Journal of Economic Literature
Vol. XXVIII (June 1990), pp. 638–648

L. V. Kantorovich: The Price Implications of Optimal Planning

By ROY GARDNER
Indiana University

The author wishes to thank Robert Becker, Andrei Belykh, Robert Campbell, Nicolas Spulber, and two anonymous referees, without whose help this paper would not have been possible. All errors of fact or interpretation are the sole responsibility of the author.

IN 1975 THE NOBEL PRIZE for economics was awarded to Tjalling C. Koopmans and Leonid V. Kantorovich for their contributions to the theory of the optimal allocation of resources. Koopmans' work in activity analysis, linear programming, and optimal growth, together with his influential doctrine of the price implications of optimality, is central to neoclassical economics. By contrast, Kantorovich's work in these areas remains poorly understood in the West. Many economists would have a hard time describing his contribution, much less justifying the award of a Nobel Prize to it. This paper aims at redressing this regrettable situation. Earlier assessments of Kantorovich's work have been made by Leif Johannsen (1976) and Aron Katsenelinboigen (1978–79). The brief entry in the *New Palgrave* by Kantorovich's coauthor V. Makarov (1987) as well as Kantorovich's Nobel autobiography and acceptance speech (Kantorovich 1976a, 1976b) should also be mentioned. However, the appearance of new material, most notably Kantorovich's own glasnost-era memoir (1987), as well as work on the nature and computation of his resolving multipliers by Roy Gardner (1988), justifies a new assessment. The argument proceeds by stages: (1) Kantorovich's life, in many ways remarkably similar to that of John von Neumann, (2) the discovery of optimal planning, the basis of the work for which he won Nobel Prize, (3) the computation of an optimal plan, in which his resolving multipliers play a crucial role, and (4) his indirect influence on the current restructuring of the Soviet economy, perestroika. In this way, the reader should come to a clearer understanding of the man and his economics.

I. *The Life of L. V. Kantorovich*

The best way to approach Kantorovich's life is by comparing him to his closest parallel in the West, John von Neumann. Both were born to bourgeois Jewish families in Eastern Europe, von Neumann in Budapest in 1908, Kantorovich in St. Petersburg four years later. Both showed mathematical genius at an early age, and both earned doctorates in mathematics by their early twenties. Both were deeply involved in functional analysis in the 1930s; indeed, their only personal meeting took place at a conference on functional analysis in Leningrad in 1935. Both applied powerful mathematical tools to economic questions in the late 1930s, von Neumann in his theory of games and his model of general equilibrium (in German), Kantorovich, in his theory of optimal planning (in Russian). Both suffered from their Jewishness—von Neumann, as a refugee from Hitler in 1938, Kantorovich, as a potential victim of Stalin throughout the 1940s. Both played major roles in atomic energy and computers after the Second World War. Finally, both were apolitical. There are some differences, however. Von Neumann's work in economics had appeared in English by 1945, while Kantorovich's only began to appear in English after 1959. Von Neumann's work entered mainstream economics within a decade, while Kantorovich's had to wait a generation for recognition. Finally, von Neumann died much earlier (1955), 31 years before Kantorovich.

Kantorovich's life can be divided both temporally and geographically into three periods: Leningrad (1912–60), Novosibirsk (1960–71), and Moscow (1971–86). During the Leningrad period, he received his education and made his greatest discoveries; during the Novosibirsk period, he founded an institute from which to propagate his ideas; and during the Moscow period, he had his greatest influence on the economic policy and economic reform of the USSR. We consider these three periods in order.

A. *The Leningrad Period (1912–60)*

Kantorovich was born the son of a doctor in 1912 in what was then called St. Petersburg, the capital of the Russian Empire. The twin revolutions of 1917 left his family destitute—he was their sole support from the age of 18 onward. Having shown signs of mathematical genius at an early age, he was admitted to Leningrad State University (LSU) at the age of 14. By the time he graduated as a mathematics major four years later, he already had eleven published articles to his credit. He continued his graduate studies at LSU also (even now, it is the norm for Soviet students to receive all their education in their hometown). He was named an assistant professor in 1932 and a full professor in 1934. The customary oral defense of his thesis on partially ordered function spaces was waived—its results were already world famous! Indeed, such spaces are now called K-spaces in his honor.[1]

Besides teaching and doing research in mathematics at the university, Kantorovich was affiliated with the Institute of Mathematics and Mechanics. It was in this capacity that he was approached by members of the laboratory of the Plywood Trust in 1937 with the problem of cutting plywood sheets in such a way as to meet a specified assortment of pieces with minimum waste. As he tells the story in his memoir, at this time he was feeling burned out by pure mathematics, which he had been working on intensively for over a decade. Moreover, he was feeling the ominous threat of German fascism to European civilization and thought that he should be doing something more practical to counter that threat. He had taken an economics course in 1929, and considered economics a good area to apply his talents. Thus, he agreed to consult on the plywood cutting problem.

His solution of this and related problems became the basis of his later work in economics (Section II). He presented his results to the engineers and also at the university in May 1939, then wrote them up an LSU working paper (Kantorovich 1960). This working paper was not widely circulated inside the USSR, much less abroad.

It is hard to imagine a political environment more inauspicious for major discoveries in mathematical economics than the Soviet Union of 1939. One of Stalin's first acts after taking dictatorial power in 1929 had been to purge the economists—a full seven years before the purges of the Communist party began. Figures such as Kondratiev, Feldman, and Groman vanished into the Gulag then, never to reappear. According to Stalin, the planned economy of the USSR was already "dizzy with success"; hence any criticism of it was anti-Soviet propaganda, a serious crime. In particular, anyone openly suggesting that waste could be cut substantially was at great personal risk. Nevertheless, Kantorovich, who already realized that his methods, if adopted, could do just that, wrote a letter to Gosplan suggesting a reform of the price system used in planning. Gosplan wrote back saying that no such reform was necessary. This outcome was rather fortunate for its author, as similar letters critical of the authorities—for example, one by Solzhenitsin—landed their authors promptly in jail.

Kantorovich's work in economics came to an abrupt halt in 1943. As he tells the story, he

[1] It is worth pointing out that such partially ordered spaces are used for the formal representation of the commodity space in abstract general equilibrium analysis. Kantorovich seemed quite surprised when he was told of this development while attending the World Congress of the Econometric Society (of which he was a Fellow) in Boston in 1985.

had been evacuated to Yaroslavl to escape the Nazi siege of Leningrad. He continues:

> Much has been said of the fact that it was essential at that time to abandon this work. Its continuation became dangerous—as I found out later on, my suppositions were not without foundation. This was a terrible blow to me, for I had placed great hopes on it. For a long time I suffered from serious depression—I wasn't able to concentrate on science at all. I had to give this work up, and from that time on I stuck with mathematics.

Even though Kantorovich was no longer involved in economics, applications of his work still got him in trouble. As Katsenelinboigen (1978–79) tells the story, one of the major materials handling operations at the Leningrad E. I. Egorov Railroad Car Building Plant was the cutting of sheet metal for railroad cars. Ordinarily, this cutting produced tremendous quantities of scrap. After introducing Kantorovich's solution technique to the problem of minimizing waste, officials were able to reduce the amount of scrap by 50 percent. This had the unfortunate side effect of greatly reducing the amount of scrap metal available to steel plants in the region, and Kantorovich was ordered to appear at Leningrad party headquarters for allegedly sabotaging the economy. In this instance, he was rescued by the military, which needed him for its atomic program. Most of the work that Kantorovich did for the Soviet military remains classified to this day. We do know that Kantorovich applied his technique to the problem of cutting metal for tanks, and to the problem of laying mine fields.

Even before Stalin's death in 1953, Kantorovich began to receive official Soviet recognition for his work in mathematics. His book *Functional Analysis and Applied Mathematics* was awarded the State Prize in 1949. In the late 1950s, Kantorovich was allowed to teach a yearly seminar at LSU entitled "Economic Calculation." This course covered foundations of higher mathematics, methods of optimization, and the theory of optimal planning. In 1958, he was made a corresponding member of the Soviet Academy of Sciences in mathematics. Finally, in 1959 he was allowed to republish his original 1939 working paper, as well as an expanded book-length version of his results on optimal planning (Kantorovich 1960, 1965). The latter had been substantially completed 16 years earlier.

B. *The Novosibirsk Period* (1960–71)

As part of the Khrushchev thaw, Kantorovich and his fellow economist V. S. Nemchinov were empowered in 1958 to begin planning and recruiting for a new laboratory for the application of statistical and mathematical methods in economics. This laboratory, to be affiliated with the Siberian Branch of the Soviet Academy of Sciences in Novosibirsk, was to work with the newly created Central Mathematical Economics Institute in Moscow. The geography of the decision is telling. Novosibirsk, some 5,000 kilometers east of Moscow, was a safe place for ideas too radical for the nation's capital. As a nonparty member, Kantorovich was not eligible to direct such a laboratory, but he was named its deputy director. He recruited heavily from his students and colleagues in Leningrad, and soon assembled a talented group to staff the new laboratory. Among those at Novosibirsk, probably the most illustrious today (1989) is Abel Agenbegyan, an important economic adviser to Mikhail Gorbachev.

The new laboratory opened with great fanfare in April of 1960. The opening conference assembled most of the Soviet Union's mathematical talent—men like Kolmogorov, Lyapunov, Markov, and Pontryagin, as well as the major economists Novozhilov, Nemchinov, and Kantorovich, all of whom spoke. The proceedings were often rancorous (USSR Academy of Sciences 1960). Kantorovich presented part of his newly published book, and criticized rather severely the mainstream Soviet economics profession for its aversion to optimization and other mathematical techniques. Hardliners, on the other hand, attacked the application of mathematics to economics, finding it a thinly disguised form of bourgeois economics. Kantorovich insisted that these methods were, however, fully consistent with Marxian orthodoxy, including the labor theory of value. At a similar conference in 1963, Kantorovich hit upon a very clever defense. According to the report of the proceedings (USSR Academy of Sciences 1963):

> In conclusion, L. V. Kantorovich took up the critical attempts to counter mathematical meth-

> ods in Marxist economic science, characterizing their application to the USSR as though it were some kind of "departure" from Marxism, made in the bourgeois press by Campbell and others. Such attempts, hardly unexpected, should not lead to a slandering of economists and mathematicians. Comrade Kantorovich asserted that Soviet economists and mathematicians stand and will continue to stand for Marxist principles. They see the application of mathematical methods as a real means of realizing the principles of Marxist-Leninist economic science, essential for the enormous measures and complexity of socialist construction.

The reference is clearly to the classic article by Robert Campbell (1961). By making Western authorities the spokesmen for criticism that was emanating mainly from the Communist party itself, Kantorovich found the perfect foil for his counterthrust.

At any rate, the opposition to mathematical economics by the authorities did relax. In 1964 Kantorovich was elected a full member of the Soviet Academy of Sciences, this time in economics. A year later he received the Lenin Prize (the highest honor available to Soviet civilians) along with Nemchinov and Novozhilov. Besides forming and training a cadre of mathematical economists in Novosibirsk, Kantorovich continued an active research program. His bibliography from this period lists over 100 articles in economic theory, many of them dealing with questions of capital investment and accumulation, paralleling the growth theory so popular at that time in the West. A complete bibliography of Kantorovich's work in economics can be found in Kantorovich (1989).

Kantorovich's work in Novosibirsk led to an entire school of thought in the Soviet economics profession which is still active today—the Optimal Functioning of the Socialist Economy. This school has consistently employed mathematics to avoid political and ideological constraints on their research. It has also come under continual attack from the orthodox economics profession, which is mostly associated with the Institute of Economics in Moscow, as well as the top echelons of Gosplan, Goskomtsen, and so on. To a large degree, Kantorovich's school is responsible for the existence of serious research in economic theory in the USSR. Without the brilliance and dynamic personality of Kantorovich, this school might not otherwise have come into existence.

C. *The Moscow Period (1971–86)*

The last 15 years of his life, Kantorovich lived and worked in Moscow. It was during this period that he had the most direct personal influence on the economic policy of the USSR. Again the geography is significant. Far from being on the periphery of economic thinking, optimization methods began to be somewhat more mainstream, even if implementation of explicit programming methods continued to be confined mostly to transportation and related problems. Kantorovich directed the laboratory of the Institute for the Management of the National Economy, an organ of Gosplan. From 1976 on, he also headed the scientific directorate VNIISI, which was responsible for working out methods of systems analysis and evaluations of effectiveness for progress in science and technology. Kantorovich also served on councils devoted to computerized optimization processes and transport, and on the State Commission on problems of price formation. In all of these capacities, he was, in the words of a colleague N. Petrakov (1987) "an uncompromising fighter," one who always stood for a scientific approach to economic questions. Once, when a decision went completely against him, he said: "It's all right. Now my hands are clean!"

Besides his official duties, Kantorovich maintained his research program, now devoted mostly to questions of technical progress, its modeling and evaluation. This, for instance, was the theme of his Nobel acceptance paper (Kantorovich 1976c). He also reviewed developments in Western economics for the Soviet audience. Of temporary general equilibrium theory, he said, "The new conceptions of equilibrium are already leading to concrete calculations and promise in the near future to become the foundation of mathematical microeconomics" (Kantorovich et al. 1982, p. 18). This judgment is especially telling when one considers that one particular regime studied by general temporary equilibrium theory, repressed inflation, was characteristic of the Brezhnev years (Gardner and Jonathan Strauss 1981).

The mounting stagnation and poor economic performance of the Soviet economy under Brezhnev and his immediate successors gave further credence to the forces seeking reform of the Soviet economy. According to Petrakov, Kantorovich never gave up on the possibility of "a restructuring of the price system on a scientific basis—the very restructuring whose necessity was later declared by the 27th Party Congress."[2] But what is a price system on a scientific basis? To this question we now turn.

II. *The Discovery of Optimal Planning*

The problem that the plywood engineers brought to Kantorovich in 1937 was the following: We have eight different peeling machines, and five different kinds of wood to peel. Our peeled wood has to be in certain fixed proportions. Each machine has a certain peeling productivity for the various types of wood per unit of time. How should we plan our machine use in such a way as to produce the most peeled wood in the given proportions? As far as the data are concerned, all productivities are nonnegative, and we are required to operate all the machines at all times.

There are several things to note about this problem at once. First, it is a standard problem in microeconomics involving the allocation of scarce resources (here, machine time) to competing ends (the various types of peeled wood). And this, even though no mention is made of value categories such as prices, costs, or profits. The problem as stated is value free—although presumably some value idea lies behind the planned output mix. At any rate, the latter is not the engineers' concern. Second, it is a well-defined optimization problem. The objective and constraints are clearly stated. Unfortunately, none of them is differentiable, as we shall see, so that the only optimization tool available in 1937—the classical method of Lagrange multipliers—is not applicable. The solution will require a new kind of mathematics.

Now the solution would be almost immediate if prices were given for each type of peeled wood. Armed with such prices, the plant man-

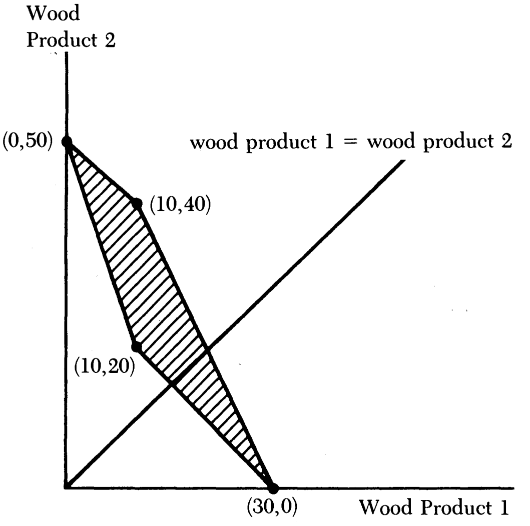

Figure 1. The Production Possibility Set

ager's task would be to maximize the value of the wood peeled by his plant. He could achieve such a maximum by *maximizing the value of the wood peeled by each machine.* But because such prices aren't given, they have to be found. In the language of Kantorovich, we seek the *resolving multipliers* of the problem. Those multipliers are mathematically different from Lagrange multipliers; but once we have found them, we will be able to resolve the given planning problem at the individual machine level.

A simple example here may help. Suppose there are only two peeling machines and two types of wood. The first peeling machine can peel 10 units of either type of wood. The second machine, which is more productive, can peel either 20 units of the first type of wood, or 40 units of the second. Equal amounts of the two types of peeled wood are desired. The production possibilities set is portrayed in Figure 1. Notice the nondifferentiability. Devoting all machine time to the first type of wood yields 30 units; to the second type of wood, 50 units. Using the first machine only on the first type of wood, the second machine only on the second type of wood, yields (10,20), and so on. Assuming machine time is perfectly divisible (an assumption implicit in Kantorovich's entire approach), then one has a convex set of production possibilities. It is clear from the diagram that no point between (0,50) and (10,40) can deliver

[2] A caveat is in order here. Kantorovich's school of thought has never been able to account for the plethora of information problems that plague the planning process of the Soviet economy. To address these problems requires a reform far more radical than any that the Optimal Functioning of the Socialist Economy ever proposed.

equal amounts of wood. Therefore, the optimal solution must lie on the line segment between (10,40) and (30,0). The slope of this line segment is -2. Taking this as a ratio of prices, we have $-\lambda_1/\lambda_2 = -2$ or $\lambda_1 = 2\lambda_2$ where λ_i is the price, or resolving multiplier, for wood type *i*. No generality is lost by setting $\lambda_2 = 1$. Then machine 1 produces the most value, 20 units, by peeling only wood type 1. Machine 2 produces the maximum value, 40 units, by producing any combination of wood types. The combination that delivers the right assortment is 20 units of type 1 and 10 units of type 2. We have found the optimal plan, yielding 20 units of each type of peeled wood.

Several purely mathematical questions arise at once. Do such resolving multipliers always exist no matter how complicated the problem? If so, are they unique? And can we find them? Kantorovich gives a positive answer to the first question, existence of resolving multipliers, in the following fashion. The constraint set for such a problem is convex. The set of outputs that exceed the optimal plan in every dimension is also convex, and disjoint from the constraint set. Therefore, by the separation theorem for disjoint convex sets, there exists a line that separates the two sets: All points in the constraint set lie on one side of the line, all points in the better-than-optimal set, on the other side of the line. The coefficients of such a line are the resolving multipliers. This existence argument is illustrated in Figure 2, and can be found in Kantorovich (1960, Appendix 3.) By the same token, there is no guarantee that the resolving multipliers are unique. Take the same production possibilities, only with the desired mix being 1:4. Then the optimal solution is (10,40), and any line with a slope between -1 and -2 will generate a ratio of resolving multipliers. Discussion of the computation question will be deferred to the next section. See the Appendix for the productivity data for the problem that Kantorovich solved in 1939.

To see the power and generality of Kantorovich's answer to this class of problems, one need only consider a (roughly) contemporary advanced theory book in the West, Samuelson's *Foundation of Economic Analysis*, where everything in sight is differentiable and the only multipliers ever used are those of Lagrange. Probably the only economist in the West capable of such a proof at that time (Kantorovich tells us in his memoir that he found this proof in January 1939) was von Neumann, who was at work on the theory of games. Indeed, one way to prove the minimax theorem is via resolving multipliers—although there was no way for either of them to know this then.

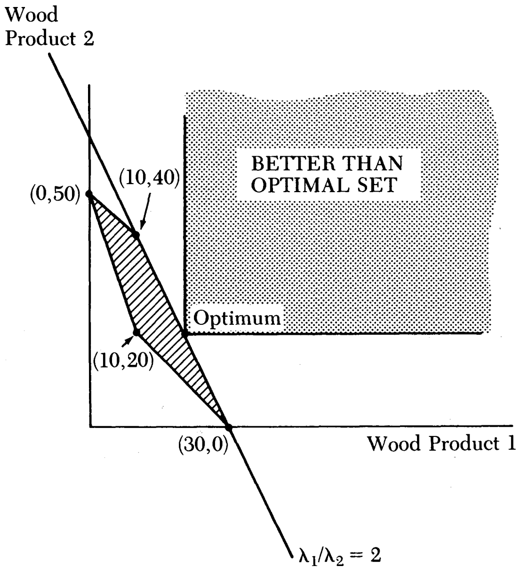

Figure 2. The Separation Argument

We are now at Kantorovich's fundamental economic insight: *An optimal plan is inseparable from its prices.* Even if a plan was entirely in quantities, and said nothing about prices, if that plan was optimal, it would imply the existence of resolving multipliers that function just like prices. The other side of this is that if the wrong prices are used and managers attempt to maximize value, then optimality will not be achieved. Finally, with a suboptimal plan, no such resolving multipliers exist because there is no separation of convex sets. There are no price implications of suboptimality.

Now the argument at the level of a plywood factory is just as valid for the economy as a whole, provided that aggregate production possibilities are a convex set. One way for this to happen is if every plant's production possibilities are a convex set, and the aggregate production possibility set is simply the sum of plant production possibility sets. Next, represent planners' preferences on final output by the concave objective function $u(x) = \min/i\ (k_1x_1,$

$\ldots, k_n x_n)$, where $x = (x_1, \ldots, x_n)$ is the vector of final output and the coefficients k_i are chosen to satisfy the assortment condition. The upper contour sets of $u(x)$ are again convex. Then there will exist an optimal plan for the entire economy, with its attendant resolving multipliers, separating the set of what is possible to produce from the set of what would be even better than optimal to produce. Thus, what we have sketched above—once one adds a proof of the boundedness of the economy—is a prototype theorem for the existence of a general economic equilibrium for a planned economy. I must caution, however, that I have not found an explicit statement and proof of such a theorem in Kantorovich's work.

Optimal planning is a planning process that finds an optimal plan on the basis of its resolving multipliers. It is somewhat ironic that at practically the same time that the great debate over the feasibility of socialism was taking place between Lange and Hayek in England, a Russian unknown to them had proved the mathematical existence of planned socialist prices. Kantorovich himself really showed very little interest in the actual institutions that would have to be present to realize an optimal plan in practice. This latter point, of course, was at the heart of Lange versus Hayek. What was clear to both sides of this debate was that Stalinist planning methods were not optimal. Thus, although he himself supported the regime, Kantorovich nevertheless found himself cast in the role of a reformer, hence by Stalinist logic an adversary of its existing planning institutions. The institutional questions of Lange versus Hayek concerning information, incentives, and decentralization have yet to receive a satisfactory answer to this day. Perestroika shows the Soviets grappling with the questions of optimality as never before.

Between the discovery of resolving multipliers and the decision to give up economics four years later, Kantorovich solved a large number and a great variety of optimization problems. Among these were transportation problems and other network problems central to linear programming, and intertemporal optimization problems, including problems in resource economics. In intertemporal problems, he found a resolving multiplier which for all intents and purposes is a rate of interest. In resource problems, he found resolving multipliers for resources such as land, water, and forests, which planners had previously taken to be free goods. Kantorovich always considered the adoption of this piece of price reform to be one of his greatest successes.

Before the publication of his major book in economics (Kantorovich 1960), Kantorovich did make a rather savvy change in nomenclature, from *resolving multipliers* to *objectively determined valuations*. This change he made mainly for political reasons. He wanted to defend himself on the one hand against the charge that these were objects akin to those of subjective value theory—hence the words *objectively determined*. On the other hand, he wanted to be sure that these were not confused with the values arising in Marx' value theory, hence *valuations*. Sometimes Kantorovich puts (*multipliers*) immediately afterward, to signal to readers that these are the same objects originally designated. Because those political considerations are not binding in this discussion, I will retain the term *resolving multipliers* in what follows.

III. *Kantorovich and Linear Programming*

The claim is often made, especially by the Soviets, that Kantorovich discovered linear programming. For example, D. M. Kazakevich, a former colleague at Novosibirsk, says "L. V. Kantorovich was the first discoverer of linear programming in the scientific world. The priority of Soviet science in this subject is generally recognized" (Kazakevich 1987, p. 73). Doubts about such claims have been raised before in the West by Abraham Charnes and William Cooper (1960), and the Nobel citation is silent on this question. Exploring this question anew will actually help with the difficult question of calculating resolving multipliers.

As we saw before, the existence of resolving multipliers rested on the separation property of disjoint convex sets. No appeal to underlying linearity was actually made. Indeed, in his most general proof of existence, Kantorovich (1940) shows the existence of a linear function passing through the optimal solution, and having the same value as the objective function. Clearly, in such a case, he would seem to have a nonlinear objective function in mind. At any rate, the resolving multipliers characterize the separating hyperplane. Nevertheless, there is a rather intimate connection between Kantorovich's solution to optimization problems and linear programming.

One way to see this is to return to the example of the last section. If the objective is to maximize output, and two quantities are desired in equal proportions, then the objective function is to maximize the minimum of the two quantities, max min (x_1,x_2), where x_i denotes the quantity of wood type. The max min function is indeed nonlinear, a typical indifference surface being shown in Figure 2. This is how Kantorovich describes the objective function in his 1939 existence proof (Kantorovich 1960, Appendix 3). However, the conditions describing the constraint set in this case all happen to be linear equalities or inequalities. Two constraints are the production functions (linear) for each of the two outputs. Two more constraints, again linear, say that each machine must be used fully. Finally, there are four nonnegativity constraints, saying that each machine must be used at least 0 percent of the time on each of the two possible tasks. This makes a total of 8 linear constraints. On the constraint side of the problem, then, the problem does look like a linear programming problem. As it turns out, the max min objective function can be written as a linear function with two additional constraints: namely, maximize Θ, subject to $x_1 \geq \Theta$ and $x_2 \geq \Theta$. At this point, one has a linear objective function, subject to 10 linear constraints.

Now the problem can be solved using the techniques of linear programming. The solution to the original primal problem will be the same as before. The dual linear programming problem will have 10 dual variables, 1 for each constraint. Of special interest are the dual variables attached to the constraints $x_i \geq \Theta$; these dual variables will be the resolving multipliers of Kantorovich, with the same ratio (2) that we see in Figure 2. Thus—and this conclusion is completely general—the resolving multipliers of Kantorovich in an optimization problem that can be rewritten as a linear programming problem are a *subset* of the dual variables of the problem.[3]

There are two central results in linear programming: the duality theorem, which establishes the relationship between a linear programming problem and its dual, and the simplex algorithm, which shows that a linear programming problem can be solved in a finite number of steps. At no time before 1943, when he temporarily gave up economics, did Kantorovich discover either of these remarkable results. Von Neumann, who was aware of the duality theorem, suggested to David Gale in 1947 that he try to prove it. This suggestion worked, and appeared in Gale et al. (1951). One might speculate that if Kantorovich had continued working on economics he would indeed have discovered the duality theorem, but as we have seen, circumstances did not permit this. Again, it was George Dantzig who invented the simplex algorithm and showed that it would solve any linear programming problem in a finite number of steps. The term *linear programming* itself was also first used in print by Dantzig. Thus, the situation of Kantorovich is rather like that of the discoverer Columbus. He really never touched the American mainland, and he didn't give it its name, but he was the first one in the area.[4]

At any rate, armed with this insight we now have a technique for computing resolving multipliers in any problem that can be transformed into a linear programming problem. First transform the problem into a linear programming problem, then solve its dual using the simplex algorithm (or one of its descendants). The dual solution will contain the resolving multipliers for the problem. This seems somewhat roundabout, however. Solving the primal directly would save a step; indeed, that is how linear programming problems have always been solved in the Soviet Union since the simplex algorithm became available.

[3] The resolving multipliers of a general programming problem can also be compared to Kuhn-Tucker multipliers. In the case of a linear programming problem, the dual variables of the problem are equivalent to the Kuhn-Tucker multipliers; hence, the resolving multipliers are a subset of the Kuhn-Tucker multipliers in this case. This set inclusion also holds true for the nonlinear case, because the hyperplane separating the optimal solution from the better-than-set can be constructed using a subset of the Kuhn-Tucker multipliers.

[4] Indeed, if the Nobel Prize committee had wanted to award a prize for linear programming, then Gale and Dantzig should have been included.

This, however, was not the method Kantorovich himself used to solve such problems. As far as is known, Kantorovich never wrote down an algorithm that would handle any problem that could be transformed into a linear programming problem. The method he did use, which turns out to be fairly effective, at least on his examples, involves iterative updating of the resolving multipliers. Although the initial values of the multipliers are arbitrary, steps can be saved by taking as initial values those given by the maximum possible production of the various types of outputs. Returning to the example one last time, this means starting with $\lambda_1/\lambda_2 = 50/30 = 5/3$, Set $\lambda_1 = 5$, $\lambda_2 = 3$. Maximizing value produced on each machine, machine 1 produces $x_1 = 10$ for a value of 50; machine 2, $x_2 = 40$ for a value of 120. Now there is too little good 1 being produced. Therefore raise its resolving multiplier until it just pays to produce good 1 on at least one more machine. The least value of λ_1 that will do this is $\lambda_1 = 6$. The resolving multipliers are now $\lambda_1 = 6$ and $\lambda_3 = 3$. After one step the optimal solution is reached, because $\lambda_1/\lambda_2 = 2$ is the correct ratio of resolving multipliers at the optimal solution. Kantorovich gives several other illustrations of this sort.[5] The reason this process is not yet an algorithm is that it is not so clear in a multigood problem which multipliers to change and which to leave alone. Nor is it clear that the process of adjusting multipliers may not get stuck in some sort of cycle rather than converging to a solution. If such an algorithm were worked out, it would fall into the class of primal-dual algorithms, because it uses information from both the primal and dual parts of the problem at each iteration.

Once optimization methods became respectable in the Soviet Union in the 1960s, the hope was widespread that it would be possible to compute the optimal plan at planning headquarters in Moscow. This hope was dubbed *computomania* by its opponents. Kantorovich himself was no computomaniac, although he did realize the potential of machines for solving large practical problems, especially at the firm level, and devoted a great deal of his later research to integrating computing into the planning process. Even with today's machines, it is not possible to store and process all the data that would go into a detailed optimal plan, requiring as output literally hundreds of thousands of resolving multipliers. Of all the institutions devised by man, markets are still the best (if imperfect) means of generating prices. The restructuring of the Soviet economy now under way makes clear that markets will have to generate an increasing number of prices that the planning process is not able to compute.

[5] For instance, solving the problem in the Appendix requires 7 iterations.

IV. *Conclusion*

At the end of his life, Kantorovich said, "A major achievement of the mathematical economic direction was the elaboration of a series of problems of planned pricing, as was the substantiation of the thesis of the inseparability of the plan and prices" (Kantorovich, M. Albegov, and V. Bezrukov 1987). In this single sentence, we see the cornerstones of Kantorovich's economic thought: (1) optimality has price implications, (2) an optimal plan is therefore inseparable from its correct prices, and (3) reform of the planning process includes price reform. This is part of the microeconomic content of perestroika. There is also a macroeconomic content to perestroika (Nicolaus Spulber 1989) to which Kantorovich, as a microeconomist, did not contribute. By opening a channel to Western ideas through the mathematization of economics in the USSR, Kantorovich had an important effect on professional economic thought there. This, in turn, has had an impact on perestroika, however the latter ultimately turns out.

Appendix

The productivity data for the problem that Kantorovich solved in 1939 are given in Table 1:

TABLE 1

Machine	Wood Product				
	1	2	3	4	5
1	4.0	7.0	8.5	13.0	16.5
2	4.5	7.8	9.7	13.7	17.5
3	5.0	8.0	10.0	14.8	18.0
4	4.0	7.0	9.0	13.5	17.0
5	3.5	6.5	8.5	12.7	16.0
6	3.0	6.0	8.0	13.5	15.0
7	4.0	7.0	9.0	14.0	17.0
8	5.0	8.0	10.0	14.8	18.0

Thus, machine 1 can produce 4 units of wood product 1, or 7 units of wood product 2, and so on. The objective is to produce as much wood as possible, in the proportions 10:12:28:36:14. Thus, the objective function is max min ($z_{1/10}$, $z_{2/12}$, $z_{3/28}$, $z_{4/36}$, $z_{5/14}$) where z_i denotes wood product *i*. The resolving multipliers for this problem are λ_1 = .3678, λ_2 = .2287, λ_3 = .1839, λ_4 = .1226, and λ_5 = .0970. Multiplying machine productivity for each wood product by the resolving multiplier for that wood product, one has the machine values given in Table 2:

TABLE 2

Machine	Wood Product				
	1	2	3	4	5
1	1.471	1.601	1.564	1.594	1.601
2	1.655	1.784	1.784	1.680	1.680
3	1.839	1.830	1.839	1.814	1.746
4	1.471	1.601	1.655	1.655	1.649
5	1.287	1.487	1.563	1.557	1.552
6	1.103	1.372	1.471	1.655	1.455
7	1.471	1.601	1.655	1.706	1.659
8	1.839	1.839	1.839	1.814	1.746

Thus, machine 1 should produce only wood products 2 and 5, machine 7, only wood product 4, and so on. There is a continuum of optimal output plans, one of which is on Table 3:

TABLE 3

Machine	Wood Product				
	1	2	3	4	5
1	0	2.325	0	0	11.020
2	0	7.121	0.845	0	0
3	3.936	0	2.128	0	0
4	0	0	8.442	0.837	0
5	0	0	8.5	0	0
6	0	0	0	13.5	0
7	0	0	0	14.0	0
8	3.936	0	2.128	0	0
TOTAL	7.872	9.446	22.043	28.337	11.020

The final pattern of wood products meets the required proportions to 10^{-3} accuracy.

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