## REFLECTIONS ON OPTIMAL PUNISHMENT, OR:

## SHOULD THE RICH PAY HIGHER FINES?

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Economic analysis of law seems to suggest that the pecuniary equivalent of the combination of probability and punishment imposed on a criminal ought to be equal to the damage done by the crime and independent of the criminal's characteristics, thus preventing all "inefficient" crimes (for which the benefit to the criminal is less than the cost imposed) and only such crimes. It is shown that both this rule and the alternative principle of "enough punishment to deter" are special cases of a more general rule which may be stated as "punishment equal to the net costs of altering the level of punishment so as to generate one more crime'; the result may be higher or lower than the damage done by the crime. The analysis is applied to show that for crimes where either the "demand" for the crime (by potential criminals) or the cost of imposing punishment varies with the criminal's income, the pecuniary equivalent of the punishment should also vary with income. Whether rich or poor should pay higher fines turns out

[^0]to be indeterminate save in special cases. Some of the empirical impli-
cations of the analysis are explored, and some attempt is made to use data
to estimate some of the parameters of the models that are used in the
analysis.

## I. INTRODUCTION

In discussing economic analysis of law with someone outside the field, one frequently encounters the argument that it is somehow wrong for a rich man and a poor man to be punished, as economic analysis seems to suggest that they should be, by equal fines for the same crime. This is seen as an argument for either using nonpecuniary punishments, such as imprisonment, or imposing higher fines on the rich. After defending the principle of equal fines for some years in the classroom, I came to the conclusion that it was in part mistaken and that the intuition behind the argument against equal fines was in part correct; this article began as an attempt to explain that conclusion. It became not so much an investigation of whether the rich should pay higher fines as an investigation of the theory of optimal punishment inspired by that question. One of my conclusions was that for many crimes economic efficiency requires that the punishment imposed on the criminal, measured in dollar terms, ought to vary with his income. Whether the rich should pay higher fines or lower ones turned out, however, to be a more complicated question than I had expected.

In considering what the punishment for a crime should be, two different criteria suggest themselves, one based on the damage done to the victim, and one based on the benefit to the criminal. The former has been suggested by Richard Posner (5): "the optimum penalty is simply the social cost of the unlawful act divided by the desired probability that the penalty will in fact be imposed'' or, in other words, that the expected punishment' ought to be equal to the total social cost of the crime. The argument for this criterion is that if the benefit to the criminal is greater than the total social cost of the crime, the crime is on net desirable and ought not to be deterred; by imposing an expected punishment equal to the social cost we deter all crimes which impose net social cost and only those crimes, leaving the criminal free to evaluate the benefit to himself of the crime as in an ordinary market and commit or not commit it accordingly. ${ }^{2}$ If this results in a rich man "buying" more crimes than a poor man, that is no more objectionable, in terms of conventional welfare analysis, than that selling Cadillacs at the same price to rich and poor results in the rich buying more of them. In each case, the rich man pays for what he gets-although in the case of crime, he may pay it to the wrong person.

This criterion would be correct if the costs of imposing an expected punishment on the criminal were independent of the level of expected punishment imposed [see Becker and Landes (2)]. Usually they are not. In order to impose a higher level of expected punishment, one must either increase the fraction of criminals caught, which in general is costly, or increase the punishment imposed. Increasing the level of punishment is likely to require a shift, at least for many convicted criminals, from relatively inexpensive punishments (fines) to more expensive punishments (imprisonment or execution). ${ }^{3}$

This argument brings us back to the second criterion-punishment according to the benefit received by the criminal. If a criminal steals $\$ 10,000$ worth of goods but his net benefit (after subtracting the cost of his labor, the fence's cut, and other expenses of his business) is only $\$ 3,000$, then an expected punishment of just over $\$ 3,000$ will deter as well as a fine of $\$ 10,000$-and more cheaply if, as I argued above, cost rises with expected punishment.

Since the first criterion tells us which crimes we wish to deter and the second tells us the cheapest way to do so, it might at first seem as though we could get an ideal system by combining them, making the expected punishment either the damage done or the cost incurred, whichever was* less. We would thus permit all efficient crimes and deter all inefficient crimes at the lowest possible cost. Unfortunately, the solution is more complicated than this. Each of the criteria is correct for a special case: the first for the case where cost of enforcement and punishment is independent of both the level of expected punishment and the number of crimes, and the second for the case where all crimes are known to be inefficient, cost is an increasing function of the total of all punishments imposed, and it is possible to discriminate perfectly among different criminals who receive different benefits from commission of similar crimes. In the more general case typical of the real world, neither set of assumptions holds. The costs of enforcement and punishment rise with the level of punishment for a given number of offenses and rise with the number of offenses for a given level of punishment. While laws and courts can (and do) discriminate to some degree between similar offenses committed by different criminals, perfect discrimination is as impractical as it is for the discriminating monopolist of conventional price theory. As a result, the calculation becomes complicated; the optimal punishment may be lower or higher than the result suggested by Posner. This may be seen most easily by going over some simple examples.

Consider a crime which imposes damage on the victim of $\$ 10,000$, and for which the cost of imposing any particular expected punishment (via an optimum combination of level of punishment and probability of detection) is 10 percent of the expected punishment. Following the rule
suggested by Posner, the optimum expected punishment is then $\$ 11,111$ (including the cost of imposing the punishment in the total social cost of the crime). But let us further assume that there are 101 potential criminals, that for 100 of them the return from crime is $\$ 1,000$ each, while for the remaining one it is $\$ 20,000$. If we set the level of expected punishment at $\$ 11,11 I$ one crime will be committed, and the cost of punishing it will be $\$ 1,111$. If we instead set the level of expected punishment at $\$ 1,001$, one crime will still be committed and the cost of punishing it will only be $\$ 100$. Clearly, there is a net gain to decreasing the punishment.

Consider, on the other hand, a crime for which the cost of catching and convicting criminals is low and independent of the level of punishment imposed, the cost of punishment is again 10 percent of the amount of punishment, and the damage imposed is again $\$ 10,000$ per crime. Further assume that with an expected punishment of $\$ 10,000$, a thousand (efficient) crimes a year occur, whereas with a punishment of $\$ 10,500$ the rate is reduced to zero. Raising the fine $\$ 500$ above the damage does eliminate a thousand efficient crimes, whose net gain would have been somewhere between zero and $\$ 500,000$ (since the criminals were unwilling to commit the crimes at a price of $\$ 10,500$ ), but it saves a million dollars in net punishment costs. In this case, the expected punishment again differs from the damage done because of the cost of raising the expected punishment. But this time that cost is negative. Because of a highly elastic demand for crimes, the reduction in the number of crimes to be punished more than makes up for the increase in the punishment per crime.

What I have shown by these examples is that the optimal punishment depends not only on the costs imposed by the crime but also on the elasticity of demand (if you think of crimes being "purchased" by criminals at a "price" corresponding to the expected punishment) for the crime. This suggests the possibility of improving a system of uniform punishments by dividing the population into groups with differing elasticities of demand for a particular crime and charging them different "prices"-or, in other words, having different punishments for different groups. The advantage of such discrimination can be seen by another simple example.

Suppose there are two groups of criminals: blonds and brunets. There exist 100 blonds, each of whom can benefit if he commits the crime (stealing a case of bottles of hair bleach?) by $\$ 1,000$, and one who can benefit by $\$ 20,000$. There exist 100 brunets, each of whom can benefit by $\$ 2,000$, and one who can benefit by $\$ 20,000$. The cost of the crime to the victim is again $\$ 10,000$. All undesirable offenses could be deterred by a fine of $\$ 2,001$-but they could be deterred more cheaply by fining blonds $\$ 1,001$ and brunets $\$ 2,001$.

To create an argument for varying punishment according to the income of the criminal, consider that set of crimes for which the return to the criminal rises, ceteris paribus, with his income. These would presumably be of three sorts: (1) such as speeding, where the payoff is in terms of time saved by the offender; (2) such as killing one's wife (instead of bribing her to consent to a divorce) in order to marry someone else, where the cost of purchasing the good is higher to a wealthy person; and (3) such as rape or killing someone for motives of dislike, where the good is not available on the market and where there is no obvious reason why it should be less pleasurable to the rich man, for whom the marginal pleasure that he can purchase with a dollar in other ways is presumably lower, than for the poor man. In the second of the cases, the criterion suggested by Posner presumably implies a higher fine for the rich man (since the cost of the crime to the victim includes the loss of the money that could have been extorted from the criminal); in the other two cases it does not.

In this case, just as in the case of blonds and brunets, potential criminals can be divided into groups (rich and poor) with different "demand functions" for crime. I will show in the next section that only under special (and improbable) circumstances will it be optimal to charge the same "price" to both groups. Whether the higher price should be charged to the rich criminal or the poor one turns out to be a more complicated question, depending on the details of demand and cost functions. While the higher (dollar) punishment required to deter a richer criminal is an argument for charging higher fines to the rich, the higher (punishment and possibly enforcement) costs of those higher fines are an argument for ignoring the rich and concentrating on the easier task of deterring the poor. In the next section, I show that if demand functions and punishment costs are linear, and if the only difference between rich and poor criminals is in their demand functions, then the optimum pattern of fines (if one exists for which amount of crime and expected punishment are both non-zero) requires that poor criminals pay much higher fines than rich ones.

A second argument for differential fines comes from looking at cost functions instead of demand functions. The motivation for imposing an expected punishment less than the social cost of the crime is that it is costly to raise the expected punishment, and that the resulting increase in deterrence may be insufficient to justify the cost. The cost of increasing the expected punishment comes in part from the fact that higher penalties are likely to have higher net social costs. Given that fines have a low net social cost and that people with higher incomes can generally pay higher fines, punishment costs are likely to start increasing (reflecting the transition from fines to imprisonment) at a lower (dollar) level of punishment for lower income criminals. Furthermore, even where both
high income and low income criminals are unable to pay fines and must be imprisoned, equal (dollar) punishments imply longer imprisonment, and hence larger net social costs of punishment, for the lower income criminal. Hence, even if the supply of crimes as a function of expected punishment were independent of the income of the criminal (as l have argued above that, for certain sorts of crimes, it is not), the higher net cost of imposing any particular punishment on the lower income criminal would imply a different optimal punishment.

While the optimal punishment for the poor criminal in this case is different from the optimal punishment for the rich, it is not, as one might at first suppose, necessarily lower. This is most easily seen by considering the case where the demand for crime is very elastic with price. Raising the amount of the punishment reduces the number of criminals to such a degree that total punishment cost falls as punishment increases. In this case, the inefficiency of collecting fines from those too poor to pay them is an argument for raising the level of the fine-in order to reduce the frequency with which it must be collected.

So far I have used verbal arguments and simple numerical examples to establish two propositions: that the optimal punishment depends on both the damage done by a crime and the elasticity of supply of crimes, and that there may be efficiency gains to imposing different punishments on different sorts of people, in particular on people of different incomes. In the next section of this paper I will set up and explore the consequences of a formal model of crime and prevention, in order to make more precise statements of both propositions.

## II. ANALYSIS

## List of Symbols 1

Q: Number of occurrences of the crime per year.
H: Harm done to the victim by one occurrence of the crime.
p: Probability that an occurrence of the crime will result in apprehension and punishment of the perpetrator.
f: Punishment imposed upon any criminal who is punished.
P: The certainty equivalent to the criminal (in dollars) of a probability $p$ of punishment $f$.
F: The amount of punishment $\equiv \mathrm{P} / \mathrm{p}$.
$\mathrm{E}(\mathrm{p}, \mathrm{Q}, \mathrm{F})$ : The cost of policemen, courts, etc., necessary to maintain a probability p of a punishment amount F for a given Q . (For any p and $\mathrm{F}, \mathrm{f}$ is assumed chosen to minimize C.)
$F^{\prime}$ : The amount received by the court system when it imposes punishment f. (More generally, $\mathrm{F}^{\prime}$ might be received by

$$
\begin{aligned}
& \begin{array}{l}
\text { anyone other than the criminal-for example, the victim in } \\
\text { a system of civil law or the accuser in a bounty system.) }
\end{array} \\
& \mathrm{Z}(\mathrm{p}, \mathrm{P}): \begin{array}{l}
\text { The punishment inefficiency } \equiv\left(\mathrm{F}-\mathrm{F}^{\prime}\right) / \mathrm{F} .
\end{array} \\
& *: \text { On any variable denotes its optimal value. } \\
& \mathrm{C}(\mathrm{p}, \mathrm{Q}, \mathrm{~F}): \text { The total cost of a given level of enforcement }= \\
& \mathrm{E}(\mathrm{p}, \mathrm{Q}, \mathrm{~F})+\mathrm{PQZ}(\mathrm{p}, \mathrm{~F}) .
\end{aligned}
$$

I will consider a single crime, assumed homogeneous; the quantity is $Q$ crimes per year. Each occurrence imposes harm $H$ on the victim, hence total harm is QH. Enforcement agencies impose on criminals a probability p of being captured and convicted; throughout the analysis $I$ assume $p$ is the same for all criminals. A convicted criminal receives a punishment $f$. I define $F$, the "amount" of the punishment, as that sum such that the criminal would be indifferent between a certain fine of pF and a probability p of suffering punishment f (which may or may not be a fine). Note that if $f$ is a fine $F>f$ if the criminal is risk averse, $\mathrm{F}<\mathrm{f}$ if the criminal is a risk preferrer. Criminals are assumed to have identical tastes for risk. Since different combinations of $p$ and $f$ which impose the same $P \equiv p F$ are equivalent to the criminal they will have the same deterrent effect. Hence $Q=Q(P)$.
$E(p, Q, F)$ is the enforcement cost, the cost of policemen, courts, etc., necessary to maintain a probability $p$ of imposing a punishment amount $F$ given that $Q$ crimes are being committed. $F$ is included as one of the variables on the assumption that the efforts taken by criminals to avoid capture may in part depend on it; ${ }^{4}$ that assumption will be dropped later for purposes of simplicity. $\mathrm{Z}(\mathrm{p}, \mathrm{P})$ is the punishment inefficiency, defined by:

$$
\mathrm{Z}(\mathrm{p}, \mathrm{P}) \equiv \frac{\mathrm{F}-\mathrm{F}^{\prime}}{\mathrm{F}},
$$

where $\mathrm{F}^{\prime}$ is the amount the court system receives when it imposes punishment $f$. It is assumed that the court system, for obvious reasons, chooses for any $F$ that $f$ which minimizes $Z$. In the case of a fine imposed (with no collection cost) on a risk-neutral criminal, $Z=0$. Note that $Z$ incorporates "inefficiencies" associated with risk aversion as well as collection costs, salary of prison guards ( $\mathrm{F}^{\prime}$ may be negative), and the like. A fine imposed with no collection costs on a risk-averse criminal is still "inefficient"; a similar fine on a risk-preferring criminal has a negative Z . I will assume that the latter does not occur and that Z is always nonnegative.

The total cost $C$ of a given level of enforcement is the enforcement cost $E$ plus the punishment cost, pQFZ . Q depends on $\mathrm{P} \equiv \mathrm{pF}$ via a
"demand function" $\mathrm{Q}(\mathrm{P})$; one may think of P as the price charged for a crime and Q the quantity of crime that criminals choose to purchase at that price. For a given $P$ and $Q(P)$, the harm done to the victims and the benefit received by the criminals (gross of punishment) are independent of the particular $p, F$ corresponding to that $P$. Hence the enforcement agency chooses $p^{*}, F^{*}$ to minimize $C=E(p, Q, F)+P Q Z(p, P)$. We may then define $C(\mathrm{P}, \mathrm{Q})=\mathrm{C}\left(\mathrm{p}^{*}, \mathrm{Q}, \mathrm{F}^{*}\right)$.

The enforcement agency then chooses $P$ to maximize net benefits in the conventional sense. Benefit to the criminals of any quantity Q is measured by the area under the demand curve up to Q ; marginal benefit is simply the inverse function $\mathrm{P}(\mathrm{Q}) .{ }^{5}$ Marginal harm to the victims is H ; the cost of a given level of enforcement and punishment (including costs to punished criminals and benefits or costs of punishment to the enforcers) is given by $C(\mathrm{P}, \mathrm{Q})$ and marginal cost by

$$
\frac{\mathrm{d} C}{\mathrm{dQ}}=C_{1} \frac{\mathrm{dP}}{\mathrm{dQ}}+C_{2} .
$$

The situation is shown in the figure; the optimal price $P^{*}$ occurs where the demand curve $P(Q)$ intersects the sum of marginal harm plus marginal enforcement and punishment cost. dC/dQ may be positive or negative; lowering the price of a crime lowers enforcement costs through lowering p but raises them by increasing Q , and similarly with punishment cost. For $\mathrm{P}=0$, it can be shown that $\mathrm{d} C / \mathrm{dQ}<0$ for any reasonable form of $Q(P)$, and $I$ have drawn the figure accordingly.

Note that I follow Becker rather than Tullock (7) in giving the same weight to costs imposed on or benefits received by criminals as to costs and benefits associated with the victims or the enforcement apparatus. This follows from a general framework in which the specification of what is or is not a crime-that is to say, the definition of property rightsfollows from efficiency considerations rather than being a constraint upon them.

In examining the figure, one can see in what sense it is and in what sense it is not true that the punishment should equal the cost imposed by the crime. It is not true that P equals the cost imposed by the criminal's decision to commit one more crime; such a decision corresponds to an upward shift in the demand curve, and the resulting cost is $\mathrm{H}+\mathrm{C}_{2}>\mathrm{H}$. It is true that P should equal the cost imposed by the society's decision to alter its control variable ( P ) so as to permit one more crime to occur. That cost is $\mathrm{H}+C_{2}+C_{1} \mathrm{dP} / \mathrm{dQ}$ which may be greater or less than H .

I have now made precise the argument sketched in the preceding section, and shown how the optimal punishment is related to the damage done by the crime, the cost of catching and punishing the criminal, and the demand function for the crime (by the criminal). I will now apply

the analysis to the question of whether punishments should be different for different income groups. In doing so, I will consider the two arguments given above: first, that demand functions may differ with income, so that for certain crimes it requires a higher punishment to deter a given fraction of rich criminals than to deter the same fraction of poor criminals; second, that it may be cheaper to impose a given amount of punishment on a rich criminal than on a poor one.

## List of Symbols 2

$\mathrm{Q}_{\mathrm{i}}$ : Number of occurrences of the crime per year committed by members of group i.
$f_{i}$ : Punishment imposed upon any member of group $i$ who is punished.
$P_{i}$ : Certainty equivalent to member of group $i$ of a probability $p$ of punishment $f_{i}$.
p: Probability that an occurrence of the crime will result in apprehension and punishment of the perpetrator (independent of which group he is a member).
m : The ratio of the number of members of group 2 (nonrich) to the number of members of group 1 (rich).
n : The ratio of the punishment necessary to deter any given fraction of members of group 1 from the crime to the punishment necessary to deter the same fraction of members of group 2 .
B: A net benefit function consisting of the benefit to criminals of the crimes they commit minus the cost to the victims of having the crimes committed upon them minus enforcement costs minus net punishment costs.
$\eta_{\mathrm{i}}$ : The inverse demand elasticity for committing the crime by members of group $i$.
$\alpha(p): \quad Z(p, P) / P$, assumed independent of $P$.
*: On any variable denotes its optimal value, defined as that which maximizes $B$.

Accordingly, I divide the population into two groups and let

$$
\begin{equation*}
\mathrm{Q}(\mathrm{P})=\mathrm{Q}_{1}\left(\mathrm{P}_{1}\right)+\mathrm{Q}_{2}\left(\mathrm{P}_{2}\right) ; \tag{1}
\end{equation*}
$$

subscript I denotes rich, subscript 2 denotes nonrich. The enforcement agency can choose to impose different fines on members of the two different groups; $p$ is the same for both. Initially I assume, with Marshall, ${ }^{6}$ that

$$
\begin{equation*}
\mathrm{P}_{1}(\mathrm{Q})=n \mathrm{P}_{2}(\mathrm{mQ}) . \tag{1'}
\end{equation*}
$$

This amounts to assuming that rich and poor have the same utility functions (or distributions of utility functions) for the benefits of committing the crime, and differ only in their marginal utility for income. m is the ratio of the number of not rich to the number of rich and $n$ the ratio of marginal utility of income (or the inverse ratio of the dollar value of time, if we consider a crime such as speeding) for the two groups. Initially, I also assume that $\mathrm{Z}(\mathrm{p}, \mathrm{P})$, the inefficiency of punishment, is the same function for the two groups. Later in this section I reverse the assumptions, giving the two groups similar demand functions but different punishment costs.

I wish to maximize a net benefit function:

$$
\begin{aligned}
B & =\int_{0}^{\mathrm{Q}} \mathrm{I}_{1}(x) d x+\int_{0}^{\mathrm{Q}}{ }_{2} P_{2}(x) d x-H Q-E(p, Q) \\
& -Z\left(p, P_{1}\left(Q_{1}\right)\right) P_{1}\left(Q_{1}\right) Q_{1}-Z\left(p, P_{2}\left(Q_{2}\right)\right) P_{2}\left(Q_{2}\right) Q_{2}
\end{aligned}
$$

For simplicity, I assume that Z is linear in P and that $\mathrm{Z}(\mathrm{p}, 0)=0$ (small fines can be collected costlessly). Hence $Z(p, P)=\alpha(p) P$. Henceforward I will write $P_{i}$ for $P_{i}\left(Q_{i}\right), \alpha$ for $\alpha(p)$.

Varying $p, Q_{1}$ and $Q_{2}$ we get the following first-order conditions:

$$
\begin{gathered}
\left.\frac{\mathrm{dB}}{\mathrm{dp}}\right|_{\alpha_{1}^{*} Q_{2}^{*}}=-\mathrm{E}_{1}-\alpha^{\prime}\left(\mathrm{P}_{1}^{* 2} \mathrm{Q}_{1}^{*}+\mathrm{P}_{2}^{* 2} \mathrm{Q}_{2}^{*}\right)=0 \\
\left.\frac{\mathrm{~dB}}{\mathrm{dQ}}\right|_{\mathrm{i}} \mathrm{P}^{*} \cdot Q_{j}^{*}
\end{gathered}=\mathrm{P}_{\mathrm{i}}^{*}-\mathrm{E}_{2}-\mathrm{P}^{* 2} \alpha\left(1-2 \eta_{i}\right)-\mathrm{H}=0 \quad \mathrm{i}=1,2 . .
$$

Here $\eta_{i} \equiv-\mathrm{P}_{\mathrm{i}}^{\prime} \mathrm{Q}_{\mathrm{i}} / \mathrm{P}_{\mathrm{i}}$ is the inverse of the demand elasticity. Note that (from 1')

$$
\begin{equation*}
\eta_{1}(\mathrm{P})=\eta_{2}(\mathrm{P} / \mathrm{n}) . \tag{2}
\end{equation*}
$$

I will assume that

$$
\begin{equation*}
\frac{\mathrm{d} \eta_{\mathrm{i}}}{\mathrm{dP}}<0 \tag{3}
\end{equation*}
$$

(that is, demand elasticity increases with price), as seems reasonable. Two of the second-order conditions will also prove useful; they are (after some rearrangement):

$$
\frac{\mathrm{E}_{22}}{\mathrm{P}_{\mathrm{j}}^{* \prime}}+2 \alpha \mathrm{P}_{\mathrm{i}}^{* \prime \prime} \frac{\mathrm{P}_{\mathrm{i}}^{*} \mathrm{Q}_{\mathrm{i}}}{\mathrm{P}_{\mathrm{i}}^{\prime *}}<1-2 \alpha \mathrm{P}_{\mathrm{i}}^{*}\left(2-\eta_{\mathrm{i}}\right) \quad \mathrm{i}=1,2 .
$$

Note that

$$
\mathrm{P}_{\mathrm{i}}^{\prime} \equiv \frac{\mathrm{dP}_{\mathrm{i}}}{\mathrm{dQ}}<\mathbf{0} .
$$

From the first-order conditions it follows that

$$
\begin{equation*}
\eta_{\mathrm{i}}<1 / 2 \rightarrow \mathrm{P}_{\mathrm{i}}^{*}>\mathrm{H} \quad \text { (I will use this in Section III) } \tag{4}
\end{equation*}
$$

and

$$
\begin{equation*}
\mathrm{P}_{1}^{*}-\mathrm{P}_{2}^{*}=\mathrm{P}_{1}^{* 2} \alpha\left(1-2 \eta_{1}\right)-\mathrm{P}_{2}^{* 2} \alpha\left(1-2 \eta_{2}\right) . \tag{5}
\end{equation*}
$$

It follows immediately that for the optimal fine to be the same for both groups ( $\mathrm{P}_{1}^{*}=\mathrm{P}_{2}^{*} \equiv \mathrm{P}^{*}$ ) the elasticities must be the same at that fine ( $\eta_{1}\left(\mathrm{P}^{*}\right)=\eta_{2}\left(\mathrm{P}^{*}\right)$ ) which is ruled out by Eqs. (2) and (3). In order to learn more about the relation between the two fines, I rearrange Eq. (5) to give:

$$
\begin{align*}
\left(P_{1}^{*}-P_{2}^{*}\right)\left(1-\alpha\left(P_{1}^{*}+P_{2}^{*}\right)\left(1-\eta_{1}-\eta_{2}\right)\right) & \\
& =-\alpha\left(P_{1}^{* 2}+P_{2}^{* 2}\right)\left(\eta_{1}-\eta_{2}\right) . \tag{6}
\end{align*}
$$

If the second parenthesis on the left-hand side is positive (as it must be
unless elasticities are high and punishment inefficiencies are high) we can deduce from Eqs. (2), (3), and (6) that:

$$
\mathrm{P}_{2}^{*}>\mathrm{P}_{1}^{*} \quad \text { or } \quad \mathrm{P}_{1}^{*}>\mathrm{nP}_{2}^{*}
$$

If we assume the parenthesis is negative, then:

$$
\mathrm{P}_{1}^{*}>\mathrm{P}_{2}^{*}>\mathrm{P}_{1}^{*} / \mathrm{n}
$$

To get a more definite result, I now assume that the demand functions are linear:

$$
\begin{aligned}
& P_{1}=A-B Q_{1} \\
& P_{2}=A / n-(B / n m) Q_{2}
\end{aligned}
$$

It follows that:

$$
\begin{aligned}
& \eta_{1}=\left(\mathrm{A} / \mathrm{P}_{1}\right)-1 \\
& \eta_{2}=\left(\mathrm{A} / \mathrm{nP}_{2}\right)-1
\end{aligned}
$$

Plugging these into the first-order conditions yields quadratic equations for $P_{i}$; the solutions are:

$$
\begin{array}{r}
P_{1}=\frac{1+x \pm \sqrt{(1+x)^{2}-y}}{6 \alpha} \\
P_{2}=\frac{1+\frac{x}{n} \pm \sqrt{\left(1+\frac{x}{n}\right)^{2}-y}}{6 \alpha}
\end{array}
$$

where $\mathrm{x} \equiv 2 \alpha \mathrm{~A}, \mathrm{y} \equiv 12 \alpha\left(\mathrm{E}_{2}+\mathrm{H}\right)$.
From the second-order conditions given above, plus the assumption $\mathrm{E}_{22}$ $<0$ (net economies of scale in catching criminals), it follows that:

$$
\begin{gather*}
0<1-2 \alpha\left(3 P_{1}-A\right)  \tag{8}\\
0<1-2 \alpha\left(3 P_{2}-A / n\right)
\end{gather*}
$$

Solving these for $P_{1.2}$ yields a pair of inequalities which eliminate the upper roots of the quadratic equations. Hence:

$$
\frac{P_{1}}{P_{2}}=\frac{1+x-\sqrt{(1+x)^{2}-y}}{1+\frac{x}{n}-\sqrt{\left(1+\frac{x}{n}\right)^{2}-y}}
$$

Since $\mathrm{n}>1$, this is unambiguously positive, provided that it is real.

Hence $P_{1}<P_{2}$. Further (and tedious) manipulations lead to the result that:

$$
\begin{equation*}
\text { iff } 3 \alpha\left(\mathrm{P}_{1}+\mathrm{P}_{2}\right)>1 \text {, then } \mathrm{P}_{2}>\mathrm{nP}_{1} . \tag{9}
\end{equation*}
$$

If $[1+x / n]^{2}-y$ is negative the root for $P_{2}$ becomes complex; the maximum benefit then occurs at a corner solution, where either a price or a quantity is zero.

## List of Symbols 3

$\mathrm{Z}^{\mathrm{Z}}(\mathrm{p}, \mathrm{P})$ : Punishment inefficiency (as in List 1) for punishing members of group i .
$\mathrm{k}(\mathrm{p})=\mathrm{Z}^{2}(\mathrm{p}, \mathrm{P}) / \mathrm{Z}^{\prime}(\mathrm{p}, \mathrm{P})$
$g_{i}=1-2 \eta_{i}$
$\alpha(p)=Z^{\prime}(p, P) / P$, assumed independent of $P$.
*: On any variable denotes its optimal value.
I next consider the case where rich and poor criminals have the same demand functions but different punishment costs. Maintaining my previous assumption on Z , I have:

$$
\begin{equation*}
Z^{\prime}(p, P)=\alpha(p) P \quad Z^{2}(p, P)=\alpha(p) k(p) P \tag{10}
\end{equation*}
$$

Since I assume it is less expensive to impose any given punishment on the rich, $\mathrm{k}>1$.

The first-order conditions then yield, with some manipulation,

$$
\left(\mathrm{P}_{1}^{*}-\mathrm{P}_{2}^{*}\right)\left(1-(\alpha / 2)\left(\mathrm{P}_{1}^{*}+\mathrm{P}_{2}^{*}\right)\right)=\left(\mathrm{P}_{1}^{* 2}+\mathrm{P}_{2}^{* 2}\right)\left(\mathrm{g}_{1}-\mathrm{kg}_{2}\right)
$$

where

$$
\mathrm{k}=\mathrm{k}(\mathrm{p}), \mathrm{g}_{\mathrm{i}} \equiv 1-2 \eta_{\mathrm{i}}, \alpha=\alpha(\mathrm{p}) .
$$

Note that here, unlike the previous case, the subscripts on $\mathrm{P}, \mathrm{g}$, and $\eta$ represent not different functions but the same functions at different values of Q . As in the previous case, $\mathrm{P}_{1}^{*} \neq \mathrm{P}_{2}^{*}$. If the g 's are small or negative (that is, the $\eta$ 's are not much less than $1 / 2$ ) we have: $g_{1}>\mathrm{kg}_{2}$ or $g_{2}>g_{1}$. Since $g_{i}$ increases with $P_{i}$ this suggests that $P_{i}^{*}$ is again either less than $\mathrm{P}_{2}^{*}$ or much larger, although in this case the exact constraints on the $\mathrm{P}^{*}$ 's depend on the functional form that links them to the g 's via the $\eta$ 's.

If I assume that $P$ is linear in $Q$, I get, following essentially the same procedure as before:

$$
\begin{equation*}
\frac{P_{1}^{*}}{P_{2}^{*}}=\frac{1+x-\sqrt{(1+x)^{2}-y}}{\frac{1}{k}+x-\sqrt{\left(\frac{1}{k}+x\right)^{2}-y}}<1 . \tag{11}
\end{equation*}
$$

If $((1 / k+x))^{2}>y$, the roots are real and $P_{2}^{*}>P_{1}^{*}$ otherwise there is only a corner solution.

The result so far is that if rich and poor criminals are distinguished only by different demand functions, the punishment imposed on rich criminals is either lower or much higher ( $\mathrm{P}_{1}^{*}>\mathrm{nP}_{2}^{*}$ ) than that imposed on poor criminals, save in the case where both elasticities and punishment inefficiencies are high $\left(\left(P_{1}^{*}+P_{2}^{*}\right)\left(1-\eta_{1}-\eta_{2}\right)>1\right)$. If the demand functions are linear and yield an interior solution, the optimal punishment for rich criminals is lower than for poor. If rich and poor criminals have identical demand functions but different punishment costs, it again follows that optimal punishments are different for the two groups; but in this case, if the demand functions are linear and yield an interior solution, the optimal punishment is higher for rich criminals than for poor.

While it would be interesting to investigate further models in which both demand functions and cost functions varied with income, or to redo the analysis parameterizing Z to keep it between zero and one, as is suggested by some of the arguments below, I have done enough to make the essential conclusion clear. An optimal pattern of fines will not, save in very special circumstances, charge the same fine to rich and poor. Which fine ought to be higher will depend on the details of enforcement and punishment cost and demand for crime functions.

## III. THE REAL WORLD

In relating my analysis to the real world, two sorts of questions are worth asking. The first is what I can learn from the real world about the various functions that go into the analysis-punishment costs, demand for crime, enforcement costs, and the like. The second is whether or not existing arrangements for controlling crime are what I say they should be. If they are not, that may be interpreted as a criticism either of existing arrangements or, if one accepts the Posner (Thompson) thesis that common law (statutory law) is optimal, of my analysis. Insofar as the two sorts of questions can be separated, I will discuss them in the order I have described, beginning with the question of how high is Z . In trying to answer that question, however, a puzzle arises which forces me to consider whether or not our system of punishments is an efficient one.

Suppose there exist two punishments, $f$ and $g$, of which $g$ is the more severe. There is some $p$ such that the criminal is indifferent between a certainty of $f$ and a probability $p$ of $g$. If the inefficiency of $g$ is less than that of $f$, $f$ will never be used; by substituting a probability $p$ of $g$, enforcement costs and punishment costs are lowered while all other costs remain the same. Hence (assuming existing arrangements are efficient) inefficiency should never be observed to decline with increasing level of punishment.

We observe that imprisonment, for which inefficiency appears to be substantially greater than 1 , is a common punishment despite the existence of more severe punishments (such as execution) for which inefficiency appears approximately equal to 1 . One explanation is that existing arrangements are suboptimal. Alternatively one can argue that imprisonment really has a lower inefficiency than it appears (because it lowers future enforcement costs, either by rehabilitation or by keeping criminals off the streets ${ }^{7}$ ) or that execution has a higher cost than it appears (presumably because it upsets third parties). If one accepts the first argument, it follows that inefficiency ( Z in my equations) never goes substantially above 1 . If one accepts the second, $Z$ is bounded by the inefficiency of imprisonment, save for levels of punishment requiring execution, for which it is larger. In 1976 state correctional expenditures per state prisoner were about $\$ 10,000$ a year ${ }^{8}$ which suggests $Z$ on the order of 2.
Total police, prosecution, and court expenditures in 1974 were roughly comparable to the sum of the value of property stolen from private individuals plus business losses from crime; in 1965, the cost of crimes against property was somewhat larger than the total expenditure for police, prosecution, and courts. Since the value of property stolen is surely a gross underestimate of the total cost of all crime (or, alternatively, the total cost of police, courts, and prosecutors is a gross overestimate of public expenditures on prosecuting crimes against property), this suggests that the value of $\mathrm{E} / \mathrm{QH}$ is on average substantially below 1. If marginal and average expenditures are roughly comparable, this suggests that $\mathrm{E}_{2}<\mathrm{H}$ on average.

Ehrlich's estimates of elasticity for various crimes ${ }^{9}$ run from .36 to 1.3 , with only one crime (robbery) having an elasticity greater than 1 . Hence my $\eta$, which is an inverse elasticity, should usually, at existing levels of punishment, averaged over rich and poor criminals, be between 1 and 3 ; for crimes against persons (elasticities between .72 and .99 ) it should be slightly above 1 .

Ehrlich also reports negative (demand) elasticities with wealth; states with higher median family income have higher levels of crime. While I could interpret this as evidence that higher income criminals have a higher demand function for crimes, it could just as easily mean that higher income victims are more tempting targets; this is supported by the fact that the effect is stronger for crimes against property than for crimes against persons.

There is one more feature of the real world of crime which is relevant to my analysis, although it does not embody itself in statistics or regressions. Crimes can be roughly divided into two categories. The first consists of those crimes which simply transfer property from one person to another without the owner's consent-theft, robbery, and the like. For
such crimes there is a strong presumption that the injury done to the victim is at least as great as the benefit to the criminal; the criminal could, after all, have simply bought the property from its owner instead of stealing it. There may be some cases-the starving hunter who breaks into a cabin in the woods-where a voluntary transaction is physically impossible, but they are surely rare. Hence, for such crimes any expected punishment higher than the damage will reduce the quantity of crime to near zero $(Q)(H) \cong 0)$.

A second category of crimes for which no such presumption exists includes speeding, gambling, prostitution, drug trafficking, and the like. While the first category contains crimes which could be converted into legal transactions by a simple bilateral exchange, ${ }^{10}$ so that there is a strong presumption for assuming the transaction would occur if there were net gains, the second contains crimes which could become legal acts only by changing the law, a much more complicated process and one for which there is, pace Posner and Thompson, no strong presumption of efficient outcomes. And even if the law is efficient, it may be unable to distinguish between efficient offenses (speeding by a skillful driver in a hurry) and inefficient ones, save by setting a price and letting potential offenders choose. Hence, for these crimes, an expected punishment equal to the harm done may result in a substantial crime rate.

We observe quite a lot of crime in both categories. It follows that for crimes in the first category the existing level of expected punishment is below the harm caused per crime. By Eq. (4) this implies that elasticity is less than 2, which is consistent with Ehrlich's results.

Applying the above observations to my formulae produces the following results for linear demand and cost functions with $Z(P, 0)=0$.

Assuming that $\mathrm{A}=\mathrm{H}$ for the first category of crimes, observed elasticities imply that $H / 2=A / 2>P>A / 4=H / 4$. If all criminals are identical, linear models give interior solutions if and only if $(1+2 \alpha A)^{2}$ $-12\left(\mathrm{E}_{2}+\mathrm{H}\right)>0$. If $\mathrm{H}=\mathrm{A}$, the opposite inequality (implying only corner solutions) corresponds to $3 \gamma-1+\sqrt{(3 \gamma-1)^{2}-1}>2 \alpha A>$ $3 \gamma-1-\sqrt{(3 \gamma-1)^{2}-1}$, where $\gamma \equiv\left(\mathrm{E}_{2}+\mathrm{H}\right) / \mathrm{H}$. If, as I argued above, $0<\mathrm{E}_{2}<\mathrm{H}$, then $1<\gamma<2$. The table shows the ranges of $\mathrm{Z}\left(\mathrm{P}^{*}\right)$ which imply corner solutions for several values of $\mathrm{P}^{*}$ and $\gamma$. If we assume that punishment inefficiency never exceeds 1 , we cannot expect interior solutions.

If there are two groups of criminals with linear demand curves related by Eq. (1'), or with identical linear demand curves and punishment inefficiencies related by Eq. (10), and if we still assume $Q(H)=0$, interior solutions become even more unlikely.

For most crimes in the first category, we should expect the demand functions of rich and poor to be similar, since they involve the transfer

Table 1. Values of $\mathrm{Z}\left(\mathrm{P}^{*}\right)$ for which Interior Solutions Do Not Exist

of a given (dollar) value; the only reason why they might be different is differential efficiency in stealing. The rich have a higher value for time but are also, presumably, more skillful; there is no obvious reason why the net effect should give one side or the other a (comparative) advantage for theft or robbery. There is therefore no particular reason to assume Eq. (1'). On the other hand, the argument about differential punishment costs still holds. Hence linear demand and inefficiency functions are unlikely to yield interior solutions, but if they do the optimal fines have $P_{1}>P_{2}$. In this case at least, the rich should pay higher fines.

There is less to be said about crimes in the second category. Where $\mathrm{A}>\mathrm{H}$ we may say that corners are, ceteris paribus, less likely than where $\mathrm{A} \leqslant \mathrm{H}$ (as in category 1 crimes). Where imprisonment is employed as a punishment, arguments similar to those applied to category 1 crimes imply again that if demand functions are the same and cost functions different, the rich should be punished more severely (assuming linear demand and cost functions). For offenses such as speeding which are usually punished by fines no such inference holds.

In summary, my results suggest that if demand and cost of punishment functions are approximately linear, offenses for which rich and poor have different demand functions [as per Eq. (1)] and the same cost function should have higher (dollar) punishments for the poor, and offenses for which they have the same demand functions but different punishment cost functions should have higher (dollar) punishments for the rich. Both conclusions hold only in the absence of corner solutions, which are likely to occur. If we drop linearity-an assumption justified more by mathematical convenience than inherent plausibility-the only general conclusion we can draw is that optimal fines for rich and poor must be different as long as either demand elasticities are different at all prices, or punishment inefficiencies are different at all (non-zero) prices. Which optimal fine is higher depends on details of demand and cost functions.

It is difficult to say how well these results on optimality correspond to the observed pattern of punishments. We may begin by noting that
most corner solutions may be ignored; where $P=0$ we do not call it a crime and where $Q=0$ we generally do not notice it (racing horses through Times Square). The exception is the case where $P$ or Q is zero for one group but not for another. It is frequently asserted that upperincome people can get away with things for which lower-income people are jailed (and occasionally the other way around). And it seems likely that there are some crimes (purse snatching, for example) which are rarely committed by the rich, and others (embezzlement) rarely committed by the poor.

The only formal pattern of discriminatory penalties in our legal system is that between fines (equal dollar punishments) and imprisonment (higher dollar punishments for those with high time value). But since fines are also a more efficient punishment which (assuming an efficient system) would be consistently favored where collectible, with imprisonment (and execution) reserved for defendants who cannot pay fines, I cannot reasonably interpret punishment as a device for punishing rich criminals more heavily (in dollar terms) although it does (for equal sentences) have that effect. Insofar as there exists systematic discrimination in punishment with income, it almost certainly takes the form of discretionary decisions by judges and prosecutors. I am not aware of any studies from which the pattern of such discrimination, if it exists, could be deduced."

## IV. CONCLUSION

It is easier to say what I have not done in this paper than what I have done. I have not determined whether the rich should pay higher or lower fines than the poor. I have not determined whether the actual system of punishments that exists corresponds to the optimal system of punishments suggested by my analysis.

I have, I hope, made somewhat clearer what the optimal level of punishments is and how it is related to our intuitions about "punishment equal to damage done" and "enough punishment to deter." In the process of doing so, I have constructed a formalism which incorporates attitudes toward risk into the set of cost and benefit functions associated with punishment, and so, I believe, corrected an incorrect treatment of that problem in Becker (1)-the paper on which my work in particular, and the theory of optimal punishment in general, is based. While my reformulation of Becker's analysis has not, in other regards, altered its conclusions, it has made the analysis more accessible to my intuition, and hopefully to that of my readers.

I have used the analysis to identify and analyze two categories of crime for which optimal punishments are different for rich and poor. I have then tried to examine the real world to see what can be learned
from it about the functions that go into the model. While the results are not very substantial, they do, I think, demonstrate how both statistical data and general information about the real world (such as the fact that stolen goods are worth more to the victim than to the thief) can be incorporated in economic models of crime prevention.

Despite its limitations, I hope this paper may help to convince legal scholars that economic analysis provides an interesting and potentially productive way of analyzing problems of crime prevention, and economists that crime prevention involves interesting and difficult economic problems.

## NOTES

1. For purposes of simplicity, I assume throughout this section that criminals are risk neutral, thus avoiding the distinction between the expected value of the cost of the punishment and the expected utility of the punishment. It is worth noting that if criminals are risk averse then fines, being uninsurable risks, no longer have a net cost (aside from administrative costs) of zero. In the analysis of Section II, risk aversion (and preference) is included among the factors affecting the efficiency of punishments.
2. Those readers who are unhappy with the idea of "efficient" crimes which ought not to be deterred may alter the arguments below by replacing the criminal who receives a high benefit from the crime with one who, being very skilled, has an atypically low probability of being caught-or thinks he does. The atypical criminal is necessary in the examples in order to eliminate the paradoxical (and unrealistic) result that a sufficiently high expected punishment deters everyone and is hence costless, since no crimes occur to be detected and no punishment need ever be imposed.
3. Indeed, if it does not the optimum is a corner solution-an infinite punishment imposed with infinitesimal probability. Enforcement costs are lower than for any other combination with the same expected value; all other costs are the same. Becker (1) in attempting to avoid the corner while assuming that punishment inefficiency is independent of level, concludes that criminals (in an equilibrium with optimal enforcement) must be risk preferrers. Since his inefficiency is calculated using the nominal value of the punishment (the number of dollars for a fine) rather than its certainty equivalent (allowing for risk preferences), his risk preference is (in my terminology) a way of making inefficiency vary with probability and level of punishment. As probability falls, the nominal punishment must rise more than proportionally in order to maintain a constant level of deterrence for a risk-preferring criminal. If punishment cost is a fixed proportion of nominal punishment, punishment cost per crime then rises. This provides a cost to balance the enforcement savings and so may prevent the corner solution.

Becker's definition of efficiency (his b) combined with his assumption that it is constant appears at first to be a natural way of describing a situation where increasing the level of punishment means a larger amount of the same punishment-a bigger fine or more years in prison. It is not. Social welfare calculations must include preferences with regard to risk; if for every dollar of fine imposed the state collects fifty cents, the efficiency in Becker's sense of imposing a hundred-dollar fine with probability one-tenth on a riskpreferring criminal is not the same as the efficiency of imposing a thousand-dollar fine on him with probability one-hundredth; the criminal prefers the latter and his benefit from receiving an attractive lottery along with his punishment must be included in calculations of total social cost. Becker ignores this because he has defined his social loss function in
terms of real income, and thus assumed away the set of nonpecuniary costs which he assumed into existence in not making his criminals neutral toward risk. If one includes the costs or benefits of risk in the social loss function, it then follows that for Becker's b to remain constant, technical efficiency (dollars collected by the state per dollar paid by the criminal) must increase with increasing level of punishment for risk-averse criminals and decrease with increasing level of punishment for risk-preferring criminals-in order to cancel the costs or benefits of the lotteries implicit in various combinations of probability and punishment. This seems a very unnatural assumption. If one assumes that technical efficiency is independent of probability and level it is possible to avoid the corner either by making criminals risk averse (if technical efficiency is positive) or by making them risk preferrers (if it is negative).

In the final paragraph of Section III of his article, Becker appears to recognize the problem, at least to the extent of pointing out that his loss function only involves real income, and that a loss function which increased as probabilities fell and punishments rose would be consistent with risk-avoiding criminals. He fails to note that the fact of nonneutral attitudes toward risk is itself a reason why a correct loss function would involve more than real income, and also why such a loss function would be affected by "compensating" changes in probability and punishment. While his results are correct in the sense of following from his assumptions, they are not (in this regard) correct in the sense of correctly describing what, on reasonable simplifying assumptions, an optimal system of punishment would look like.
4. Strictly speaking, the analysis should also include the cost to criminals of avoiding conviction, which should be an increasing function of severity of punishment. Defensive expenditures by criminals, defensive expenditures by victims, and varying probabilities of apprehension and conviction (depending on the skill of the criminal) are three important elements of a complete analysis which are omitted in most of the literature on economics of crime. This paper is no exception.
5. $\mathrm{Q}(\mathrm{P})$ should be an income-compensated demand curve; I shall neglect the difference between the income-compensated demand curve and the ordinary demand curve whose elasticity is actually measured. Considering the imprecision of both the measurements and the predictions they test, that seems a harmless simplification.
6. Marshall (4), III, p. iv, Sec. 8. The assumed relation between utility functions is not given explicitly, but is an essential part of the argument.
7. Of course, executing criminals also keeps them off the street, but since the number executed would be less than the number imprisoned (for equivalent levels of punishment) it is at least possible for imprisonment to be a superior way of doing so.
8. These numbers and similar statistics following are from the U.S. Department of Justice (6).
9. The figures are from Ehrlich (3), Tables 4 and 5 . I take his $b_{1}$. which measures elasticity with probability of conviction, length of sentence held constant, as a measure of elasticity with level of punishment. His $b_{2}$, which measures elasticity with length of sentence, probability of conviction held constant, is presumably an underestimate of elasticity with amount of punishment; doubling the length of sentence corresponds to a less than doubling of the amount of punishment, both because of discounting and because of threshold effects such as the effect of conviction on future employment opportunities. If criminals are not risk neutral, his elasticity with probability does not precisely correspond to my elasticity with amount of punishment, but I use it as the best approximation available.
10. One can imagine exceptions; an employee stealing paper clips from a large corporation or covertly using company equipment for private purposes might be an efficient transaction which could not be arranged voluntarily because of transaction costs.
11. I would welcome correspondence from anyone knowing of such studies.

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[^0]:    Research in Law and Economics, volume 3, pages 185-205.
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