A Simple Model of Crime Waves, Riots, and Revolutions

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Standard economic models of criminal behavior analyze the criminal's decision in a partial equilibrium context. The standard model does not recognize that the probability of being punished is a function of the total amount of crime that occurs. As the total amount of crime increases, police resources become strained, courts become congested, and prisons become overcrowded. As a result, proportionately fewer criminals are apprehended, convicted, and imprisoned. The feedback effects from one criminal's decision to participate in crime to another criminal's decision can be highly significant. In one parameterization of the model developed here, the individual commits twice as many crimes for a given parameter shift than is implied by the standard model. The model also sheds light on other areas where criminal actions are interdependent such as riots, crime waves, and revolutions. (JEL K42)

Introduction

Crime costs Americans over $400 billion dollars a year, not including the misery caused by violent crime.[1] Understanding of the causes and consequences of crime has been greatly improved by the economic theory of criminal behavior. Standard economic models of criminal behavior [Becker, 1968; Ehrlich, 1974], however, analyze the criminal's decision in a partial equilibrium context. These models do not recognize that the probability of being punished is a function of the total amount of crime which occurs. As the total amount of crime increases, police resources become strained, courts become congested, and prisons become overcrowded.[2] As a result, proportionately fewer criminals are apprehended, convicted, and imprisoned. Those who are punished may receive lighter sentences because the quality of investigation and prosecution is likely to decline as police and prosecutor caseloads increase. Furthermore, prison officials have little choice but to release criminals early when prisons become unreasonably overcrowded. Thus, when an individual commits more crimes, the probability is reduced that other criminals are apprehended, convicted, and imprisoned. Other criminals optimize their choices appropriately. Each individual is, therefore, involved in a system of decisions.

This paper analyzes the general equilibrium effects of an individual's decision to participate in crime. The paper shows that feedbacks effects can be highly significant. In one parameterization, a single criminal commits twice as many crimes for a given parameter shift than is implied by the standard model. The aggregate effects are even

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larger. In the same parameterization, the aggregate effect on crime is n times as large as that implied by the standard model (where n is the number of agents in the model).

The model applies not only to crime strictly but also to phenomena like riots, strikes, and revolutions. In each of these cases, the probability of being punished is a decreasing function of the total amount of the activity. The probability that a rioter is apprehended falls the more rioters there are. The probability that a striker or a revolutionary is punished is less the greater the number of strikers and revolutionaries. This is true even if the revolution or strike fails.

This paper is most closely related to Sah [1991] and Kuran [1989]. Sah assumes that agents do not know the true probability of punishment—rather they estimate this probability from local information. If, for example, an agent observes a lot of crime, he is likely to assume that the probability of punishment is lower than his prior estimate. Thus, the observation of significant amounts of crime can increase the amount of crime. Sah analyzes the dynamics of actual and estimated probabilities and the effect of these on crime rates. Kuran asks why major events—like revolutions—are often not anticipated. His explanation focuses on what he calls "preference falsification," the hiding of one's true preferences. A majority of the public may not be in favor of the current government without a majority of the public knowing that a majority is not in favor of the current government. Kuran analyzes a process whereby a slight parameter shift can cause one individual to reveal his true preferences which causes another individual to reveal his true preferences and so forth until a revolution occurs which appears to come "out of nowhere." Sah [1991] and Kuran [1989] differ from this paper in assuming that agents are misinformed or imperfectly informed in some important manner. In this paper, agents are well informed and perfectly rational.[3]

The Model

There are n agents who are indexed by i and come in different types denoted by θ. Agent utility is a function of wealth, $U_i(W_i \mid \theta_i)$, where $U'(W) > 0, U''(W) < 0$. Higher values of θ will represent a greater tendency towards crime. An agent's type could represent moral convictions, risk aversion, productivity in the legitimate sector of the economy, productivity in the criminal sector, and so forth. The particular formulation is not important for the purposes of this paper.

Labor supply is determined by Ehrlich's [1974] state-preference model.[4] Each agent chooses how much time to devote to crime, c_i, and how much time to devote to legitimate work, l_i. Time spent at leisure is held constant so assume that $c_i = 1 - l_i$ or $c_i \in [0,1]$. If an agent devotes positive time to crime, then with probability $p(C)$ he will be punished and with probability $1 - p(C)$ he will not punished. C is a measure of the total amount of crime, i.e., $C = C(c_1,..c_i,...c_n)$ such that $C_i > 0$ (where the subscript denotes a partial derivative). It is assumed that the probability a criminal is punished decreases as the total amount of crime increases, $p'(C) < 0$. Furthermore, each criminal is small relative to the market and so takes $p(C)$ as fixed.

If an agent is punished, his wealth is written as:

$$W_p = W_o + W_c(c) + W_l(l) - F(c) .$$

Where W_o is initial wealth, $W_c(c)$ is wealth from criminal activity as a function of c, W_l (l) is wealth from working in the legitimate sector of the economy, and $F(c)$ is the fine or punishment (measured in monetary terms). If the individual is not punished, his wealth is written as:

$$W_{np} = W_o + W_c(c) + W_l(l) .$$

Wealth increases with time spent in either the criminal activity or the legitimate activity but in either case at a diminishing rate. Punishment is assumed to increase with time spent in criminal activity at an increasing rate. These assumptions can be stated as:

$$\frac{dW_c}{dc} , \frac{dW_l}{dl} , \frac{dF}{dc} > 0 , \frac{d^2 W_c}{dc^2} , \frac{d^2 W_l}{dl^2} < 0 \text{ and } \frac{d^2 F}{dc^2} > 0 .$$

For simplicity, the paper focuses on interior solutions for all types.

Equilibrium

The strategy choices $(c_1^*,\ldots c_i^* \ldots c_n^*)$ constitute a Nash equilibrium if for every agent i, c_i^* solves the following maximization problem:

$$\underset{c_i}{\mathrm{Max}}\, EU_i = \underset{c_i}{\mathrm{Max}}\, p(C)U(W_p) + (1 - p(C))U(W_{np}) .$$

The first-order conditions for this problem are that for each agent $i : i = 1 \ldots n$:[5]

$$\frac{p(C)U'(W_p)}{(1 - p(C))U'(W_{np})} = - \frac{(W'_c - W'_l)}{(W'_c - W'_l - F')} . \tag{1}$$

In general, there will be many Nash equilibria. Some of these equilibria will be trivially identical in the sense that the aggregate amount of crime is the same but the individual criminals are different. These equilibria are not discussed. Other equilibria may involve large differences in the total amount of crime even though underlying parameters are identical. These types of equilibria are discussed below.

Note that the criminal takes $p(C)$ as given. If each criminal is small relative to the market, this assumption is both reasonable and innocuous. It is illegitimate, however, to assume that other agents ignore the effect of c_i on $p(C)$. The reason is that there are many other agents. Even if each of these agents makes only a marginal adjustment in his crime rate, the total adjustment may be large. If the total adjustment is large, agent i will reoptimize once the effects of his initial response make their way through the system. Thus, although the initial impact of a change in c_i on $p(C)$ is ignored, the model does take into account the far more important indirect effects. The following results expand on these themes.

Following the language of Cooper and John [1988], this game exhibits positive spillovers, strategic complementarity, and multiplier effects. Positive spillovers occur when an agent's utility is increasing in the strategic choices of the other agents. In this game, an agent's expected utility is increasing in the amount of crime committed by other agents:[6]

$$\frac{dEU_i}{dc_j} = \frac{dp(C)}{dC}\frac{dC}{dc_j}(U(W_p) - U(W_{np})) > 0 .$$

The game also exhibits strategic complementarity—an agent's optimal strategy, here, the time spent in criminal activity, is increasing in the strategy choices of other agents. Taking the derivative of the first-order condition with respect to the choices of the other agents, one finds:

$$\frac{dEU_i}{dc_i c_j} = \frac{dp(C)}{dC}\frac{dC}{dc_j}[U'(W_p)(W'_c - W'_l - F') - U'(W_{np})(W'_c - W'_l)] > 0 \quad (2)$$

$$= -\frac{dp(C)}{dC}\frac{dC}{dc_j}\frac{1}{p(C)} > 0 , \quad (3)$$

where inequality (3) makes use of inequality (2) and the first-order condition.

A game exhibits multiplier effects when the aggregate response to a parameter shock (θ) exceeds the individual's initial response and when the individual's equilibrium response exceeds the individual's initial response. Strategic complementarity is a necessary and sufficient condition for multiplier effects [Cooper and John, 1988].[7] The intuition in this model is simple. An increase in θ_i increases the time agent i devotes to crime. As crime increases, police resources become strained at the margin and the probability of punishment falls, causing other criminals $j \neq i$ to increase their criminal activities. As other agents turn to crime, the probability of punishment falls even further, giving agent i an additional reason to increase his criminal activities.

Comparative Statics and Stability

There are no real dynamics in this model since the first-order conditions (1) are assumed to hold exactly at all times. It has become traditional, however, to appeal to notions of stability to sign comparative static relations [Varian, 1992; Dixit, 1986]. Without apology, that procedure is followed here. It is convenient to assume that near the equilibrium, agents adjust their labor choices in the direction of increasing expected utility, assuming that other agents hold their choices fixed. This type of dynamic system corresponds to that assumed by Cournot [1838] in his analysis of duopoly. Thus, assume Cournot dynamics near the Cournot-Nash equilibrium. Cournot dynamics have the useful property that stability conditions allow one to sign the Hessian needed for comparative statics. Write the first-order conditions (1) as $\pi_1(c_1, c_2 \ldots c_n, \theta_1) = 0, \ldots, \pi_n(c_1, c_2 \ldots c_n, \theta_n) = 0$. The Cournot dynamic system can be written as:

$$\frac{dc_1}{dt} = \alpha_1 * \pi_1 \quad (c_1, c_2 \ldots c_n, \theta_1)$$

$$\vdots$$

$$\frac{dc_1}{dt} = \alpha_n * \pi_n \quad (c_1, c_2 \ldots c_n, \theta_n) \quad .$$

A sufficient condition for stability is that the matrix:

$$A = \begin{pmatrix} \pi_{11} & ..\pi_{1i}.. & \pi_{1n} \\ : & ..\pi_{ii}.. & : \\ \pi_{n1} & ..\pi_{ni}.. & \pi_{nn} \end{pmatrix}$$

be negative definite at the equilibrium point,[8] but this matrix is just the Hessian matrix which one needs to sign to get comparative statics. Assume that the dynamic system is stable at the equilibrium point. Below a sufficient condition for stability will be discussed.

Comparative Statics

The comparative statics of a change in θ_i are of primary interest. Recall that an increase in θ_i indicates an increase in the propensity to commit crime. In the past 20 years, for example, legitimate earnings opportunities for low-skilled men have deteriorated substantially, leading to a relative increase in the attractiveness of crime as an occupation [Freeman, 1996].[9] Following this interpretation, the comparative statics results indicate the system-wide or total effects of an exogenous decrease in legitimate-sector earnings for agent *i*. Other interpretations are, of course, possible (as shown below).

It will be useful to define some terms. The partial effect, $\partial c_i / \partial \theta_i$ shows how much agent *i* would increase his criminal activity following an increase in θ_i if no other agents altered their criminal activity. The total effect, $dc_1 / d\theta_1$ gives *i*'s increase in criminal activity, taking into account the initial increase in θ_i and all of the secondary effects that flow from the increase in the criminal activities of other agents.

Proposition 1

At a stable equilibrium, the total effect is positive, $dc_1 / d\theta_1 > 0$.

Proof

Using Cramer's rule, the comparative static is given by (4). From the stability condition, the denominator has sign $(-1)^n$ and the numerator has the same sign as $-\pi_{1\theta_1} (-1)^{n-1} = \pi_{1\theta_1} (-1)^n$. Since $\pi_{1\theta_1} > 0 \Rightarrow dc_1 / d\theta_1 > 0$:[10]

$$\frac{dc_1}{d\theta_1} = \frac{\begin{vmatrix} -\pi_{1\theta_1} & \cdots & \pi_{1n} \\ \vdots & ..\pi_{ii}.. & \vdots \\ 0 & \cdots & \pi_{nm} \end{vmatrix}}{|A|} \, . \tag{4}$$

Proposition 2

At a stable equilibrium, $dc_i / d\theta_j > 0, j \neq i$.

Proof

The same reasoning as above follows. Proposition 1 indicates that the total effect of an increase in θ_i has the same sign as the partial effect. Proposition 2 indicates that other agents increase their criminal activities in response to an increase in activity by agent i. Further assumptions on the structure of the model can be used to strengthen and illustrate Propositions 1 and 2. Let $\rho_{ij} = \pi_{ij} / \pi_{ii} \, \forall \, i, j$. (Note that $\rho_{ij} < 0$.) ρ_{ij} is the slope of agent i's reaction function with respect to agent j's actions. Now assume that every individual contributes equally, on the margin, to the aggregate measure of crime, i.e., an increase in c_j has the same effect on $p(C)$ as an increase in c_i. This implies $C = C(c_1 + c_2 + \ldots c_n)$. It follows that agent i reacts the same way to an increase in crime by any other agent or $\rho_{ij} = \rho_i \, \forall j$.

A sufficient condition for stability is that $\rho_i \leq -1 / n \, \forall \, i$ (see the Appendix). An increase in crime by agent i reduces the probability that agent j is punished, which causes agent j to increase his criminal activities, which causes agent i to increase his criminal activity. In order for the equilibrium to be stable, this process must be damped. The condition $\rho_i \leq -1 / n \, \forall \, i$ is a sufficient condition for the process to be damped. A necessary condition for stability (see the Appendix) is that $-\sum_{i=1}^{n} \rho i / 1 - \rho i < 1$. The necessary condition indicates that some agents can have highly reactive reaction functions, $\rho_i > -1 / n$, so long as other agents have sufficiently damped reaction functions, $\rho_i < -1 / n$. The sufficient condition is easier to work with, however. For the next set of results, assume that $\rho_i = -1 / n \, \forall \, i$.

The difference between agent i's total and partial effects, $dc_1 / d\theta_1 - \partial c_i / \partial \theta_i$, is the own contagion effect, the increase in i's criminal activity which occurs because other criminals increase their activity in response to i's increased activity. The author also defines the general contagion effect as the increase in agent i's criminal activity when agent j increases criminal activity, $dc_i / d\theta_j$. Finally, the aggregate effect of a change in θ_i is the change in the aggregate amount of crime, which is found by summing up all of the individual total effects.

Using the above stability condition, it is shown in the Appendix that the total effect is nearly twice as large as the partial effect:

$$\frac{dc_i}{d\theta_i} = 2\,\frac{n}{n+1}\,\frac{\partial c_i}{\partial \theta_i}\ . \tag{5}$$

By definition, the direct effect can be decomposed into the partial and own contagion effects so the above can also be written as $dc_i/d\theta_i = \partial c_i/\partial\theta_i + (n - 1/n + 1)\ \partial c_i/\partial\theta_i$. It can also be shown that the general contagion effect, the increase in i's criminal activity when agent j increases criminal activity, is nearly as large as the own partial effect:

$$\frac{dc_i}{d\theta_j} = \frac{n}{n+1}\,\frac{\partial c_i}{\partial \theta_j}\ . \tag{6}$$

Noting that $\partial c_j/\partial\theta_j = \partial c_i/\partial\theta_i$, rewrite this as $dc_i/d\theta_j = (n/n + 1)\ \partial c_i/\partial\theta_i$. The above results can be put together to give Proposition 3.

Proposition 3

For large n, the aggregate increase in crime is n times the initial increase in crime.

Proof

The aggregate effect of an increase in θ_i is the total effect aggregating over all agents. Thus:

$$\frac{dC}{d\theta_i} = \frac{\partial c_i}{\partial \theta_i} + \left(\frac{n-1}{n+1}\right)\frac{\partial c_i}{\partial \theta_i} + (n-1)\,\frac{n}{n+1}\,\frac{\partial c_i}{\partial \theta_i}$$

or:

$$\frac{dC}{d\theta_i} = n\,\frac{\partial c_i}{\partial \theta_i}\ . \tag{7}$$

Proposition 3 indicates that a small increase in a parameter can cause large changes in the aggregate amount of crime because of the overwhelming influence of the contagion effect.

The contagion effect is often ignored. It is well known, for example, that age-specific arrest rates tend to peak between 16 and 18 years of age.[11] Blumstein et al. [1980] use this fact in combination with predictable demographic changes to predict future crime rates. Unfortunately, they treat arrest, conviction, and imprisonment rates as exogenous, estimating the future values of these parameters by simple extrapolation. Proposition 3 indicates that such a procedure may greatly underestimate changes in crime. As crime increases due to an increase in the number of young men, for example, arrest, conviction, and imprisonment rates will fall, causing crime rates to be higher than demographic factors alone would predict. Similarly, a fall in the crime rate will be underpredicted because as crime rates fall, arrest, conviction, and imprisonment rates will rise, causing crime rates to fall further than the initial prediction.

Of course, this paper too misses an endogenous factor—as crime increases, more resources will be put into crime prevention. Two responses to this point may be noted. First, public reaction to increased crime appears to be slow. Second—and more importantly—one needs to know how the criminal system works before analyzing the appropriate response of the law enforcement system. If law enforcement uses the standard model to predict the demand for their services, they are likely to be woefully unprepared when demographic changes interact with the contagion effect to produce a large increase in crime.

Multiple Equilibria

The set of n first-order conditions (1) can have more than one solution. Thus, multiple Nash equilibria with very different aggregate crime rates are possible. Suppose there are two stable equilibria—one with high crime rates and the other with low crime rates. Which equilibrium is played will depend on factors outside of the model. Agents may look for events, which may or may not be rationally connected to the underlying game, to coordinate their actions. Thus, a huge increase in crime may take place because of the occurrence of a seemingly minor event. In different contexts, such huge increases in crime are called riots or revolutions.

Revolutions, riots, and crime are often said to come in waves. The 1989 revolutions in Poland, Hungary, East Germany, Czechoslovakia, Bulgaria, and Romania are but the most recent example of a revolutionary wave. One reason that revolutions come in waves is that underlying parameters shift at the same time. In this case, the weakening of the Soviet Union probably played a role. Another reason may be that a revolution in one country acts as a signal that helps to coordinate revolutionaries in another country. Communication among revolutionaries is costly. An exogenous signal can coordinate action even without communication. Riots may be transmitted from borough to borough and city to city in much the same way.

Standard comparative statics results can be seriously misleading in models with multiple equilibria. The impact of anti-crime measures, for example, can be strikingly different depending on whether the underlying system has a unique equilibrium or multiple equilibria. The governor of Indiana, for example, sent state troopers into Gary, Indiana to try to control the rampant crime in that city. It was widely acknowledged that the political will did not exist to keep the state police in Gary permanently. The governor, therefore, was widely criticized for adopting what his critics concluded could only be a temporary solution. The critics implicitly assumed that the high crime equilibrium was the unique equilibrium and that once the troops left then the crime rate would return to its previous level. Temporary measures can have permanent effects when there are multiple equilibria. Once controlled, crime in Gary may stabilize at a new, lower level.

In New York City, piecemeal efforts to erase the graffiti on subway cars proved ineffective. As soon as a car was cleaned, it was vandalized again. A big push, however, which cleaned nearly all of the cars in a short amount of time, proved to be more effective and long-lasting. Thus, although New York's crime rate was locally stable, the big push drove the system to a new equilibrium at a lower level of crime.

Conclusions

The standard model of crime analyzes a criminal's decisions as if they were unrelated to the decisions of other criminals. The game-theoretic approach, adopted in this paper, examines the entire system of criminal decisions. The comparative statics of the game theoretic model are considerably different than those of the standard model. Small changes in parameters, for example, have much larger effects on total crime rates. Unlike the standard model, the game-theoretic model allows for such phenomena as riots, revolutions, and crime waves—all of which may be triggered by seemingly minor events. The game-theoretic model also provides a better understanding of the difficulties and potentials of anti-crime efforts. History matters in models with multiple equilibria because of its importance in establishing signals or sunspots that coordinate criminal behavior. Once a community is stuck in a high crime equilibrium, it may be very difficult to exit without a large increase in resources. A low crime equilibrium can be sustained if anti-crime forces are perceived to be alert and responsive to any potential shocks to the system.

APPENDIX

Proposition A1: proof that $dc_i/d\theta_i = 2\ (n/n + 1)\ \partial c_i/\partial\theta_i$. Let the payoff functions be written $\pi(c_1, c_2 \ldots c_n, \theta_1) \ldots \pi(c_1, c_2 \ldots c_n, \theta_n)$. The first-order conditions are, therefore:

$$\pi_1\ (c_1, c_2 \ldots c_n,\ \theta_1) = 0$$
$$\vdots$$
$$\pi_n\ (c_1, c_2 \ldots c_n,\ \theta_n) = 0\ .$$

Using Cramer's rule:

$$\frac{dc_1}{d\theta_1} = \frac{\begin{vmatrix} -\pi_{1\theta_1} & \cdots & \pi_{1n} \\ \vdots & ..\pi_{ii}.. & \vdots \\ 0 & \cdots & \pi_{nm} \end{vmatrix}}{\begin{vmatrix} \pi_{11} & \cdots & \pi_{1n} \\ \vdots & ..\pi_{ii}.. & \vdots \\ \pi_{n1} & \cdots & \pi_{nn} \end{vmatrix}}\ .$$

Let $\rho_{ij} = \pi_{ij}/\pi_{ii} \ \forall\ i, j$. (Note that $\rho_{ij} = \pi_{ij}/\pi_{ii} < 0$). To simplify, assume that $C = C(c_1+c_2+\ldots c_n)$, which says that every individual contributes equally, on the margin, to the aggregate measure of crime, i.e., an increase in c_j has the same effect on $p(C)$ as an increase in c_i. It follows that $\rho_{ij} = \rho_i \ \forall\ j$.

Dividing each row by π_{ii} and using standard results on determinants the denominator can be written as:

$$\begin{vmatrix} \pi_{11} & \ldots & \pi_{1n} \\ : & ..\pi_{ii}.. & : \\ \pi_{n1} & \ldots & \pi_{nn} \end{vmatrix} = \pi_{11}\ \pi_{22}\ldots\pi_{nn} \begin{vmatrix} 1 & \ldots\rho_1\ldots & \rho_1 \\ : & ..1.. & : \\ \rho_n & \ldots\rho_n.. & 1 \end{vmatrix}.$$

Dividing each row by ρ_i, this can be rewritten as:

$$\pi_{11}\ \pi_{22}\ldots\pi_{nn*}\ \rho_1\ \rho_2\ldots\rho_n * \begin{vmatrix} \frac{1}{\rho_1} & \ldots & 1 \\ : & ..\frac{i}{\rho_i}.. & : \\ 1 & \ldots & \frac{1}{\rho_n} \end{vmatrix}.$$

A standard formula from Berck and Sydsaeter [1993] indicates that:

$$\begin{vmatrix} a_1 & ..1.. & 1 \\ : & ..a_i.. & 1 \\ 1 & ..1.. & a_n \end{vmatrix} = (a_1 - 1)\ (a_2 - 1)\ldots(a_n - 1)\left[1 + \sum_{i=1}^{n} \frac{1}{a_i - 1}\right].$$

Using this formula, the determinant of the above expression is equal to:

$$(\pi_{11}\ \pi_{22}\ldots\pi_{nn})\ (\rho_1\ \rho_2\ldots\rho_n) * \left((\frac{1}{\rho_1} - 1) *\ldots\left(\frac{1}{\rho_i} - 1\right) *\ldots\left(\frac{1}{\rho_n} - 1\right)\right)$$
$$* \left[1 + \sum_{1}^{n} \frac{1}{\frac{1}{\rho_i} - 1} \right].$$

The first three terms have sign $(-1)^n$. For stability, this is the sign needed in the denominator. Thus, the final term must be positive or rewriting that $1 > -\Sigma_{i=1}^{n} \rho_i / 1 - \rho_i$. A sufficient condition for $1 > -\Sigma_{i=1}^{n} \rho_i / 1 - \rho_i$ is $\rho_i \leq -1/N \ \forall\ i$. The determinant of the numerator is:

$$\begin{vmatrix} -\pi_{1\theta_1} & \dots & \pi_{1n} \\ \vdots & ..\pi_{ii}.. & \vdots \\ 0 & \dots & \pi_{nm} \end{vmatrix} = -\pi_{1\theta_1} \begin{vmatrix} \pi_{ii} & ..:.. \\ ..:.. & \pi_{nm} \end{vmatrix} .$$

The determinant of the numerator, following the above, can be rewritten (where the previous 2 × 2 matrix has been expanded to a 3 × 3 matrix for easier readability):

$$-\pi_{1\theta_1} * (\pi_{22}\dots\pi_{nn}) * (\rho_2\dots\rho_n) * \begin{vmatrix} \frac{1}{\rho_2} & \dots & 1 \\ \vdots & ..\frac{1}{\rho_i}.. & \vdots \\ 1 & \dots & \frac{1}{\rho_n} \end{vmatrix}$$

$$= -\pi_{1\theta_1} (\pi_{22}\dots\pi_{nn}) (\rho_2\dots\rho_n) * \left((\frac{1}{\rho_2} - 1) * \dots \left(\frac{1}{\rho_i} - 1\right) * \dots \left(\frac{1}{\rho_n} - 1\right)\right) * \left[1 + \sum_{1}^{n-1} \frac{1}{\frac{1}{\rho_i} - 1} \right] .$$

Thus:

$$\frac{dc_1}{d\theta_1} = \frac{-\pi_{1\theta_1} (\pi_{22}\dots\pi_{nn}) (\rho_2\dots\rho_n) *}{(\pi_{11}\pi_{22}\dots\pi_{nn}) (\rho_1\rho_2\dots\rho_n) *}$$

$$\frac{\left((\frac{1}{\rho_2} - 1) * \dots \left(\frac{1}{\rho_i} - 1\right) * \dots (\frac{1}{\rho_n} - 1)\right) * \left[1 + \Sigma_{i=1}^{n-1} \frac{1}{\frac{1}{\rho_i} - 1} \right]}{\left((\frac{1}{\rho_1} - 1) * \dots \left(\frac{1}{\rho_i} - 1\right) * \dots (\frac{1}{\rho_n} - 1)\right) * \left[1 + \Sigma_{i=1}^{n} \frac{1}{\frac{1}{\rho_i} - 1} \right]}$$

$$= \frac{dc_1}{d\theta_1} = \frac{-\pi_{1\theta_1}}{\pi_{11}} \frac{1}{\rho_1} \frac{1}{\left(\frac{1}{\rho_1} - 1\right)} \frac{\left[1 + \sum_{i=1}^{n-1} \frac{1}{\frac{1}{\rho_i} - 1}\right]}{\left[1 + \sum_{i=1}^{n} \frac{1}{\frac{1}{\rho_i} - 1}\right]} .$$

Using the fact that $\partial c_1 / \partial \theta_1 = -\pi_{1\theta_1} / \pi_{11}$ and evaluating at $\rho_i = -(1/n)$ the above is equal to:

$$\frac{\partial c_1}{\partial \theta_1} \frac{1}{-\frac{1}{n}} \frac{1}{\left(\frac{1}{-\frac{1}{n}} - 1\right)} \frac{\left[1 + \sum_{i=1}^{n-1} \frac{1}{\frac{1}{\frac{-1}{n}} - 1}\right]}{\left[1 + \sum_{i=1}^{n} \frac{1}{\frac{1}{\frac{-1}{n}} - 1}\right]}$$

$$= \frac{\partial c_1}{\partial \theta_1} \frac{n}{-1-n} \frac{1 + \frac{n-1}{-1-n}}{1 + \frac{n}{-1-n}} = 2 \frac{\partial c_1}{\partial \theta_1} \frac{n}{n+1} .$$

Thus:

$$\frac{dc_1}{d\theta_1} = 2 \frac{n}{n+1} \frac{\partial c_1}{\partial \theta_1} .$$

Proposition A2: proof that $dc_1 / d\theta_j = (n / n + 1)\, \partial c_j / \partial \theta_j$. Using Cramer's rule, the comparative statics of $dc_1 / d\theta_2$, for example, is given by:

$$\frac{dc_1}{d\theta_2} = \frac{\begin{vmatrix} 0 & ..\pi_{1i}.. & \pi_{1n} \\ -\pi_{2\theta_2} & ..\pi_{ii}.. & : \\ 0 & ..\pi_{ni}.. & \pi_{nn} \end{vmatrix}}{\begin{vmatrix} \pi_{11} & .... & \pi_{1n} \\ : & ..\pi_{ii}.. & : \\ \pi_{n1} & .... & \pi_{nn} \end{vmatrix}} .$$

To solve for this comparative static, use the following lemma:

Lemma A1: $\lim_{a_1 \to 1} \quad (a_1 - 1)(a_2 - 1)....(a_n - 1) \left[1 + \Sigma_{i=1}^{n} \frac{1}{a_i - 1} \right] = (a_2 - 1)....(a_n - 1).$

Proof: let $k = (a_2 - 1)....(a_n - 1)$. Then upon rewriting the above, it is clear that:

$$\lim_{a_1 \to 1} \left((a_1 - 1)k + k + (a_1 - 1)k \left[\sum_{i=2}^{n} \frac{1}{a_i - 1} \right] \right) = k .$$

The numerator of the comparative static can be written (where the author has eliminated the first column and second row in the usual manner but then expanded the resulting 2×2 matrix to a 3×3 matrix for easier readability):

$$\pi_{2\theta_2} * \begin{vmatrix} \pi_{12} & ..\pi_{1i}.. & \pi_{1n} \\ : & ..\pi_{ii}.. & : \\ \pi_{nn} & ..\pi_{ni}.. & \pi_{nn} \end{vmatrix} = \pi_{2\theta_2} * (\pi_{11}\,\pi_{33} \cdots \pi_{nn}) * \begin{vmatrix} \rho_1 & ..\rho_1.. & \rho_1 \\ : & ..1.. & : \\ \rho_n & ..\rho_n.. & 1 \end{vmatrix}$$

$$= \pi_{2\theta_2} * (\pi_{11}\,\pi_{33} \cdots \pi_{nn}) * (\rho_1\,\rho_3 \cdots \rho_n) * \begin{vmatrix} 1 & ..1.. & 1 \\ : & ..\frac{1}{\rho_i}.. & : \\ 1 & ..1.. & \frac{1}{\rho_i} \end{vmatrix} .$$

Using Lemma A1 and placing the result over the denominator:

$$\frac{dc_1}{d\theta_2} = \frac{\pi_{2\theta_2} (\pi_{11}\, \pi_{33} \ldots \pi_{nn})\, (\rho_1\, \rho_3 \ldots \rho_n) * \left((\frac{1}{\rho_3} - 1) * \ldots (\frac{1}{\rho_i} - 1) * \ldots (\frac{1}{\rho_n} - 1) \right)}{(\pi_{11}\, \pi_{22} \ldots \pi_{nn})\, (\rho_1\, \rho_2 \ldots \rho_n) * \left((\frac{1}{\rho_1} - 1) * \ldots (\frac{1}{\rho_i} - 1) * \ldots (\frac{1}{\rho_n} - 1) \right) * \left[1 + \Sigma_{i=1}^{n} \frac{1}{\frac{1}{\rho_i} - 1} \right]}$$

$$\frac{dc_1}{d\theta_2} = \frac{\pi_{2\theta_2}}{\pi_{22}\, \rho_2 * \left((\frac{1}{\rho_1} - 1)(\frac{1}{\rho_2} - 1) \right) * \left[1 + \Sigma_{i=1}^{n} \frac{1}{\frac{1}{\rho_i}} - 1 \right]} .$$

Using the fact that $\partial c_2 / \partial\theta_2 = -\pi_{2\theta_2} / \pi_{22}$ and evaluating at $\rho_i = -(1/n)$, the above is equal to:

$$\frac{dc_1}{d\theta_2} = \frac{\frac{\partial c_2}{\partial \theta_2}}{\frac{-1}{n} * \left((\frac{1}{\frac{-1}{n}} - 1)(\frac{1}{\frac{-1}{n}} - 1) \right) * \left[1 + \sum_{i=1}^{n} \frac{1}{\frac{1}{\frac{-1}{n}} - 1} \right]}$$

$$\frac{dc_1}{d\theta_2} = \frac{n}{n+1} \frac{\partial c_2}{\partial \theta_2} .$$

Footnotes

1. The figure is a calculation of losses and expenditures made by *Business Week* [Mandel, 1993].
2. Sherman [1995] notes that every arrest takes police off the street to do paperwork and, thus, reduces police visibility and effectiveness. In New York City, arrest paperwork can take one or two police officers off the street for an entire shift. Blumstein [1995] discusses how overcrowding can lead to reduced sentences.
3. This paper also differs from Sah [1991] and Kuran [1989] in setting up the problem explicitly as a well-specified game.
4. Pyle [1983] gives a concise exposition of the model.
5. Subscripts have been dropped for notational ease.
6. In Cooper and John [1988], positive spillovers and strategic complementarity are defined at a symmetric Nash equilibrium. In this model, these effects are global.

7. Strategic complementarity is required only for the more important part (a) of the multiplier definition. Part (b) occurs under much weaker conditions.
8. The matrix being negative definite is equivalent to the real portion of the characteristic roots being negative.
9. In this context, θ_i could represent (the inverse of) the individual's wage in the legitimate sector or (the inverse of) a parameter indicating individual productivity in the legitimate sector.
10. An increase in θ_i represents an (*ceteris paribus*) increased tendency towards crime, so the partial derivative $\partial c_i / \partial \theta_i = -(\pi_{i\theta_i} / \pi_{ii}) > 0$ by assumption. Since $\pi_{ii} < 0$, it follows that $\pi_{i\theta_i} > 0$.
11. Sixteen- to 18-year-olds, for example, commit about twice as many aggravated assaults and murders and about five times as many robberies and burglaries as those twice their age [Blumstein, 1995].

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