

date to be the lender of last resort for the U.S. banking system. The book may be appreciated, nonetheless, despite this terminal chapter.

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Foundations: Essays in Philosophy, Logic, Mathematics and Economics. By F. P. RAMSEY, edited by D. H. MELLOR.
Atlantic Highlands, N.J.: Humanities Press, 1978. Pp. viii+287. \$20.50.

Frank P. Ramsey must be one of the most remarkable minds of all times, though the contributions to human knowledge of which he was capable were so severely rationed by the chance of an early death, not yet at his twenty-seventh birthday. In the Cambridge manner of the great period, his brilliance was quickly recognized, and he was acknowledged as a peer by such intellectual leaders as Bertrand Russell, C. K. Ogden, G. E. Moore, Ludwig Wittgenstein, and John Maynard Keynes. Also in the Cambridge tradition, his fame was based as much on personal interactions and unpublished manuscripts as on his seven published articles (apart from reviews and encyclopedia articles). But four of those seven papers have been profoundly influential; one, indeed, rather quickly, but the others have found their successor literatures only in the last quarter century.

The first of these, and Ramsey's second apart from reviews, was "The Foundations of Mathematics," a critical examination of the theory of types introduced by Russell to overcome some basic paradoxes in the foundations of thought. Russell and his collaborator, Alfred North Whitehead (1910-13), following on the pioneering formulation of Gottlob Frege ([1879] 1970), had the program of deriving all mathematics from (a properly formulated) pure logic. The paradoxes, which threatened the entire program, occurred when generalizations referred to themselves. The theory of types was a systematic way of excluding such self-reference. However, it restricted the scope of the system so much as to endanger the basic program. To remedy this, Whitehead and Russell had to introduce additional assumptions which seemed much less intuitive than the basic axioms of logic. Ramsey (and, independently, the Polish logician, L. Chwistek [1921]) showed, by reformulating the way the hierarchy of types is constructed, that the objectionable axioms could be rendered superfluous.

During the 1920s, the foundations of mathematics were an object of great controversy; the reduction of mathematics to logic was merely one of three leading schools. The controversy engaged the attention of such great mathematicians as David Hilbert, L. E. J. Brouwer, and Hermann Weyl. The terms of the controversy were, however, drastically altered in 1931, the year after Ramsey's death, by Kurt Gödel's proof that every mathematical system of

sufficient richness necessarily contained undecidable statements. That is, there would always exist a proposition statable in terms of the system but which could be neither proved nor disproved within the system. It also follows that it is impossible to prove the consistency of any such mathematical system within that system.

An ironic result of this extensive discussion was that many of the logical arguments themselves became mathematical systems of great intellectual interest. Ramsey himself made a contribution to the decidability question by showing that in a certain limited system every statable proposition could be proved true or false. (The system was not rich enough in consequences to have violated Gödel's theorem.) The proof required a certain rather complicated proposition in combinatory mathematics. This proposition has, within the last quarter century, suddenly become a field of mathematical interest under the heading of "Ramsey numbers."

The remaining two important papers are in the field of economic theory, those on optimal savings and optimal commodity taxation, both published in the *Economic Journal*. Since these have become such major components of current economic analysis, I forebear from summary. Their delayed recognition is indeed a problem in *Dogmengeschichte*. The paper on optimal savings made use of the calculus of variations, a technique regarded as beyond the pale of decent usage in economics; Harold Hotelling's famous piece on exhaustible resources, also using the calculus of variations, was rejected as incomprehensible by the *Economic Journal* before publication in this journal. (Presumably, Ramsey's patron, Keynes, smoothed the way for him.) But the paper on optimal commodity taxation is by no means difficult, and yet it was not remarked on again until about 1965. Meanwhile, the basic ideas had been rediscovered, in much greater generality, by Marcel Boiteux in a classic paper of 1956 (though surely Ramsey's paper is much easier to read!). Perhaps it was that during the 1930s welfare economists (such as Harold Hotelling, Abba Lerner, and John Hicks) were gradually clarifying the meaning of first-best Pareto optima and were not ready to assimilate a proposition in second-best analysis.

A contemporary of Shakespeare's refers to "his sugar'd sonnets among his friends." A number of Ramsey's manuscripts of varying degrees of tentativeness seemed to be similarly esteemed by his friends. These dealt mostly with philosophical issues, but one, perhaps the most ready for publication, was an essay on "Truth and Probability" which set forth the subjective theory of probability. It is totally modern in spirit, with a simultaneous axiomatization of utility and probability. Although he did not cover the mathematical difficulties in all rigor, there is nothing in the fundamental concepts of the modern theory, due to Leonard J. Savage (1954), which is not in Ramsey's paper.

Upon Ramsey's death, his friends, particularly R. B. Braithwaite, rushed to put a collection of published and unpublished papers into print (Ramsey 1931). The essays on mathematical economics were not included, with no explanation, though all his other published works were, and a selection was made of the unpublished papers, including the one on "Truth and Probability." Despite this appearance in print, it was unknown to subsequent subjective probability theorists such as Bruno de Finetti (1937) and Savage (though Savage learned about Ramsey's work before publication and referred to it).

A book has now appeared which is somewhere between a new edition of the earlier work and a brand-new collection. The papers on mathematical eco-

nomics have been added; some of the unpublished pieces on philosophy have been deleted, as well as my own personal favorite, a talk given before "a Cambridge discussion society" (the Society, the secret center of Cambridge intellectual life for over a century), published in the 1931 volume as an epilogue. The theme was that there was nothing to discuss, all interchange now being reduced to matters of scientific fact, on which discussion contributed nothing, or values, on which disagreements could not be resolved. The charm and wit of the talk have not stalled with time. The graciously and wittily condescending preface of G. E. Moore and the intense, beautifully written editor's introduction by Braithwaite have also gone. (I cannot refrain from one quotation from the latter. In referring to Ramsey's willingness to change his mind, Braithwaite says: "His calmness in infanticide frequently amazed his friends.") Instead, we have four introductions (by D. H. Mellor, T. J. Smiley, L. Mirsky, and Richard Stone) commenting on Ramsey's work in the fields of philosophy, mathematical logic, mathematics, and economics, respectively. They are all workmanlike and useful, but I fear that the quality of English prose has declined in the mother country as much as it has here.

I cannot help feeling that if a new edition was called for it should at this stage have been more scholarly. Could we not have had *all* the papers worth reproducing, certainly including those that were printed in the 1931 volume? It also would have helped to have had some idea how and to whom the unpublished papers circulated. Do we know what caused Ramsey to become interested in the particular problems in economics that he did? As it stands, the major gain is the reprinting of the two articles on economics. In fairness, it should be added that the new edition has both a bibliography and an index, both lacking in the earlier version.

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