots, S-1\}$ produces with a Cobb-Douglas technology that combines labor and input bundles, with shares γ_s and $1-\gamma_s$, respectively; that is,

$$x_s = l_s^{\gamma_s} \left[\int_{i \in \Omega_s^u} m_{s-1}(i)^{\alpha_s} di\right]^{\frac{1-\gamma_s}{\alpha_s}}, \tag{3}$$

where Ω_s^u is the set of its surviving suppliers and $m_{s-1}(i)$ is the quantity purchased from supplier i. The differentiated inputs in its bundle bear a constant elasticity of substitution $\sigma_s > 1$, where $\sigma_s = \frac{1}{1-\alpha_s}$. In equilibrium, the firms in tier s have agreed to supply m_s units of their output to each of n_s^d customers. The Cobb-Douglas technology dictates how much labor they must hire to fulfill their various sales contracts in the light of their various purchase contracts. By inverting the production function with output $x_s = n_s^d m_s$, we find

$$l_s = \left[\frac{n_s^d m_s}{\left(\int_{i=0}^{n_s^u} m_{s-1}(i)^{\alpha_s} di\right)^{\frac{1-\gamma_s}{\alpha_s}}}\right]^{\frac{1}{\gamma_s}} \quad \text{for } s \in \{1, \ldots, S-1\}, \tag{4}$$

where $\int_{i=0}^{n_s^u} m_{s-1}(i)^{\alpha_s} di = n_s^u (m_{s-1})^{\alpha_s}$ in the symmetric equilibrium that arises when productivities are homogeneous.

The firms in tier 0 produce using labor alone, with constant returns to scale. Choosing units so that one unit of labor generates one unit of output, we have

$$x_0 = l_0. \tag{5}$$

These firms have agreed to provide m_0 units to each of their n_0^d clients. To fulfill its contracts, a typical tier 0 producer must employ a workforce of

$$l_0 = n_0^d m_0. \tag{6}$$

II.D. Bargaining between a Buyer in Tier 1 and a Supplier in Tier 0

Turning to the procurement stages, we begin with the last set of negotiations, those between buyers in tier 1 and their suppliers in tier 0. A typical firm in tier 1 has committed to supply m_1 units of its product to each of its measure n_1^d of downstream customers. It takes as given its agreement to purchase m_0 units of inputs from each of a measure n_1^u of suppliers other than the (infinitesimal) one with whom it now negotiates. The bargaining takes place over a quantity $\tilde{m}_0$ and a payment $\tilde{t}_0$. If the negotiation fails, the downstream firm must do without this marginal input. Instead, it would need to hire a small amount of additional labor to fulfill its own contracts. The firm's surplus from the relationship with the particular seller amounts to the savings in labor cost less the extra payment. We denote this surplus by $V_1^d\left(\tilde{m}_0, \tilde{t}_0\right)$.

In the Online Appendix, we calculate the labor-cost savings by differentiating l_1 in equation (4) with respect to n_1^u (the measure of upstream suppliers) and evaluate the derivative at $\tilde{m}_0$, the quantity provided by the marginal supplier when all other suppliers provide m_0. Then we take $V_1^d\left(\tilde{m}_0, \tilde{t}_0\right) = -\frac{\partial l_1(\tilde{m}_0; m_0)}{\partial n_1^u} - \tilde{t}_0$.[12]

Meanwhile, the supplier in tier 0 stands to gain a payment of $\tilde{t}_0$ if it manages to strike a deal with the particular customer, but it would bear an extra labor cost of $\tilde{m}_0$ to produce the required output. The seller's surplus in a deal calling for $\tilde{m}_0$ and $\tilde{t}_0$ is simply $V_0^u\left(\tilde{m}_0, \tilde{t}_0\right) = \tilde{t}_0 - \tilde{m}_0$.

As usual, the Nash bargain solves

$$\{m_0, t_0\} = \arg \max_{\{\tilde{m}_0, \tilde{t}_0\}} V_1^d\left(\tilde{m}_0, \tilde{t}_0\right)^{\beta_1} V_0^u\left(\tilde{m}_0, \tilde{t}_0\right)^{1-\beta_1},$$

where β_1 is the bargaining weight of the buyer and $1 - \beta_1$ is that of the seller. In the Online Appendix, we show that the first-order conditions for this maximization problem imply

$$m_0 = \left(\frac{1-\gamma_1}{\gamma_1}\right)^{\gamma_1} \left(n_1^u\right)^{\frac{\gamma_1-\sigma_1}{\sigma_1-1}} n_1^d m_1. \tag{7}$$

12. Specifically, we find

$$V_1^d\left(\tilde{m}_0, \tilde{t}_0\right) = \frac{1-\gamma_1}{\alpha_1\gamma_1} \left(l_1\right)^{\frac{1-\gamma_1(1-\alpha_1)}{1-\gamma_1}} \left(n_1^d m_1\right)^{\frac{-\alpha_1}{1-\gamma_1}} \tilde{m}_0^{\alpha_1} - \tilde{t}_0.$$

Intuitively, the negotiated quantity grows linearly with the volume of output, $n_1^d m_1$, that the tier 1 firm has promised to deliver to its downstream customers. The quantity m_0 falls with n_1^u, because a larger bundle of inputs into tier 1 production offers more substitutes for any particular one of them.

We also use the first-order conditions to calculate the negotiated payment, t_0, and find

$$t_0 = \mu_0 m_0,$$

where

$$\mu_0 \equiv \beta_1 + (1 - \beta_1)\frac{\sigma_1}{\sigma_1 - 1}.$$

The total payment is proportional to the quantity, so μ_0 can be interpreted as a per unit payment. If all of the bargaining power were to rest with the buyer ($\beta_1 = 1$), the per unit payment would be $\mu_0 = 1$, which is the unit production cost. Alternatively, if all bargaining power were to rest with the seller ($\beta_1 = 0$), the per unit payment would be $\mu_0 = \frac{\sigma_1}{\sigma_1 - 1}$, which is the monopoly price of a differentiated input when the elasticity of demand is σ_1. In general, the per unit payment by a tier 1 producer is a weighted average of the competitive price of the input and the monopoly price, with the Nash-bargaining shares serving as weights.

We refer to μ_0 as a markup factor, by analogy to the pricing of differentiated inputs in an economy with monopolistic competition. Here, it measures the ratio of the negotiated payment to the supplier's production cost. The Nash bargaining protocol with a continuum of buyers and suppliers generates a constant "markup," which is greater when the seller has more bargaining power ($1 - \beta_1$ is large) and when the seller's input substitutes poorly for other inputs used by the downstream customer (σ_1 is small).

II.E. Bargaining between a Buyer in Tier 2 and a Supplier in Tier 1

Consider the negotiation between a typical buyer in tier 2 and a seller in tier 1. The downstream firm has committed to supply m_2 units to each of its n_2^d customers. It takes as given its agreement to purchase m_1 units of inputs from each of a measure n_2^u of other suppliers. Using equation (4) again, with $s = 2$, we can calculate the labor savings for the buyer from expanding its set of suppliers slightly and by purchasing $\tilde{m}_1$ units from the marginal seller. The surplus for the downstream firm, $V_2^d\left(\tilde{m}_1, \tilde{t}_1\right)$ is the difference between the marginal wage savings and the payment to the supplier, as before.

However, the calculation of the surplus for the seller is slightly different, because now the firms must anticipate subsequent negotiations, in keeping with the requirements for subgame perfection. The seller in tier 1 stands to gain the payment $\tilde{t}_1$ under the proposed contract. To fulfill such a contract, it will choose to hire marginally more labor. But it will also choose to purchase additional inputs from its other suppliers, which will necessitate a marginally larger bill for its input bundle. In the Online Appendix, we calculate the marginal wage bill, $\frac{\partial l_1}{\partial n_1^d}$, and the marginal input bill, $\frac{\partial\left(n_1^u t_1\right)}{\partial n_{1,}^d}$ and evaluate both at $\tilde{m}_1$. We find that the extra cost of producing $\tilde{m}_1$ units for a marginal buyer amounts to $c_1\tilde{m}_1$, where c_1 is defined in equation (A.19) in the Online Appendix as

$$c_1 = \gamma_1^{-\gamma_1}\left(1-\gamma_1\right)^{-(1-\gamma_1)}\left(n_1^u\right)^{-\frac{1-\gamma_1}{\sigma_1-1}} B_1 \quad (8)$$

and

$$B_1 \equiv \gamma_1 + \left(1-\gamma_1\right)\mu_0. \quad (9)$$

We interpret c_1 as the marginal cost to a tier 1 producer of providing an additional unit of its input to one of its customers. The marginal cost decreases with n_1^u, because a more diverse set of tier 0 inputs makes its own input bundle more productive. The marginal cost increases with B_1, which is a cost-share weighted average of the wage and the anticipated, per unit payment for inputs by the tier 1 supplier. Importantly, the marginal cost of producing tier 1 inputs grows with the markup μ_0 that the firm expects to emerge from its negotiations with its own suppliers.

Using the expressions for $V_2^d\left(\tilde{m}_1, \tilde{t}_1\right)$ and $V_1^u\left(\tilde{m}_1, \tilde{t}_1\right) = \tilde{t}_1 - c_1\tilde{m}_1$, we can solve for the Nash bargain,

$$\left\{m_1, t_1\right\} = \arg\max_{\left\{\tilde{m}_1, \tilde{t}_1\right\}} V_2^d\left(\tilde{m}_1, \tilde{t}_1\right)^{\beta_2} V_1^u\left(\tilde{m}_1, \tilde{t}_1\right)^{1-\beta_2}.$$

In the Online Appendix, we show that the first-order conditions imply

$$m_1 = c_1^{-\gamma_2}\left(\frac{1-\gamma_2}{\gamma_2}\right)^{\gamma_2}\left(n_2^u\right)^{\frac{\gamma_2-\sigma_2}{\sigma_2-1}} n_2^d m_2. \quad (10)$$

The solution implies that the typical seller in tier 1 delivers a smaller quantity of inputs to a typical customer when it perceives the marginal cost of producing those inputs to be higher. In other words, when a tier 1 seller and a tier 2 buyer choose the size of their transaction, they take account of the per unit payment for tier 0 inputs that will result from the subsequent negotiations. Apart from this, equation (10) has the same form and interpretation as equation (7).[13]

We can also calculate the payment implied by Nash bargaining and find

$$t_1 = \mu_1 c_1 m_1, \tag{11}$$

where

$$\mu_1 \equiv \beta_2 + (1 - \beta_2)\frac{\sigma_2}{\sigma_2 - 1}.$$

Here, $\mu_1 c_1$ is the per unit payment that emerges from the negotiations between the tier 1 producer and the tier 2 producer. It is a (constant) markup μ_1 over the unit cost c_1, where the markup reflects the bargaining shares of the two sides and the substitutability of tier 1 inputs in the production function for x_2.

II.F. Bargaining between a Buyer in Tier s (1 < s < S) and a Supplier in Tier s − 1

We proceed in a similar fashion to solve for all of the remaining Nash bargains between nonextreme buyers and sellers. A typical supplier in tier $s-1$ sells a quantity

$$m_{s-1} = c_{s-1}^{-\gamma_s}\left(\frac{1-\gamma_s}{\gamma_s}\right)^{\gamma_s}\left(n_s^u\right)^{\frac{\gamma_s-\sigma_s}{\sigma_s-1}} n_s^d m_s \tag{12}$$

to a typical buyer in tier s in exchange for a payment of

$$t_{s-1} = \mu_{s-1} c_{s-1} m_{s-1}, \tag{13}$$

where $\mu_{s-1} \equiv \beta_s + (1-\beta_s)\frac{\sigma_s}{\sigma_s-1}$ is the markup factor that results from negotiations between the firms in tier $s-1$ and tier s,

$$c_{s-1} = \prod_{j=1}^{s-1} \gamma_j^{-\gamma_j \Gamma_{j+1}^{s-1}}\left(1-\gamma_j\right)^{-(1-\gamma_j)\Gamma_{j+1}^{s-1}}\left(n_j^u\right)^{-\frac{\Gamma_j^{s-1}}{\sigma_j-1}}\left(B_j\right)^{\Gamma_{j+1}^{s-1}} \tag{14}$$

is the unit cost of production for the firm in tier $s-1$, $\Gamma_j^s \equiv \Pi_{i=j}^s (1-\gamma_i)$ is the product of the input shares for all stages

13. The marginal cost of producing the tier 0 input is $c_0 = 1$.

between j and s, and $B_j \equiv \gamma_j + \left(1-\gamma_j\right)\mu_{j-1}$ is defined analogously to B_1. We obtain equation (14) from equation (A.26) in the Online Appendix by using the recursive structure of c_s.

The negotiated quantity m_{s-1} in equation (12) depends on the marginal production cost c_{s-1}, the measure of competing inputs n_s^u, and the total amount of downstream demand, $n_s^d m_s$, much as for m_1. But now the marginal cost reflects the diversity in the input bundles and the input-share weighted averages of the wage and the price of input bundles in all stages further upstream. The per unit payment in equation (13) is the product of the marginal cost and a markup factor, μ_{s-1}, that emerges from the negotiation at hand.[14]

Evidently, the per unit payment by tier s producers to their suppliers in tier $s-1$ reflects not only the division of surplus between the two negotiants but also the markups they anticipate will emerge from bargaining further upstream. This outcome is the analog under sequential bargaining to the double marginalization that results from monopoly pricing of inputs in a market setting. With sequential bargaining, as with successive rounds of markup pricing, cost premia cumulate along the supply chain.

II.G. Bargaining between a Lead Firm and a Supplier in Tier $S-1$

Finally, we come to the negotiation between a typical final producer in tier S and a typical one of its suppliers in tier $S-1$. According to the sequencing outlined in Figure I, these negotiations happen first, ahead of all the other bargaining. But they take place in anticipation of all that will follow.

The final producer expects to employ labor so as to maximize profits in equation (2). This gives the usual markup pricing over marginal cost, as in Dixit and Stiglitz (1977) and elsewhere. Substituting the resulting employment, l_S, into the expression for profits gives a relationship between profits net of labor costs, the size and productivity of the firm's input bundle, and the total payment to suppliers. Profits increase with the measure of input suppliers, all else the same, because the CES aggregator implies a love of input variety.

We can calculate the surplus of a lead producer in its relationship with one of its suppliers by taking the marginal gain in prof-

14. Note that equation (12) yields equation (7) with $c_0 = 1$.

its with respect to a marginal seller that provides input quantity $\tilde{m}_{S-1}$ and subtracting from this amount the payment $\tilde{t}_{S-1}$ to that marginal supplier. The marginal profit gain can be computed by differentiating π_S with respect to n_S^u and evaluating the quantity provided by the marginal firm at $\tilde{m}_{S-1}$. This gives $V_S^d(\tilde{m}_{S-1}, \tilde{t}_{S-1})$.

As for the seller in this relationship, the calculus is the same as for any other supplier in a tier $s > 1$. The potential sale offers a gain of $\tilde{t}_{S-1}$, but at the expense of additional labor costs and additional input costs. The total additional costs are captured by $c_{S-1}\tilde{m}_{S-1}$.[15] The surplus is given by $V_{S-1}^u(\tilde{m}_{S-1}, \tilde{t}_{S-1}) = \tilde{t}_{S-1} - c_{S-1}\tilde{m}_{S-1}$. The Nash bargain, $\{m_{S-1}, t_{S-1}\}$ maximizes the geometric average of $V_S^d(\tilde{m}_{S-1}, \tilde{t}_{S-1})$ and $V_{S-1}^u(\tilde{m}_{S-1}, \tilde{t}_{S-1})$, with β_S and $1 - \beta_S$ as geometric weights.

The first-order conditions for the bargaining problem imply

$$m_{S-1} = A\left(c_{S-1}\right)^{\gamma_S(\varepsilon-1)-\varepsilon}\left(\frac{\gamma_S}{1-\gamma_S}\right)^{\gamma_S(\varepsilon-1)} \times \left[\frac{(1-\gamma_S)(\varepsilon-1)}{\varepsilon}\right]^{\varepsilon}\left(n_S^u\right)^{\frac{(1-\gamma_S)(\varepsilon-1)}{\sigma_S-1}} \tag{15}$$

and

$$t_{S-1} = \mu_{S-1} c_{S-1} m_{S-1}. \tag{16}$$

The lead producer buys more inputs from a typical supplier when aggregate demand for inputs (as captured by A) is great, when the perceived marginal cost of producing those inputs, c_{S-1}, is small, and when inputs are productive thanks to their diversity. It negotiates a payment for its inputs that is a multiple $\mu_{S-1} = \beta_S + (1-\beta_S)\frac{\sigma_S}{\sigma_S-1}$ of the production costs.

II.H. Recursive Solution for Quantities, Payments, and Employment Levels

We can now use the various bargaining solutions to express the input quantities $\{m_{s-1}\}$, the payments $\{t_{s-1}\}$, and the employment levels $\{l_s\}$ as functions of the aggregate demand shifter A and the numbers of active input suppliers per firm $\{n_s^u\}$ in every tier. First, we eliminate from the equations the number of customers for a typical firm in tier $s-1$ using the fact that every transaction involves one customer and one supplier. The $\phi_{s-1}(r_{s-1})N_{s-1}$ active firms in tier $s-1$ each have n_{s-1}^d customers,

15. Here, c_{S-1} can be calculated using the formula for c_{s-1} in equation (14).

which gives a total of $\phi_{s-1}(r_{s-1})N_{s-1}n^d_{s-1}$ customer relationships. Meanwhile, the $\phi_s(r_s)N_s$ active firms in tier s each have n^u_s suppliers, for a total of $\phi_s(r_s)N_s n^u_s$ supply relationships. Since each customer relationship corresponds to one supply relationship, we have $\phi_{s-1}(r_{s-1})N_{s-1}n^d_{s-1} = \phi_s(r_s)N_s n^u_s$, or

$$n^d_{s-1} = \frac{\phi_s(r_s)N_s}{\phi_{s-1}(r_{s-1})N_{s-1}} n^u_s.$$

Now we solve the system of equations for $\{m_s\}$ recursively. We use equation (15) to solve for m_{S-1} as a function of A and the numbers of suppliers per firm in tiers S and above.[16] Then, given any m_s and the numbers of suppliers per firm in tier s and above, we use equation (12) to solve for m_{s-1}. Finally, given m_1 and the number of suppliers to firms in tier 1, we use equation (7) to solve for m_0.

Once we have all of the input quantities, we use equations (11), (13), and (16) to solve for the payments for each transaction and use the (inverted) production functions (4) and (6) to solve for the employment levels.[17]

II.I. *Protective Capabilities and Network Thickness*

We turn finally to the initial stage of the game, when firms choose their protective capabilities and those in tier 1 and beyond form their supply networks.[18] We consider the problem facing a firm in tier $s > 0$ that takes the investment decisions of all other firms as given. The firm in question chooses $\tilde{r}_s$ and $\tilde{\eta}_s$ to maximize its expected net profits,[19]

$$v_s\left(\tilde{r}_s, \tilde{\eta}_s\right) = \phi_s\left(\tilde{r}_s\right)\pi_s\left(\tilde{\eta}_s\right) - \tilde{r}_s - k\tilde{\eta}_s N_{S-1},$$

where $\pi_s(\cdot)$ denotes the firm's operating profits conditional on avoiding a supply disruption and $\tilde{r}_s + k\tilde{\eta}_s N_{s-1}$ represents the total costs of its investments in resilience.

16. The number of suppliers per firm, n^u_s, for all $s \leqslant S-1$ figure in the expression for c_{S-1}.

17. We also need the first-order condition for profit maximization by final producers to solve for l_S.

18. A firm in tier 0 faces a similar problem when choosing its protective capabilities, r_0, but it has no relationships with input suppliers.

19. Note that $\pi_s(\cdot)$ depends on the protective capabilities, $\{r_s\}$, and network links, $\{\eta_s\}$, of all other firms. In the Online Appendix, where we admit heterogeneity in ex post productivity, v_s is the expected value of net profits over possible realizations of productivity z.

Notice that conditional on survival, a firm's prior investment in protective capabilities has no influence on its operating profits. A firm in any tier s (including $s = 0$) chooses $\tilde{r}_s$ to maximize $v_s(\tilde{r}_s, \tilde{\eta}_s)$, which gives the first-order condition

$$\phi_s'(\tilde{r}_s)\,\pi_s(\tilde{\eta}_s) = 1. \tag{17}$$

Naturally, investments in protective capabilities are larger when the prospective profits for operating are greater.

The thickness of a firm's network does affect its subsequent operating profits, because it determines the variety of its inputs after supply shocks are realized. This, in turn, determines the firm's productivity and thus the outcomes in its negotiations with suppliers and customers. The first-order condition for the choice of $\tilde{\eta}_s$ can be written as

$$\phi_s(\tilde{r}_s)\,\pi_s'(\tilde{\eta}_s) = kN_{S-1}. \tag{18}$$

Clearly, we need to derive $\pi_s'(\tilde{\eta}_s)$, the marginal effect of a thicker network on a firm's operating profits.

Consider a firm in a middle tier, that is, $s \in \{1, 2, \ldots, S-1\}$. The firm's operating profits are the difference between its receipts from all downstream customers and its total production costs. Production costs comprise the sum of payments to all suppliers and the firm's wage bill. We write

$$\pi_s(\tilde{\eta}_s) = n_s^d t_s(\tilde{\eta}_s) - n_s^u(\tilde{\eta}_s)\,t_{s-1}(\tilde{\eta}_s) - l_s(\tilde{\eta}_s).$$

The number of a firm's supplier links has no bearing on the size of its customer base, n_s^d, which is determined by decisions of downstream firms. But more links means more surviving suppliers and having more suppliers spells higher productivity. With higher productivity, the firm achieves a lower unit cost and sells more to each of its customers. It receives a payment per customer of $t_s(\tilde{\eta}_s) = \mu_s \tilde{c}_s(\tilde{\eta}_s)\,\tilde{m}_s(\tilde{\eta}_s)$. Notice that $\mu_s \equiv \beta_{s+1} + (1 - \beta_{s+1})\frac{\sigma_{s+1}}{\sigma_{s+1}-1}$ depends on the bargaining weight of the firm vis-à-vis its customers and the elasticity of substitution between the firm's output and that of other suppliers to the same buyer. Neither of these depends on the thickness of a firm's own supplier network. But $\tilde{c}_s\tilde{m}_s$ grows at a constant rate with $\tilde{\eta}_s$, because the firm negotiates larger sales to each of its customers, who substitute its product for other inputs to take advantage of their lower cost; see equations (A.68) and (A.69) in the Online Appendix.

Meanwhile, the firm's total costs rise with $\tilde{\eta}_s$, because the firm makes larger commitments to its customers. We find that production costs also increase at a constant rate as the number of supplier links grows.

In the Online Appendix, we show in deriving equation (A.72) that

$$\pi_s\left(\tilde{\eta}_s\right) = Q_{\pi_s} \tilde{\eta}_s^{\frac{(1-\gamma_S)\left(\sigma_{s+1}-1\right)}{\sigma_S-1}}, \tag{19}$$

where Q_{π_s} is a constant from the firm's point of view. The elasticity of expected profits with respect to the firm's investment in relationship links is greater when having a more diverse set of inputs contributes more to productivity, that is, when inputs are a larger share of production costs for firms in tier s (higher $1-\gamma_s$) and when the inputs used by these firms are more differentiated (smaller σ_s). A given productivity gain is more beneficial to a firm in tier s when its competitors produce inputs that are closer substitutes for its own in the eyes of its downstream customers (higher σ_{s+1}).

The power function on the right-hand side of equation (19) reflects the CES technology for the input bundle and the Cobb-Douglas combination of inputs and labor. Indeed, the profit elasticity here is reminiscent of that in settings with monopolistically competitive input markets. Although our payments and quantities result from sequential bilateral bargaining in a complex supply chain, the mechanism by which input variety raises profits is similar to what happens in a setting with unilateral price setting. In a model with monopolistic competition and CES technology, an increase in the number of inputs makes the inputs more productive while leaving markups unchanged. With greater productivity and unchanged prices, a firm sells more inputs and earns greater profits. Here, firms negotiate with each of their customers and then with their suppliers. An increase in productivity has no effect on the negotiated "markups," but it does increase the profits that can be shared in each pairwise negotiation. A more productive firm negotiates a larger volume of sales with each customer and larger purchases from each of its suppliers, which generates increased profits all along its supply chain.

We can use a similar procedure to find how π_S, the operating profits of a final producer in equation (2), vary with the firm's investment in supply links. We need to calculate how revenues and costs vary with $\tilde{\eta}_S$, which is tedious but straightforward. The calculations leading to equation (A.74) in the Online Appendix yield

$$\pi_S\left(\tilde{\eta}_S\right) = Q_{\pi_S} \tilde{\eta}_S^{\frac{(1-\gamma_S)(\varepsilon-1)}{\sigma_S-1}}. \tag{20}$$

For interior solutions to the optimization problem in equation (18), we need that $\pi_s\left(\tilde{\eta}_s\right)$ and $\pi_S\left(\tilde{\eta}_S\right)$ are concave functions. Concavity of these functions is ensured by the following assumption.

ASSUMPTION 1. $\sigma_1 \geqslant \sigma_2 \geqslant \cdots \geqslant \sigma_S \geqslant \varepsilon$.

Assumption 1 says that a good becomes more and more differentiated as it proceeds down the supply chain. This seems a reasonable assumption about the multistage transformation of raw materials into ever-more-customized inputs and finally into consumer products.

II.J. General Equilibrium

A labor-market clearing condition closes the model. Labor is used to produce intermediate inputs, produce final goods, form supply networks, and acquire protective capabilities at every level in the supply chain. Production labor in a typical firm in tier s must satisfy equation (4) for $s \in \{1, \ldots, S-1\}$ and equation (6) for $s = 0$. Final producers hire labor l_S to maximize operating profits in equation (2). In addition, each firm in tier s employs r_s workers to protect against its own supply disruption and each firm in tier $s \neq 0$ employs $k\eta_s N_{s-1}$ workers to form supply relationships with firms upstream. There are $\phi_s\left(r_s\right)N_s$ active firms in tier s after the resolution of the supply shocks. Therefore, the general equilibrium requires

$$\sum_{s=0}^{S} N_s r_s + \sum_{s=1}^{S} N_s k \eta_s N_{s-1} + \sum_{s=0}^{S} \phi_s\left(r_s\right) N_s l_s = L.$$

This condition determines the demand shifter A that appears in equations (2) and (15); see equation (A.57) in the Online Appendix and the discussion there.

III. FIRST-BEST ALLOCATION AND OPTIMAL POLICY

In this section, we characterize the optimal allocation of resources in an economy with ongoing risks of supply disturbances. First, we formulate and solve the social planner's direct control problem. Then, in Section III.B, we derive the fiscal policies that would eliminate the wedges between social and private incentives when firms in successive tiers negotiate their input transactions. In principle, these policies could depend on the numbers of surviving firms and the networks that have been built. In fact, however, we find that optimal transaction subsidies are independent of the numbers of suppliers and customers, and thus independent of the government's policies toward investments in protective capabilities and network thickness. In Section III.C, we derive the subsidies or taxes for spending on protective capabilities and for the formation of supplier links that would eliminate the wedges between private and social incentives for these investment decisions. We show that the optimal policies reflect the government's choice of transaction policies and that, in fact, the subsidy or tax rates for the two types of policies are the same. Finally, in Section III.D, we combine the results from the prior sections to describe the policy package that could implement the first-best allocation. Although the informational requirements for implementing such a package would be immense, finding the optimal taxes and subsidies helps us understand where inefficiencies can arise in a multitier supply chain and how these inefficiencies interact.

III.A. The Social Planner's Direct Control Problem

The planner allocates resources to maximize welfare of the representative household. The constant-elasticity demand facing each final producer derives, as usual, from a CES utility function,

$$W = \left[\int_{j \in \Omega_S} x_S(j)^{\frac{\varepsilon-1}{\varepsilon}} \, dj \right]^{\frac{\varepsilon}{\varepsilon-1}},$$

where Ω_S is the set of differentiated products available to consumers. With homogeneous production functions for final goods, the symmetry of the utility function implies that the planner should provide households with equal quantities x_S of all available consumer goods, so we can rewrite the planner's objective as

$$W = (n_S)^{\frac{\varepsilon}{\varepsilon-1}} x_S, \tag{21}$$

where $n_S = \phi_S(r_S) N_S$ is the measure of final producers that avoid supply disturbances.[20]

20. As with the market equilibrium, we solve the planner's problem in the Online Appendix allowing for Hicks-neutral productivity differences in all tiers of the supply chain.

With homogeneous production technologies for inputs in a given tier, the symmetry of equation (3) also dictates that equal quantities m_s be provided to a typical producer in tier $s+1$ by every one of its input suppliers, considering the relationships that have been formed and the suppliers that survive. A typical final producer has $n_S^u = \eta_S \phi_{S-1}\left(r_{S-1}\right) N_{S-1}$ suppliers. So equation (1) implies

$$x_S = l_S^{\gamma_S}\left(m_{S-1}\right)^{1-\gamma_S}\left[\eta_S \phi_{S-1}\left(r_{S-1}\right) N_{S-1}\right]^{\frac{1-\gamma_S}{\alpha_S}}.$$

Then, substituting for x_S in equation (21), we can write the planner's problem as choosing investments in protective capabilities, $\{r_s\}$, the thickness of supply networks, $\{\eta_s\}$, the input quantities, $\{m_s\}$, and the manufacturing employment levels, $\left\{l_s\right\}$, to maximize

$$W = \left[\phi_S\left(r_S\right) N_S\right]^{\frac{\varepsilon}{\varepsilon-1}} l_S^{\gamma_S}\left(m_{S-1}\right)^{1-\gamma_S}\left[\eta_S \phi_{S-1}\left(r_{S-1}\right) N_{S-1}\right]^{\frac{1-\gamma_S}{\alpha_S}} \tag{22}$$

subject to the various resource constraints. First, labor employed in all uses should not exceed the inelastic supply, or

$$\sum_{s=0}^{S} N_s r_s + \sum_{s=1}^{S} N_s k \eta_s N_{s-1} + \sum_{s=0}^{S} \phi_s\left(r_s\right) N_s l_s \leqslant L. \tag{23}$$

Second, the m_s units of inputs provided to the $\phi_{s+1}\left(r_{s+1}\right) N_{s+1}$ downstream producers by each of their $\eta_{s+1}\phi_s\left(r_s\right) N_s$ suppliers in tier s should not exceed the aggregate amount of tier s inputs produced, or

$$\begin{aligned}
&\left[\phi_{s+1}\left(r_{s+1}\right) N_{s+1}\right]\left[\eta_{s+1}\phi_s\left(r_s\right) N_s\right] m_s \leqslant \phi_s\left(r_s\right) N_s l_s^{\gamma_s}\left(m_{s-1}\right)^{1-\gamma_s} \\
&\quad \times \left[\eta_s \phi_{s-1}\left(r_{s-1}\right) N_{s-1}\right]^{\frac{1-\gamma_s}{\alpha_s}}, \\
&\text{for } s \in \left\{1, \ldots, S-1\right\},
\end{aligned} \tag{24}$$

where we have taken into account the Cobb-Douglas technology equation (3) available to the $\phi_s\left(r_s\right) N_s$ suppliers. Finally, the planner must not allocate more of the tier 0 input than can be produced by the $\phi_0\left(r_0\right) N_0$ surviving firms, or

$$\left[\phi_1\left(r_1\right) N_1\right]\left[\eta_1 \phi_0\left(r_0\right) N_0\right] m_0 \leqslant \phi_0\left(r_0\right) N_0 l_0, \tag{25}$$

in the light of the linear technology described by equation (6).

In the optimal allocation, the constraints are satisfied with equality. The first-order conditions with respect to labor l_s for all $s \in \left\{0, \ldots, S\right\}$ and input quantities m_s for all $s \in \left\{0, \ldots, S-1\right\}$

dictate that the optimal ratio of labor to aggregate inputs employed by a firm in tier s, $s \in \{1, \ldots S\}$, should equal $\frac{\gamma_s}{1-\gamma_s}\frac{\rho_{s-1}}{\omega}$, where ρ_s denotes the shadow value of a tier s input (the Lagrange multiplier on constraints (24) or (25), as the case may be), and ω denotes the shadow value of labor (the Lagrange multiplier on constraint (23)); this is the usual relationship between optimal cost shares that results from the Cobb-Douglas technology. Also, $\rho_0 = \omega$, because the planner can readily convert one unit of labor into one input of a tier 0 input. Therefore,

$$\frac{l_1^*}{n_1^u m_0^*} = \frac{\gamma_1}{1-\gamma_1}, \tag{26}$$

where asterisks indicate first-best allocations.

Next we can use the optimal input cost share in tier 1, $\rho_0 n_1^u m_0^* = (1-\gamma_1)\,\rho_1 n_1^d m_1^*$, and the fact that $\rho_0 = \omega$, to derive

$$\frac{l_2^*}{n_2^u m_1^*} = \gamma_1^{-\gamma_1}(1-\gamma_1)^{-(1-\gamma_1)}\frac{\gamma_2}{1-\gamma_2}\left(n_1^u\right)^{-\frac{1-\gamma_1}{\sigma_1-1}}, \tag{27}$$

where we have used the ratio of the optimal cost shares in tier 2, the relationship between $n_1^d m_1^*$ and $\left(m_0^*, l_1^*\right)$ implied by the production function (4), and the value of $\frac{l_1^*}{n_1^u m_0^*}$ that has been solved in equation (26). The right side of equation (27) represents the ratio of the Cobb-Douglas exponents in the production of tier 2 goods, adjusted for the productivity of the tier 1 inputs that reflect their variety. Proceeding similarly and recursively, we can compute the optimal input ratios $\frac{l_s^*}{n_s^u m_{s-1}^*}$ for $s \in \{3, \ldots, S\}$ using $\rho_{s-1} n_s^u m_{s-1}^* = (1-\gamma_s)\,\rho_s n_s^d m_s^*$ and the relationship between output $n_s^d m_s^*$ and inputs $\left(l_s^*, m_{s-1}^*\right)$ that is implied by equation (4). This gives us the optimal allocations of labor, $\left\{l_s^*\right\}_{s=0}^{S}$, and the optimal input quantities, $\left\{m_s^*\right\}_{s=0}^{S-1}$, for any numbers of active upstream and downstream relationships, $\left\{n_s^d\right\}_{s=0}^{S-1}$ and $\left\{n_s^u\right\}_{s=1}^{S}$.[21]

The first-best numbers of supply relationships at every tier result from optimal investments in protective capabilities and optimal investments in supplier links. In the Online Appendix, we show that the first-order conditions with respect to η_s, l_s, and m_{s-1}

21. Using the solutions for l_S^* and m_{S-1}^*, we can recover the optimal sales of a typical final good, x_S^*, from the production function.

together imply (see equations (A.111) and (A.112))

$$\frac{kN_sN_{s-1}\eta_s^*}{L-\sum_{j=0}^{S}N_jr_j^*-\sum_{j=1}^{S}kN_{j-1}N_j\eta_j^*}=\frac{\Gamma_s^S}{\sigma_s-1}$$

$$\text{for } s=\left\{1,2,\ldots,S-1\right\}, \tag{28}$$

and

$$\frac{kN_SN_{S-1}\eta_S^*}{L-\sum_{j=0}^{S}N_jr_j^*-\sum_{j=1}^{S}kN_{j-1}N_j\eta_j^*}=\frac{1-\gamma_S}{\sigma_S-1}, \tag{29}$$

where we recall that $\Gamma_s^S\equiv\Pi_{i=s}^S(1-\gamma_i)$ represents the product of the input shares in stages s and beyond. The left-hand side of equation (28) is the ratio of the aggregate amount of labor optimally used for forming supplier links in tier s to the aggregate labor optimally used in manufacturing. The right side of equation (28) reflects the cumulation of cost shares beginning with tier s and the elasticity of substitution between inputs used in that tier. The greater are the input shares downstream and the less substitutable are the inputs used in tier s, the more socially valuable are links to suppliers in tier $s-1$. Similarly, equation (29) equates the ratio of labor optimally used for forming supplier links in the final tier S relative to aggregate manufacturing labor with a measure of the social value of the marginal input to the lead producers.

As for the optimal investments in protective capabilities, we combine the first-order conditions with respect to r_s with the conditions for the optimal quantities, and find in equations (A.109) and (A.110) in the Online Appendix that

$$\frac{N_sr_s^*}{L-\sum_{j=0}^{S}N_jr_j^*-\sum_{j=1}^{S}kN_{j-1}N_j\eta_j^*}=\frac{\Gamma_{s+1}^S}{\sigma_{s+1}-1}\frac{\phi_s'\left(r_s^*\right)r_s^*}{\phi_s\left(r_s^*\right)}$$

$$\text{for } s=\left\{0,1,\ldots,S-1\right\} \tag{30}$$

and

$$\frac{N_Sr_S^*}{L-\sum_{j=0}^{S}N_jr_j^*-\sum_{j=1}^{S}kN_{j-1}N_j\eta_j^*}=\frac{1}{\varepsilon-1}\frac{\phi_S'\left(r_S^*\right)r_S^*}{\phi_S\left(r_S^*\right)}. \tag{31}$$

In equations (30) and (31), the left-hand side is the ratio of the aggregate labor optimally used to promote firm survival in some tier to the aggregate labor optimally used for manufacturing, and the right-hand side reflects the social benefits of survival at that tier. In all tiers, the benefits increase with the elasticity of survival probability with respect to investment. For intermediate goods, they also increase with the cost shares of intermediates in all tiers downstream from s and decrease with the elasticity of substitution between tier s inputs when used in tier $s+1$; firm survival is more valuable when inputs constitute a greater share of costs along the supply chain and when the inputs are imperfect substitutes. The survival of final good producers is socially more valuable when their outputs are less substitutable in the eyes of consumers.

We are ready to compare the equilibrium allocation described in Section II with the first-best allocation described here. To do so, we introduce three sets of policies that would allow the planner to implement the first-best allocation as a decentralized equilibrium.[22] These policies eliminate wedges between private and social incentives for each use of resources. We let $\boldsymbol{\tau} \equiv \{\tau_s\}_{s=0}^{S-1}$ be the vector of sales policies along the supply chain, where τ_s denotes the fraction of the cost of a tier s input paid by the downstream firm in tier $s+1$. Clearly, $\tau_s < 1$ represents a subsidy to promote sales from tier s to tier $s+1$, whereas $\tau_s > 1$ represents a tax. Similarly, we let $\boldsymbol{\theta} \equiv \{\theta_s\}_{s=0}^{S}$ be a vector of investment policies, where θ_s is the fraction (or multiple) of any investment in protective capabilities that is paid by firms in tier s. Finally, we let $\boldsymbol{\psi} \equiv \{\psi_s\}_{s=1}^{S}$ denote a vector of policies directed at network formation, where ψ_s denotes the fraction (or multiple) of the cost paid by a typical tier s producer when forming links to potential suppliers in tier $s-1$. We assume that subsidies are financed by lump-sum taxation and revenues are rebated similarly. We discuss the wedges in turn.

III.B. Eliminating Wedges in Input Procurement

We take the policies directed at the two aspects of resilience as given and derive the optimal transaction policies conditional on the levels of these other policies. We let $T_s(\boldsymbol{\theta}, \boldsymbol{\psi})$ for $s = \{0, 1, \ldots, S-1\}$ denote the functional relationship between the optimal policy directed at sales by a firm in tier s to a firm in tier $s+1$ and the vectors of policies pertaining to investments in protective capabilities and link formation.

Consider first the scale of transactions between firms in tier 0 and tier 1. In the Nash-in-Nash bargaining solution, a pair of negotiants choose m_0 to maximize their joint surplus, taking as given the quantities in other relationships. When the downstream

22. The private and social incentives for resource allocation diverge on three margins, for m_s, r_s, and η_s. Therefore, three policy instruments are necessary and sufficient to implement the first-best allocation.

firm pays only the fraction τ_0 of what the upstream firm receives, the Nash bargain in equation (7) must be amended to read

$$m_0 = \left(\frac{1-\gamma_1}{\gamma_1\tau_0}\right)^{\gamma_1}\left[n_1^u\right]^{\frac{\gamma_1-\sigma_1}{\sigma_1-1}} n_1^d m_1.$$

Then, using the technological constraints in equations (4) and (6), this implies

$$\frac{l_1}{n_1^u m_0} = \frac{\gamma_1}{1-\gamma_1}\tau_0. \tag{32}$$

Now compare the left-hand side of equation (32), which is the equilibrium ratio of labor to intermediate inputs employed by a tier 1 firm, to the optimal ratio expressed in equation (26). We see that the social planner can implement the socially desirable transactions between these firms by leaving these decisions entirely to the discretion of the private parties, no matter what the levels of the investment policies. In other words, $T_0(\boldsymbol{\theta}, \boldsymbol{\psi}) = 1$ for all $\boldsymbol{\theta}$ and $\boldsymbol{\psi}$.

Why are private and social incentives aligned for these transactions between the most upstream firms? With sequential bargaining, the negotiations between tier 0 firms and tier 1 firms are the last to take place. A deal that emerges at this stage does not affect any other transactions. Because the outcome of this bargaining generates no externalities, what remains is a desire for joint efficiency in production, which the firms share with the social planner. Put differently, when the most upstream firms bargain, the potential surplus for the pair reflects the private marginal cost of producing the tier 0 input. But the private marginal cost mirrors the social marginal cost, because only labor is used in its production. It follows that the planner need not intervene in these upstream transactions.

Next consider the private incentives in a negotiation between a tier 1 firm and a tier 2 firm. The joint-surplus maximization in the Nash bargaining implies

$$\frac{l_2}{n_2^u m_1} = \frac{\gamma_2}{1-\gamma_2}c_1\tau_1, \tag{33}$$

where we recall that c_1 is the marginal cost of a unit of the tier 1 input, including both the labor cost and the cost of acquiring the tier 0 input bundle, and the product $c_1\tau_1$ is the cost per unit borne by the buyer after the subsidy (or tax). The left-hand side of equation (33) represents the ratio of physical quantities of labor to produced inputs in tier 2 production, and the right-hand side is the ratio of the private factor costs multiplied by the ratio of the optimal factor shares implied by the Cobb-Douglas technology.[23] Using the expression for c_1 in equation (8), we see that the planner must intervene in these transactions to induce the efficient techniques in equation (27). The efficient factor ratio requires $B_1\tau_1 = 1$, or

$$T_1(\boldsymbol{\theta}, \boldsymbol{\psi}) = \frac{1}{\gamma_1 + (1-\gamma_1)\,\mu_0} < 1, \qquad \text{for all } \boldsymbol{\theta} \text{ and } \boldsymbol{\psi}. \tag{34}$$

The required subsidy on sales of tier 1 inputs to tier 2 producers reveals a divergence between private and social incentives. In the absence of any policy, the pair will negotiate based on an anticipated private marginal cost of producing the tier 1 input that reflects the markup that will ensue when the tier 1 firm purchases inputs from its tier 0 suppliers. As noted, $B_1 = \gamma_1 + (1-\gamma_1)\,\mu_0$ measures how much this anticipated markup distorts the marginal cost of producing tier 1 inputs. The inflated private cost would lead the two firms to transact too little. The optimal subsidy counteracts this distortion, ensuring that the parties consider the social cost of producing tier 1 inputs when they design their procurement contracts.

Notice that the requisite transaction policy in equation (34) does not depend on the numbers of firms in tier 1 or tier 2. Therefore, it does not depend on the policies directed at investments in resilience. Intuitively, the policy only must correct the distortion introduced by markups that are anticipated in contracts that will subsequently be negotiated by firms in tier 1, which are constants in our setting.

In the Online Appendix, we show that the wedges between private and social incentives in transactions between firms in tier s and their customers in tier $s+1$ can be eliminated by a set of transaction policies that satisfy[24]

$$T_s(\boldsymbol{\theta}, \boldsymbol{\psi}) = \frac{1}{\gamma_s + (1-\gamma_s)\,\mu_{s-1}} < 1,$$

$$\text{for all } s \in \{1, \dots, S-1\} \text{ and all } \boldsymbol{\theta} \text{ and } \boldsymbol{\psi}. \tag{35}$$

The logic for all of the subsidies is similar; in each negotiation, the private parties in tiers s and $s+1$ face a distorted marginal

23. Recall that the wage is the numeraire, so $c_1\tau_1$ represents the cost of a unit of an intermediate input relative to the cost of labor.

24. In fact, we show in the Online Appendix that equation (35) gives the optimal subsidy even when firms in a tier are heterogeneous in their productivities.

cost of the good they are transacting, because the producer of the tier s good anticipates paying an elevated price for its own inputs in its subsequent negotiations. At each stage, the planner offsets the anticipated markup, thereby ensuring that the firms in s and $s+1$ choose the efficient quantities. The optimal subsidy declines with the elasticity of substitution between tier $s-1$ inputs in producing tier s goods, because greater substitutability between these inputs weakens the bargaining position of the suppliers and reduces the markup. The optimal subsidy falls with the labor share of cost in producing the tier s inputs, because a higher γ_s implies that a given markup of input prices has a smaller effect on the marginal cost of m_s. None of the optimal sales policies vary with the policies that apply to investments in protective capabilities or to investments in link formation.

We record our findings in the following lemma.

LEMMA 1. The transaction subsidies that eliminate the wedges between private and social incentives in bargaining over input sales are independent of the investment policies, $\boldsymbol{\theta}$ and $\boldsymbol{\psi}$, and are given by $T_0(\boldsymbol{\theta},\boldsymbol{\psi}) = 1$ and $T_s(\boldsymbol{\theta}, \boldsymbol{\psi}) = \frac{1}{\gamma_s + (1-\gamma_s)\mu_{s-1}}$ for $s \in \{1, \ldots, S-1\}$.

Notice that if all negotiations give identical weights to the sellers and if inputs have identical cost shares, then all subsidies for tiers $s \geqslant 1$ will be the same. Alternatively, if inputs become more specialized (and thus strictly less substitutable) as a good proceeds down the supply chain (so that μ_{s-1} rises with s), and if bargaining weights and labor shares are the same all along the chain, then the optimal transaction subsidies rise monotonically as we move downstream.

The planner need not apply any subsidy or tax to sales of the final good.[25] Although the lead producers charge prices in excess of their marginal costs, the markups are common to all final goods and do not distort any consumption decisions.

25. The planner has a degree of freedom with regard to optimal taxes or subsidies on final goods. As long as the same tax or subsidy rate applies to all final goods, the consumers' purchase decisions will not be distorted.

III.C. Eliminating Wedges in Choices of Protective Capabilities and Network Thickness

We consider policies directed at investments in resilience. We denote by $\boldsymbol{\Theta}(\boldsymbol{\tau}) \equiv \{\Theta_s(\boldsymbol{\tau})\}$ the vector of investment policies that would eliminate the wedges between private and social incentives in the choices of $\{r_s\}$ for all $s \in \{0, \ldots, S\}$ and by $\boldsymbol{\Psi}(\boldsymbol{\tau}) \equiv \{\Psi_s(\boldsymbol{\tau})\}$ the vector of policies that would eliminate wedges between private and social incentives in the choices of $\{\eta_s\}$ for all $s \in \{1, \ldots, S\}$, both conditional on an arbitrary vector of transaction policies $\boldsymbol{\tau}$.

In the Online Appendix, we show that

$$\Theta_s(\boldsymbol{\tau}) = \frac{1-\beta_{s+1}}{\tau_s} \frac{1}{J(\boldsymbol{\tau}) \prod_{j=s+1}^{S-1} B_j \tau_j}, \quad \text{for } s \in \{0, 1, \ldots, S-1\}, \tag{36}$$

and

$$\Theta_S(\boldsymbol{\tau}) = \frac{1 - (1-\beta_S)\frac{(1-\gamma_S)(\varepsilon-1)}{\sigma_S - 1}}{J(\boldsymbol{\tau})}, \tag{37}$$

where

$$J(\boldsymbol{\tau}) \equiv (1-\gamma_S)\left[\frac{\gamma_S}{1-\gamma_S} + \sum_{j=1}^{S-1} \frac{\gamma_j \Gamma_{j+1}^{S-1}}{\prod_{z=j}^{S-1} B_z \tau_z} + \frac{\Gamma_1^{S-1}}{\tau_0 \prod_{z=1}^{S-1} B_z \tau_z}\right].$$

There are four terms on the right side of equation (36) that characterize the wedge between the private and social incentives for investment in protective capabilities for a producer of an intermediate input in tier s. First, a firm in tier s garners only the fraction $1-\beta_{s+1}$ of the joint surplus in its relationship with customers in tier $s+1$. The smaller this share, the smaller the firm's incentive to invest in protective capabilities. The planner, in contrast, is concerned with the total surplus, not the division between the parties. For this reason, the surplus sharing tends to generate underinvestment in protective capabilities by firms all along the supply chain. Second, the planner applies a subsidy $1-\tau_s$ to sales by firms in tier s to customers in tier $s+1$. These subsidies artificially boost profitability for the input seller, which tends to incentivize investments in protective capabilities beyond their social value. Third, the term $\prod_{j=s+1}^{S-1} B_j \tau_j$ measures, for an arbitrary set of transaction policies $\boldsymbol{\tau}$, the distortion that remains in the subsidy inclusive per unit cost of inputs to firms in tiers downstream from s. When $B_j \tau_j > 1$ for some or all $j > s$, the derived demand for an input in tier s will be depressed relative to what it would be without any transaction distortions. The shortfall in demand diminishes the profitability of firms in tier s; see equations (A.63) and (A.64) in the Online Appendix. Accordingly, the downstream distortions reduce by this channel the incentives for firms in tier s to invest in their protective capabilities. But $J(\boldsymbol{\tau})$ captures an offsetting distortion that can arise in the aggregate labor market. When $B_j \tau_j > 1$ for some j anywhere in the supply chain, the cost distortion depresses demand for manufacturing labor in the tier that uses this input. This shortfall in labor demand reduces the real wage relative to what it would be in the absence of such a distortion, which in turn raises profitability in all tiers. To the extent that the augmented profitability reflects a real wage below the shadow value of labor, it contributes to an excessive private incentive for investment in protective capabilities.[26]

Concerning the formula for $\Theta_S(\boldsymbol{\tau})$ in equation (37), we note that there is no subsidy to sales by final producers and no activity downstream from S. All that remains is for the planner to induce producers of final goods to internalize the positive externalities for consumers generated by their presence in the marketplace and to correct any excess incentive for investment in protective capabilities that results from a depressed real wage.

We summarize these arguments in Lemma 2.

LEMMA 2. For an arbitrary vector of transaction policies $\boldsymbol{\tau}$, the government can eliminate the wedge between private and social incentives for investments in protective capabilities by having firms bear the fraction of investment costs in tier $s \in \{0, 1, \ldots, S-1\}$ given in equation (36) and the fraction of investments costs in tier S given in equation (37).

Similar considerations come into play when we consider policies directed toward investments in network thickness. Firms in tier s tend to have insufficient incentive to form links with upstream suppliers, because they capture only a fraction of the surplus created by such investments. Meanwhile, the sales by firms in tier s may be subsidized, generating private profits that are not part of social surplus. These extra profits tend to incentivize

26. In the Online Appendix, we show that the formulas in equations (36) and (37) continue to apply when firm productivities within tiers are heterogeneous.

excess investments in network formation. Also, to the extent that transaction policies do not fully correct distortions in input sales, the remaining distortions downstream from any tier s depress derived demand for the input produced by firms in tier s and thus profitability in that tier. Finally, an uncorrected distortion in transaction size in any tier upstream or downstream from s alters the aggregate demand for manufacturing labor and with it the equilibrium real wage. If the real wage falls below the shadow value of labor, this tends to promote overinvestment in links by firms in tier s.

To get a handle on whether subsidies to network formation ought to be bigger or smaller than those for investments in protective capabilities, we compare the equilibrium ratio of investments in protective capabilities relative to network thickness in tier s when the two forms of resilience are subsidized or taxed at the same rate with the ratio of investments that satisfies the planner's first-order condition for maximizing social welfare. Concerning the private incentives, firms in tier s will invest more in relationships when the cost share of inputs is large (γ_s small), when diversity adds more to productivity (σ_s small), and when their own output substitutes more closely for that of their competitors (σ_{s+1} large), which allows them to steal more sales and profits from rivals following a reduction in cost. None of these parameters directly affects a firm's incentives to invest in protective capabilities, except inasmuch as they affect the level of operating profits. Using equations (17) and (18) and the relationship between operating profits and network thickness in equation (19), we show in equation (A.121) in the Online Appendix that when $\theta_s = \psi_s$,

$$\frac{r_s}{\eta_s} = \frac{\sigma_s - 1}{(1-\gamma_s)(\sigma_{s+1}-1)} \frac{r_s \phi'(r_s)}{\phi(r_s)} k N_{s-1}. \tag{38}$$

The calculus for the social planner is seemingly different. The social benefits from relationship links for firms in tier s increase with the input share in tier s, but also with the input shares in all tiers downstream from s. Whereas imperfect substitutability of inputs used in tier s (σ_s small) raises the marginal social benefit from having additional suppliers, the substitutability between the inputs used in tier $s+1$ has no bearing on the marginal benefit, because the planner does not care about the distribution of profits among firms in tier s.[27] Meanwhile, the social benefit from investments in protective capabilities in tier s reflects the input share in tiers $s+1$ and beyond and they are larger when the tier s inputs are less close substitutes for their customers. Dividing the first-order condition for r_s^* (equation (30)) by that for η_s^* (equation (28)), we find[28]

$$\begin{aligned} \frac{r_s^*}{\eta_s^*} &= \frac{\Gamma_{s+1}^S}{\Gamma_s^S}\frac{\sigma_s-1}{\sigma_{s+1}-1}\frac{r_s^*\phi'\left(r_s^*\right)}{\phi\left(r_s^*\right)}\frac{kN_sN_{s-1}}{N_s} \\ &= \frac{\sigma_s-1}{\left(1-\gamma_s\right)\left(\sigma_{s+1}-1\right)}\frac{r_s^*\phi'\left(r_s^*\right)}{\phi\left(r_s^*\right)}kN_{s-1}. \end{aligned} \tag{39}$$

Notice that the expression in the second row of equation (39) is identical to that on the right side of equation (38). Evidently, when the two forms of investment in resilience are subsidized or taxed at the same rate, the relative private incentives to invest in the alternative forms of resilience coincide with the social imperative. For any arbitrary vector of transaction policies $\boldsymbol{\tau}$, the planner has no desire to encourage or discourage investments in network thickness relative to investments in protective capabilities. She preserves relative incentives by setting the two investment policies at the same rates; $\Psi_s(\boldsymbol{\tau}) = \Theta_s(\boldsymbol{\tau})$ for $s \in \{1, \ldots, S-1\}$ and $\Psi_S(\boldsymbol{\tau}) = \Theta_S(\boldsymbol{\tau})$.

We state Lemma 3 for future reference.

LEMMA 3. For any arbitrary transaction policies $\boldsymbol{\tau}$, the policies that eliminate the wedges between private and social incentives for investment in network thickness are identical to those that eliminate the wedges between private and social incentives for investment in protective capabilities; $\Psi_s(\boldsymbol{\tau}) = \Theta_s(\boldsymbol{\tau})$ for $s \in \{1, \ldots, S-1\}$ and $\Psi_S(\boldsymbol{\tau}) = \Theta_S(\boldsymbol{\tau})$.

In the Online Appendix, we show that this lemma continues to apply in an extended model with heterogeneous productivities.

Admittedly, Lemma 3 relies on special features of our model. First, the Nash-in-Nash bargaining protocol generates constant

27. In other contexts, the private incentive to enter or invest to capture profits at the expense of rivals has been called the "business-stealing effect," and it generally tends to cause overinvestment relative to the social optimum.

28. Equation (A.61) of the Online Appendix shows that welfare is multiplicatively separable in a term that depends on $\boldsymbol{r}$ and $\boldsymbol{\eta}$ and one that depends on $\boldsymbol{\tau}$. It follows that the optimal investment levels, r_s^* and η_s^*, are independent of the vector of transaction policies, $\boldsymbol{\tau}$.

"markups" that firms cannot manipulate by their choice of network thickness. Second, all firms in our model are small, so they cannot manipulate the general equilibrium in a way that improves their bargaining position vis-à-vis their suppliers or customers. Finally, the CES production technology creates a tight relationship between the positive externalities from investments in resilience that accrue to downstream customers and the negative externalities suffered by competing firms due to the loss of sales and profits. The offsetting "consumer-surplus" externality and "business-stealing externality" are familiar from other contexts with CES technologies (or preferences) and ex ante investments (in market entry or cost reduction).[29]

III.D. Implementing the First Best

Finally, we are ready to characterize the package of policy interventions that would implement the first-best allocation of resources. We denote the vectors of first-best policies by $\boldsymbol{\tau}^*, \boldsymbol{\theta}^*$, and $\boldsymbol{\psi}^*$. These policies must satisfy $\boldsymbol{\tau}^* = \boldsymbol{T}\left(\boldsymbol{\theta}^*, \boldsymbol{\psi}^*\right)$, $\boldsymbol{\theta}^* = \boldsymbol{\Theta}\left(\boldsymbol{\tau}^*\right)$, and $\boldsymbol{\psi}^* = \boldsymbol{\Psi}\left(\boldsymbol{\tau}^*\right)$; that is, each policy must be optimal given the optimal choices of the others.

Recall that the optimal transaction policies do not vary with the number of surviving firms or with network thickness. Therefore, $\tau_0^* = 1$ and $\tau_s^* = \frac{1}{\gamma_s + (1-\gamma_s)\mu_{s-1}}$ for $s \in \{1, \ldots, S-1\}$, per Lemma 1. Since consumption choices are not distorted, the planner can set $\tau_S^* = 1$ or at any other (uniform) level.

With the optimal transaction policies in place, $\prod_{j=s+1}^{S-1} B_j \tau_j^* = 1$ for all $s \in \{0, 1, \ldots, S-1\}$ and $J(\boldsymbol{\tau}^*) = 1$. That is, there are no distortions downstream from s to depress derived demand and profits in tier s, and no general-equilibrium effects of markups to distort the equilibrium real wage. Then Lemma 2 implies $\theta_s^* = \frac{1-\beta_{s+1}}{\tau_s^*}$ for $s \in \{0, 1, \ldots, S-1\}$ and $\theta_S^* = 1 - \frac{(1-\beta_S)(1-\gamma_S)(\varepsilon-1)}{\sigma_S - 1}$. Finally, Lemma 3 implies $\psi_s^* = \theta_s^*$ and $\psi_S^* = \theta_S^*$. We have thus proven:

PROPOSITION 1. The first-best allocation of resources can be achieved as a market equilibrium with taxes or subsidies on input transactions, on investments in protective capabilities, and on investments in network formation, with

(i) $\tau_0^* = 1$ and $\tau_s^* = \frac{1}{\gamma_s + (1-\gamma_s)\mu_{s-1}}$ for $s \in \{1, \ldots, S-1\}$,

29. See, for example, Tirole (1988, ch. 7), Matsuyama (1995), Dhingra and Morrow (2019), and Matsuyama and Uschev (2021).

(ii) $\theta_0^* = 1 - \beta_1$, $\theta_s^* = \psi_s^* = \frac{1-\beta_{s+1}}{\tau_s^*}$ for $s \in \{1, \ldots, S-1\}$ and $\theta_S^* = \psi_S^* = 1 - (1-\beta_S)\,\frac{(1-\gamma_S)(\varepsilon-1)}{\sigma_S - 1}$,

(iii) purchases of all final goods taxed or subsidized at uniform rates, including zero.

The first-best transaction policies subsidize all input purchases except those at the very top of the supply chain. The subsidies offset the markups that result from tier-to-tier bargaining. When the optimal transaction subsidies are in place, the optimal policies for resilience reflect only a trade-off between the excess incentives for investment generated by the subsidies to sales and the insufficient incentives that result from surplus sharing in the Nash bargaining. The optimal policy for investment in protective capabilities does not depend on properties of the function $\phi_s(r_s)$ that relates the probability of a disruption to the size of the investment. Although the elasticity of $\phi_s(r_s)$ affects the planner's preferred resilience (see equation (30)), that same elasticity also affects the firms' private incentives to avoid disturbances in much the same way. Moreover, the optimal investment policies depend only on the bargaining weights for firms in their negotiations with their downstream customers and on the optimal subsidy on their purchases from their upstream suppliers. Since there is no subsidy for purchases of tier 0 inputs ($\tau_0^* = 1$), the planner always wishes to promote investment in protective capabilities in the most upstream tier of the supply chain ($\theta_0^* = 1 - \beta_1 < 1$). It might be that other far upstream inputs are highly substitutable, in which case the transaction subsidies for these tiers will be small. Then, with τ_s^* close to one, the optimal policy promotes investments in protective capabilities and relationship links in other upstream tiers as well. Further downstream, inputs may become more specialized and less substitutable. If the elasticity of substitution between inputs falls monotonically (and strictly) as a good moves downstream, and if bargaining weights and labor shares do not vary along the chain, then the optimal subsidies for investment in protective capabilities and network formation will decline monotonically and may eventually turn from subsidy to tax. A tax on investments in the alternative forms of resilience will be indicated when a large markup of input costs must be offset by a large transaction subsidy, which then inflates greatly the private incentives for investment.

IV. Second-Best Policies for Resilience

The salience of recent supply-chain disruptions has directed attention to what governments might do to promote greater chain resilience. In the current environment, policies that encourage firms to invest in reducing the likelihood of disruptions or in diversifying their input sources might be politically palatable even when direct subsidies to their sales are not. To address this apparent political reality, we consider in this section a second-best setting in which the government can subsidize investments in protective capabilities and network formation but cannot bankroll firm-to-firm transactions along the supply chain.

The government's problem is the same as before, except that we impose $\tau_s = 1$ for all s. We denote by θ_s° the fraction of the cost of investing in protective capabilities paid by a firm in tier s, $s \in \{0, 1, \ldots, S\}$, in the second-best regime. Similarly, ψ_s° is the share of the cost of network formation borne by a firm in tier s, $s \in \{1, 2, \ldots, S\}$.

We can apply Lemmas 2 and 3 to find the second-best subsidies for investments in protective capabilities and for investments in network links, noting that $\theta_s^\circ = \Theta_s(\mathbf{1})$ for $s \in \{0, 1, \ldots, S\}$ and $\eta_s^\circ = \theta_s^\circ$ for $s \in \{1, \ldots, S\}$. Using equations (36) and (37), this gives

$$\theta_s^\circ = \frac{1}{J(\mathbf{1})}\left\{\frac{1-\beta_{s+1}}{\prod_{j=s+1}^{S-1}\left[\gamma_j + (1-\gamma_j)\,\mu_{j-1}\right]}\right\} \quad \text{for } s \in \{0, 1, \ldots, S-1\} \tag{40}$$

and

$$\theta_S^\circ = \frac{1}{J(\mathbf{1})}\left[1 - \frac{(1-\beta_S)(1-\gamma_S)(\varepsilon-1)}{\sigma_S - 1}\right], \tag{41}$$

and $J(\mathbf{1}) < 1$.[30]

How do we understand the expressions for the second-best subsidies (or taxes) on investments in protective capabilities? The first thing to note is that in the sequential bargaining equilibrium, the planner's objective, W, is multiplicatively separable in

30. Note that

$$J(\mathbf{1}) = \gamma_S + \sum_{j=1}^{S-1}\frac{\gamma_j\Gamma_{j+1}^S}{\prod_{s=j}^{S-1}B_s} + \frac{\Gamma_1^S}{\prod_{s=1}^{S-1}B_s} < \gamma_S + \sum_{j=1}^{S-1}\gamma_j\Gamma_{j+1}^S + \Gamma_1^S = 1,$$

because $B_s > 1$ for every s.

a term that depends on $\{r_s\}$ and $\{\eta_s\}$ and a term that reflects the sizes of the transaction subsidies, $\{\tau_s\}$ (see equation (A.61) in the Online Appendix). This separability follows from the assumption of CES technologies and preferences, with their multiplicative aggregation properties. It implies that the planner targets the same investment levels irrespective of any transaction subsidies that may be imposed. Evidently, the second-best investment levels, $\{r_s^\circ\}$ and $\{\eta_s^\circ\}$, are the same as the first-best levels, $\{r_s^*\}$ and $\{\eta_s^*\}$ that are reported in equations (28), (30), and (31).

However, the private incentives for ex ante investments do vary with the transaction policies, because these policies affect operating profits. To achieve the *same investment levels*, $\{r_s^*\}$ and $\{\eta_s^*\}$, in a second-best equilibrium, the planner must impose different policies than prescribed in part (ii) of Proposition 1.

As in the first-best setting, the planner must account for the positive externality associated with a firm's survival as a supplier. The upstream firm in every relationship captures only the fraction $1-\beta_{s+1}$ of the social surplus from any investment in protective capabilities, while the remaining fraction β_{s+1} accrues to firms downstream. The second-best subsidies induce firms to invest based on the full surplus, rather than their negotiated shares. This externality accounts for the term $1-\beta_{s+1}$ in the numerator of equation (40), just as it figures in the first-best subsidy rate in part (ii) of Proposition 1.

However, the lack of transaction subsidies leaves in place the negotiated markups that distort tier-to-tier transactions. These distortions figure in the denominator of the term in the curly brackets in equation (40). The agreed payments that exceed production costs reduce profitability at every stage; see equation (A.63) in the Online Appendix. Consequently, they dim the incentives for investments in protective capabilities. In particular, since $B_j = \gamma_j + (1-\gamma_j)\,\mu_{j-1} > 1$ for all j, the denominators in the curly brackets all exceed one and thus contribute to even larger investment subsidies for every tier than are implied by the surplus sharing. But note that the uncorrected distortions do not affect profitability equally across tiers. Since the negotiated markups cumulate as we move downstream, the upstream firms lose more in sales and profits than do their counterparts downstream. This double marginalization points to the need for larger investment subsidies upstream than downstream.[31]

Overall, the term in curly brackets suggests the desirability of second-best subsidies for investments in protective capabilities all along the supply chain. However, this conclusion may not be warranted when we consider the role of $J(\mathbf{1})$. The term $J(\mathbf{1})$ captures the fact that the cost distortions collectively depress the demand for manufacturing labor. The resulting fall in the real wage raises profitability and incentives for ex ante investment. The smaller is $J(\mathbf{1})$, the smaller are the second-best subsidies, and taxes may be needed in some downstream tiers to induce the socially efficient investment levels.

We can readily compare the second-best subsidies at different points in the supply chain. Let us begin with second-best policy for investments in tier 0. We see that $J(\mathbf{1})\,\Pi_{j=1}^{S-1}\left[\gamma_j + \left(1-\gamma_j\right)\mu_{j-1}\right] > 1$; that is, the general equilibrium effect of the subsidies cannot outweigh the strongest of the direct effects.[32] Since, with $s = 0$, the numerator in equation (40) is less than one and the denominator exceeds one, it follows that

$$\theta_0^\circ < 1;$$

that is, in the second-best regime, it is always optimal for the government to subsidize investments in protective capabilities in the most upstream tier.

Turning to the relationship between the second-best subsidies in successive tiers, we have from equation (40) that

$$\frac{\theta_{s-1}^\circ}{\theta_s^\circ} = \frac{1-\beta_s}{1-\beta_{s+1}}\left[\frac{1}{\gamma_s + (1-\gamma_s)\,\mu_{s-1}}\right].$$

Thus, if $\beta_{s+1} \leqslant \beta_s$, then $\theta_{s-1}^\circ < \theta_s^\circ$; that is, if bargaining weights are constant or decreasing along the supply chain, the second-best subsidies to investments in protective capabilities shrink as

31. The fact that $B_j > 1$ for all j implies that the denominator grows monotonically as we add more terms to the product.

32. Note that

$$J(\mathbf{1})\prod_{j=1}^{S-1} B_j = \Gamma_1^S + \sum_{j=1}^{S-1}\frac{\gamma_j}{1-\gamma_j}\Gamma_j^S\prod_{s=j+1}^{S-1} B_s + \gamma_S$$

$$> \Gamma_1^S + \sum_{j=1}^{S-1}\frac{\gamma_j}{1-\gamma_j}\Gamma_j^S + \gamma_S = 1.$$

we proceed downstream. In the absence of transaction subsidies, and with $\beta_{s+1} \leqslant \beta_s$, the social imperative for resilience is greater for the upstream firm in any supplier-buyer relationship, due to the cumulation of cost distortions.

How do the second-best policies toward investments in protective capabilities compare with the first best? Both policies address the externality that results from rent sharing, as reflected in the bargaining weight, $1 - \beta_{s+1}$. Beyond that, they address different distortions: excess private profitability created by transaction subsidies on the one hand, and contraction of downstream input demand caused by uncorrected markups on the other. As a result, these subsidies are not directly comparable. If the denominator of equation (40) exceeds one, as is mostly likely for firms that are far upstream, then $\theta_s^\circ < \theta_s^*$; that is, the optimal second-best subsidy to resilience must exceed the first-best subsidy at tier s. This is a situation in which the downstream contraction of input demand caused by the successive markups leads to a substantial underinvestment in resilience in the absence of policy. However, if the product in the denominator is sufficiently less than one, as it may be for firms far downstream, then the second-best subsidy to investments in resilience may be smaller than the first best. Comparing equation (41) with the expression for θ_S° in part (ii) of Proposition 1, we see that $\theta_S^\circ = \psi_S^\circ > \theta_S^* = \psi_S^*$; that is, the government always shaves the second-best subsidy to investments in resilience by final producers relative to the first best; for these firms, there are no downstream distortions, but the markups upstream boost their overall profitability, which tends to lead them to overinvest in resilience compared to the incentives they see in the first best.

We summarize our findings about the second-best policies:

PROPOSITION 2. For all $s \geqslant 1$, the second-best policies for link formation are equal to those for investments in protective capabilities. To achieve the second best, the government subsidizes investments in protective capabilities in the most upstream tier. If $\beta_{s+1} \leqslant \beta_s$ for all $s \in \{0, 1, \ldots, S-1\}$, the second-best subsidies decline with s, and may require a tax for the most downstream tiers. The second-best subsidies may be larger or smaller than their first-best counterparts for $s < S$, but the second-best subsidies (taxes) for investments by final producers are always smaller (larger) than the first-best subsidies (taxes).

V. Concluding Remarks

We have identified several sources of inefficiency in the market equilibrium of an economy with multitier supply chains and endogenous determination of firms' resilience to supply disturbances. First, in the absence of government policy, firms in adjacent tiers of the supply chain will not choose the socially optimal volume of input sales. Instead, they will negotiate a contract that calls for more limited sales in anticipation that the supplier will face a marked-up cost of its own inputs when it subsequently bargains with its own suppliers. The wedge between the private and social incentives for input transactions dictates an optimal subsidy on input sales in all transactions other than between the firms that are most upstream. Second, firms in every tier will not on their own choose the socially optimal investments to avoid their own supply disturbances. On the one hand, these investments tend to be socially insufficient because firms do not take account that their survival affects the profitability of their downstream customers. On the other hand, these investments may be socially excessive if the optimal subsidy for sales creates a large profit boost that comes at the expense of the public finances. If the bargaining weights and the labor shares do not vary across tiers but inputs become less substitutable as we move down the supply chain, the optimal subsidies for investments in protective capabilities will be largest upstream and decline monotonically, possibly turning to an optimal tax at some point in the chain. Neither the optimal subsidies on sales nor the optimal subsidies for investments in protective capabilities depend on the number of backward links formed by suppliers, and thus the same subsidies apply for networks of arbitrary length. Finally, we find a wedge between private and social incentives for firms to form thick supply networks as a hedge against disturbances that might befall their suppliers. As with investments in protective capabilities, firms do not take account that their relationships generate surplus for downstream partners. When firms are too small to use the number of their relationships to manipulate their bargaining position vis-à-vis their suppliers and customers, the optimal policy toward network formation coincides with the optimal policy to promote or discourage investments in protective capabilities.

Political realities may limit the scope for subsidies to firm-to-firm transactions. If so, the government's choice of whether and how to promote resilience takes on a second-best flavor. We considered optimal policies for investments in protective capabilities and for the formation of supplier relationships when a government lacks the ability to use subsidies to counteract the distortionary effects of negotiated input payments. In this setting, optimal policies reflect markups and input shares in all transactions downstream from a targeted tier. Survival and supplier relationships are more socially valuable at upstream stages than at downstream stages due to the cumulative effects of double marginalization. If bargaining weights and production parameters are common across tiers, then the second-best subsidies for investments in protective capabilities and in supplier relationships are larger for producers further upstream. This contrasts with the first-best subsidies, which are constant along the interior of the supply chain when bargaining weights and production parameters are common to all tiers.

We have modeled vertical supply chains in a stylized but realistic way that captures many of the features described in the more descriptive literature. Each firm has multiple suppliers and multiple customers. Bargaining happens sequentially, beginning with final producers that purchase intermediate goods to use in their production processes and proceeding upstream to suppliers that seek inputs to fulfill their procurement contracts. Our bilateral negotiations involve a single buyer and a single seller, not grand coalitions of producers at various stages. Firms form their networks of potential suppliers by investing in bilateral relationships. Resilience reflects deliberate investment. Yet as with all models of firm-to-firm dealings, the details matter and we recognize that a variety of alternative assumptions may be worthy of further consideration.

First, we have assumed a particular timing and a particular form of contracts. In our model, bargaining between upstream and downstream firms takes place after the realization of the supply shocks and firms negotiate only with partners that escape these disturbances. If negotiations were to occur before any disruptions, this would open a role for contingent contracts. Payments might be contingent on contract fulfillment, with penalties for failure to deliver. Payments might also be contingent on the size of an upstream firm's investment in resilience. Even more sophisticated contracts might allow payments contingent on the resilience of a supplier's own upstream suppliers, or on a firm's realized production costs. Richer contracts would allow firms to mitigate the inefficiencies of double marginalization and internalize to some extent the externalities that their resilience confers on downstream customers. Complex contracts that allow for payments based on decisions throughout the network might be needed to achieve full efficiency, especially in a second-best setting in which the government cannot subsidize firm-to-firm transactions. The externalities that we highlight would likely still be relevant even in a world with a wider menu of contracts.

Second, if downstream firms could observe investments in protective capabilities before they form their supply networks, they might seek out partners that are more likely to deliver. This would give upstream firms greater incentive to make such investments, thereby mitigating the externality associated with shared benefits. Even if firms could not observe investments before creating their supply chains, they might infer something about such investments if potential suppliers differed in some observable primitives that would affect their incentives to invest.

Finally, our model features only idiosyncratic supply shocks and only one place of production. An obvious extension would be to consider correlated shocks, based, for example, on geography. These would seem particularly important if combined with an extension to global supply chains. The presence of correlated shocks would interact with the possibilities for contract contingencies, as penalties for breach might differ for failures that are specific to a firm versus those that result from more widespread disturbances that are outside a single firm's control. Analyzing optimal unilateral policy and optimal cooperative policy toward resilience in global supply chains will require that cross-country differences in wages, production technologies, and risks of disturbances be taken into account. We regard the modeling of global supply chains with endogenous networks and resilience as an important direction for future research.

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Supplementary Material

An Online Appendix for this article can be found at *The Quarterly Journal of Economics* online.

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