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The case of the Case of Benny: Elucidating the influence of a landmark study in mathematics education

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ABSTRACT

Stanley Erlwanger's Case of Benny is seen by many as particularly influential in the mathematics education research community. This paper reports the results of a study designed to describe the nature of that influence. Through an analysis of academic references to the Case of Benny from the past 40 years, five primary purposes for citing the case were identified. These purposes revolve around the themes of student mathematical conceptions, the relationship between correct answers and understanding, the value of qualitative research, the impact of a behaviorist-based curriculum, and students as sense makers. The paper concludes by using these themes to reflect on the past 40 years and to look ahead to the future of research in mathematics education.

1. Introduction

It has been 40 years since the original publication of Erlwanger's (1973) "Case of Benny." In the ensuing years the work has been referred to as "classic" (Shulman, 1985, p. 442), "seminal"(Ernest, 1996, p. 805), and as "one of the most influential and important research studies in mathematics education" (Boaler, 2008, p. 592).[1] In addition, Erlwanger's Case of Benny was chosen as one of 17 articles to be included in *Classics in Mathematics Education Research* (Carpenter, Dossey, & Koehler, 2004a), a collection of articles judged to have "influenced the direction of mathematics education today" (Carpenter, Dossey, & Koehler, 2004, p. vii). This paper reports the results of a study designed to elucidate the nature of the article's influence by closely examining the ways the Case of Benny is referenced in research literature. Furthermore, the story of the influence of the Case of Benny is an interesting "case" in its own right—a compelling story that illuminates the history of research in mathematics education and the fundamental issues that have become its heart and soul.

2. Background

Stanley Erlwanger pursued his doctoral degree at the University of Illinois at Urbana, working with pioneers the likes of Robert Davis and Jack Easley. During this time he began working with Davis on his long-running NSF-funded Madison Project, the expressed purpose of which was to use "interview procedures to compare and contrast 'the mathematics in children's heads' with 'the mathematics in the school curriculum'" (Davis & Ginsburg, 1975, p. 5). In the fall of 1972 Erlwanger began

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[1] The Case of Benny is nowhere near the most-cited article in mathematics education literature, but it just may be one of the most read. In the author's experience, the article is routinely read by everyone who achieves an advanced degree in mathematics education.

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data collection for his dissertation, spending considerable time studying the mathematical experiences and conceptions of a 6th grade student he pseudonamed Benny. During the spring of 1973 Erlwanger conducted five additional case studies. The compilation of all six case studies constituted Erlwanger's dissertation.

Meanwhile (also in 1972), Erlwanger's dissertation advisor and mentor Bob Davis (along with Herbert Ginsburg, then at Cornell University) had begun a new journal—*The Journal of Children's Mathematical Behavior (JCMB).* In that first issue Davis and Ginsburg explained that the purpose in creating this new journal was to provide a space where open dialogue could take place around the issues of "what 'mathematical thought' means with children, how it develops, and how one might attempt to study it" (Davis & Ginsburg, 1972, p. 5). Erlwanger wrote up his original case—*Benny's Conception of Rules and Answers in IPI Mathematics*—and it was published in the second issue of *JCMB*, which appeared in the autumn of 1973 and was guest-edited by Jack Easley. This 1973 article is often referred to as the *Case of Benny*.

Erlwanger's dissertation (1974) presented the six case studies in comparison pairs. The write-up of the Case of Benny in the dissertation (paired with the Case of Mat) followed the same basic structure as the 1973 JCMB article, but was expanded by way of including more transcript excerpts and some additional analysis. In 1975 *JCMB* published the first two of those six cases (Benny and Mat) much as they appeared in the dissertation (Erlwanger, 1975), with the stated intention to subsequently publish the other four cases "because of the great interest in Erlwanger's results" (Davis & Ginsburg, 1975, p. 5). The journal never did publish these other cases, possibly because the journal itself did not publish another issue until 1977.

The Case of Benny (Erlwanger, 1973) described in great detail the mathematics-related conceptions of Benny, a 6th-grade student who "was making much better than average progress" (p. 7) in a behaviorist-based mathematics curriculum (Individually Prescribed Instruction or IPI). The IPI curriculum was based on a hierarchical sequencing of behavioral objectives. Students worked individually on exercises related to a set of objectives, asking for help when desired, then took tests to measure their competency. When students scored above 80–85% (depending on the test), they moved on to the next set of objectives; when they scored less than the threshold, they were assigned remedial exercises related to the specific items they had missed then given the opportunity to retest. As Erlwanger noted, "IPI mathematics emphasizes continuous diagnosis and assessment through pre-tests, curriculum-embedded-tests and post-tests" (p. 12).

Despite Benny's ability to attain a sufficient number of correct answers on exercises and tests related to fraction addition and decimal multiplication, Erlwanger (1973) uncovered and detailed numerous rules Benny had developed for operating on decimals and fractions that did not yield the correct answer. For example, "Benny converted fractions into decimals by finding the sum of the numerator and denominator of the fraction and then deciding on the position of the decimal point from the number obtained" (p. 8). Using this rule Benny concluded that 2/10 converts to 1.2 and that 5/10, 4/11, and 11/4 all convert to 1.5.

Beyond establishing *that* Benny was relatively successful despite his construction of erroneous rules such as this one, Erlwanger (1973) examined how such a phenomenon could exist: "How is it that Benny, with this kind of understanding of decimals and fractions, had made so much progress in IPI mathematics?" (p. 11). To answer this question he examined Benny's conception of the nature of mathematics, including his views on the nature of learning and teaching mathematics. He further explored the IPI curriculum and how it played out in Benny's classroom to illustrate how the nature of his learning environment contributed to the development of Benny's unfortunate (but seemingly sufficient for classroom success) conceptions of mathematics.

Benny's classroom success was made possible through a fascinating confluence of conceptions of mathematics and curricular design. Benny knew there were multiple equivalent representations for the fractions he was working with (he used the example of the equivalence of 1/2 and 2/4). He also knew that the answer key for his tests had a single correct answer for each problem. What Erlwanger (1973) uncovered was that Benny had combined these conceptions into "an incorrect generalization about answers" (p. 15), one that allowed him "to believe that all his answers are correct 'no matter what the key says"' (p. 15). Thus, rather than interpreting his wrong answers as wrong, he interpreted them as correct but in the wrong form. He then played a game, a "wild goose chase," (p. 16) of looking for patterns in the correct answers and "rules" that would allow him to get those answers frequently enough to get at least 80% on his mastery tests. He thus maintained numerous rules for working "different" kinds of problems, even though frequently these rules contradicted each other and resulted in numbers that actually were not equivalent. This game he played led "him to believe that the answers work like 'magic, because really they're just different answers *which we think they're different*, but really they're the same"' (p. 18).[2]

3. Methods

We began our search for references to the Case of Benny by using the Google Scholar citation reports for the original publication of the case (Erlwanger, 1973) as well as for later publications that also contain the entire case (Erlwanger, 1973/2004, 1974, 1975). We located each of these 290 publications and verified whether they truly cited Benny. Only 26 of the publications did not contain legitimate citations. We further excluded non-English-language publications as well as documents without some level of peer-review (e.g., unpublished manuscripts, class syllabi, webpages). Having relied solely on Google Scholar to create this initial collection of 221 publications, we searched for phrases like "Erlwanger Benny" and "Benny's

[2] Although these paragraphs presented a brief summary of the Case of Benny, if the reader has not already done so we recommend they read Erlwanger (1973) in order to make the current article more meaningful.

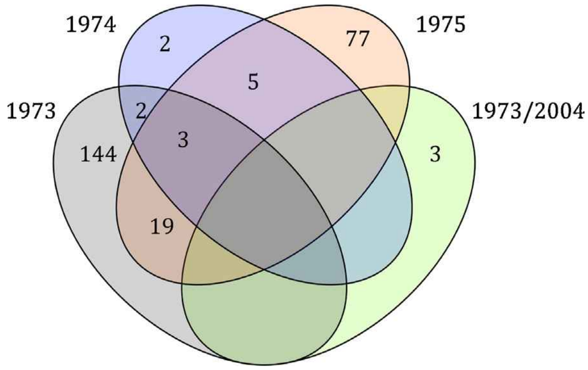

Fig. 1. Publication references to the various versions of the Case of Benny.

conception" in ERIC and at several mathematics education journal websites in order to determine how comprehensive our list was. In addition, periodically our review of the identified publications led us to previously unidentified possibilities. These searches resulted in 32 additional publications. The 253 resulting publications and their associated references to the Case of Benny make up the data corpus for the study.

We conducted a preliminary analysis that regarded the publication as the unit of analysis. We investigated the nature of these 253 publications, recording and analyzing the year, type and focus of the publications. We then searched within each publication to identify any reference to the Case of Benny. The authors' text related to the Case of Benny was collected. Thus the primary unit of analysis for this portion of the study was a direct reference to the Case of Benny, with each distinct reference (some publications contained multiple distinct references) being identified as a separate citation instance. We identified 311 citation instances across the 253 publications. Each citation instance was then analyzed according to its nature and purpose. In doing so we sought to capture how the citation instances "used" their reference to the Case of Benny—how the citation supported the argument at hand. We developed our coding scheme directly from the data, with no predetermined categories, and refined the codes through a constant comparative method. Once we had identified the primary categories we analyzed each individual category to ensure that all citation instances "belonged together" and to determine whether there were significant sub-themes within categories.

4. Results

4.1. Nature of the 253 publications

The Case of Benny is cited in a wide variety of peer-reviewed publications primarily within but also outside of the field of mathematics education. Of the 253 publications, 121 (48%) were journal articles, 89 (35%) were books or chapters in edited books (including conference proceedings), 27 (11%) were theses or dissertations, and 16 (6%) were conference papers. It is also interesting to inspect when these reports were published—21 (8%) in the 1970s, 81 (32%) in the 1980s, 69 (27%) in the 1990s, 63 (25%) in the 2000s, and 19 (8%) thus far in the 2010s.

Although the Case of Benny is about a 6th grader and his conceptions of elementary mathematics, only 54 (21%) of the publications focus specifically on the elementary level. Over half (137 or 54%) are written for a general education audience, 40 (16%) focus on the secondary level, and 14 (6%) focus on the post-secondary level. The mathematical focus of these articles ranges from arithmetic and fractions to algebra, geometry, statistics and calculus. It is also interesting to note that, although certainly primarily influential within the field of mathematics education (215 or 85%), the Case of Benny has also been influential in computer science (23 or 9%) and science (14 or 6%) (with one publication in the area of geography).

Finally, given the multiple versions of the Case of Benny—1973, 1974, 1975 and 1973/2004—we note to which version(s) each of the 253 publications refer (see Fig. 1). The 1973 version accounts for a majority of the citations; the 1975 version accounts for most of the rest. Most publications cite a single version. As would be expected, few cite the dissertation itself. Interestingly, the 1975 version adds a substantial amount of detail to the 1973 version while removing very little. Of all of Erlwanger's cases, Benny was the most compelling and, as such, the version devoted only to his case is the most valued.

4.2. Nature of the 311 citation instances

Each citation instance was analyzed according to the primary purpose or purposes for citing the Case of Benny. Taken together, these purposes give a sense for just what the Case of Benny is a "case of" to the academic community. Over 90% (all but 29) of the citation instances received at least one of the five most common purposes for citing Benny (described below). Thus, these five purposes seem to capture what the Case of Benny means to the academic community. As each purpose is discussed, evidence from Erlwanger's writing is presented to support such a purpose and examples from the publications are used to illustrate both typicality and variation.

4.2.1. Students' conceptions of mathematics

The most common purpose for citing the Case of Benny (112 or 36%) is to support the claim that students have idiosyncratic conceptions of mathematics that influence and are influenced by their experiences with mathematics. As Erlwanger stated,

> As children learn they develop their own conceptions of mathematics that influence their mathematical behavior and subsequent learning. The nature of these conceptions depends upon the learning environment, and may be quite different from the adult view of mathematics (Erlwanger, 1975, p. 158).

Erlwanger based this generalization on the compelling Case of Benny, who, having worked with IPI for 5 years, had "developed learning habits and views about mathematics" that were likely to "impede his progress in the future" (p. 25).

That researchers would use the Case of Benny to support claims related to students' conception of mathematics is certainly not surprising given the title of the 1973 paper (and of the dissertation as well). Erlwanger used *conception*, however, to encapsulate a broad range of constructs:

> In the course of learning mathematics a child develops his own ideas, views and beliefs about mathematics which can be represented as his conception of mathematics. This conception of mathematics may be regarded as a developing conceptual system of interrelated ideas, beliefs, emotions, and views concerning mathematics and learning mathematics that directs and controls his mathematical behavior, how he learns and what he understands (Erlwanger, 1975, pp. 166–167).

This broad definition of *conception of mathematics* has allowed the research community to draw on the Case of Benny to support claims related to a number of subsets of Erlwanger's conception of conception. Thus, on the one hand, we see arguments that "studies such as that of Erlwanger (1975) drew attention to the significance of a students' belief system regarding mathematics and mathematical behavior" (Clarke, Breed, & Fraser, 2004, p. 8) and, on the other hand, claims that "particular aspects of children's mathematical understandings have been investigated quite extensively" (Macmillan, 1995, p. 111).

Furthermore, in the Case of Benny, Erlwanger demonstrated the value in viewing as conceptions such constructs as misconceptions and errors. Thus researchers turn to the Case of Benny as an example of the utility of research on students' errors and misconceptions (e.g., Shulman, 1987; Stewart, 2005), illustrating that "pupils often have major misconceptions about fundamental aspects of mathematics" (Brousseau, Davis, & Werner, 1986, p. 208). It should be noted, however, that Erlwanger refined the way he viewed constructs such as "misconceptions" from the 1973 version of the Case of Benny to the 1974/75 versions. In the 1973 Case of Benny Erlwanger referred to "Benny's misconceptions." He abandoned such language in the later incarnations of the case, explaining that terms "such as 'the child's error', 'misconception', and 'lack of understanding' or 'lack of comprehension' are not used because they reflect an adult's point of view about the child and his work," (1975, p. 192) whereas the focus of Erlwanger's work was "the individual child's cognitive structure, and that is whatever it is" (p. 192).

Three of Benny's beliefs about mathematics seem to have resonated particularly with the research community. The belief to which citation instances most commonly referred is that children "see mathematics as a mass of rules from which they make random selections in an effort to achieve the required answer" (Neyland, 1995, p. 142), so finding answers in mathematics is "like a wild goose chase" (Erlwanger, 1973, p. 16). Secondly, researchers claim that other students, like Benny, "see few connections between school mathematics and reality" (Becker & Selter, 1996, p. 512). These first two beliefs are often paired together by researchers in order to claim that many students see mathematics "as a meaningless set of rules and procedures that do not relate to the everyday world" (Tytler, Osborne, Williams, Tytler, & Clark, 2008, p. 35). Finally, researchers refer to Benny to support the claim that "the need for a unique solution is not particularly important to young students" (Linchevski & Livneh, 1999, pp. 180–181).

4.2.2. Correct answers do not imply understanding

The next most common purpose for citing the Case of Benny (79 or 25%) was to draw on Erlwanger's (1973) claim that "Benny's case indicates that a 'mastery of content and skill' does not imply understanding" (p.12). There are two kindred aspects of this claim that researchers seem to value. The first is closely related to the environment in which Benny was learning, where his understanding was assessed solely by whether an acceptable percentage of his answers matched the answer key. The Case of Benny is compelling because it richly described a student with many mathematical conceptions that belied the mathematical intentions of the teacher and of the curriculum, who nevertheless was "making much better than average progress" in the IPI curriculum, and was viewed by his teacher "as one of her best pupils in mathematics" (Erlwanger, 1973, p. 7). Thus researchers claim that "Erlwanger (1973) has shown that getting an answer correct on a test does not mean that the child knows what he is doing" (Brown, Campione, Reeve, Ferrara, & Palincsar, 1991, p. 160).

This first aspect is thus that correct answers to procedural questions do not imply correct understanding of the procedures themselves. But beyond an understanding of how to carry out a particular procedure is an understanding of the mathematics that procedure embodies. Researchers have thus gravitated toward the findings of the Case of Benny because they, along with Erlwanger (1973), believe that "mathematics should be a subject in which rules are generalizations derived from mathematical concepts and principles" (p. 22). The Case of Benny presents a student for whom this is not at all what rules were. Hence the numerous citation instances that used the Case of Benny to support the claim that focusing on the mastery of procedures does not necessarily lead to an understanding of the mathematics that those procedures embody. Students can "arrive at correct answers without actually practicing the skill or concept that the assignment presumably reinforces" (Anderson, Brubaker, Alleman-Brooks, & Duffy, 1985, p. 124) and thus those correct answers "[do] not necessarily imply mathematical proficiency" (Stylianides & Stylianides, 2007, p. 105).

4.2.3. The limitations of a behaviorist-based curriculum

A fair number of articles (40 or 13%) used the Case of Benny to support the claim that there are serious limitations to behaviorist-based learning systems. The Case of Benny was set in a classroom (and school) that used IPI mathematics as its curriculum, and sixth grader Benny had "been using IPI mathematics since second grade" (Erlwanger, 1973, p. 25). IPI mathematics is "based on a carefully sequenced and detailed listing of behaviorally stated instructional objectives" (Erlwanger, 1974, p. 336). "Individualization in the program is achieved through placement tests" (p. 337) and "each pupil is guided by written prescriptions prepared to meet his needs and his interests" (p. 339). Erlwanger concluded that the "inherent weaknesses in the IPI mathematics program" stemmed "from its behaviorist approach to mathematics, its mode of instruction, and its concept of individualization" (Erlwanger, 1973, p. 25).

Authors cited the Case of Benny to support critiques of IPI or IPI-like approaches to curriculum and instruction, typically focusing on one of the three weaknesses identified by Erlwanger. It was most common for authors to focus on the weaknesses of individualization, citing Erlwanger to support the dangers of students being "left on their own too long" (Brophy, 1982, p. 529) or "learning in isolation from interaction with others" (Lappan & Ferrini-Mundy, 1993, p. 630). Authors also appealed to the Case of Benny to provide a "critical view" of behaviorism and its influence in decomposing "school curricular objectives" into "discrete, testable behaviors" (Goldin, 2003, p. 192). Finally, authors use the Case of Benny to support critiques of particular "modes of instruction," such as approaches where teachers are merely "teaching rules and conventions" (Mueller, Yankelewitz, & Maher, 2010, p. 309) or where "students are drilled frequently and then tested on their level of retention and skill mastery as demonstrated by their performance on exams" (Blumenfeld, 2006, p. 3). Instruction characterized by these approaches is seen as inherently weak because it lacks "appropriate experiences" and tends to overlook "work with children on the difficult task of interpreting these experiences" (Davis, 1986, p. 274).

4.2.4. The value of qualitative research

For some (38or 12%) the Case of Benny is important not just because of *what* its results are but also because of *how* they were achieved. Erlwanger used "the observation-interview method" (1975, p. 157) to construct the Case of Benny at a time when qualitative research methods were rarely used in mathematics education research. Quantitative approaches involving collecting and comparing scores on achievement tests were the norm. In part as rationale for his qualitative methods, Erlwanger (1975) outlined several problems with the typical quantitative research methods of the time:

> The achievement test results have described children's proficiency in mathematics, but they have not explained how and why the children got their answers; the error studies have identified typical errors, but they have not explained the causes of these errors or what the children's ideas were about their work; and the interview studies have described the patterns of thinking and the strategies children use in computation, but they have not explained why the children used particular procedures (p. 164).

Thus, quantitative methods simply are not designed to answer many important questions related to students' conceptions, and thus far even qualitative methods had not been used to their full potential to answer such questions. In the early 1970s we knew far more about what mathematics students do than we did about how and why they do it—the latter questions being exactly what qualitative research is designed to answer.

The Case of Benny is viewed by many as a good example of qualitative research in general, of case study methodology in particular. Thus the Case of Benny is used as an "illuminating example" of "nonquantitative research . . . in which the observer is not attempting to test any particular preconceptions (theoretical or personal). Rather, like an anthropologist, the researcher is attempting to observe and report what actually occurs" (Yager, 1978, p. 106). It is also seen as a good example of case study methodology, one in which Erlwanger successfully faced the common case study challenge of "convincing his readers that a close look at Benny can contribute to a much larger set of theoretical and practical issues" (Ball, 2000, p. 374).

Beyond, or perhaps because of the fact that the Case of Benny is viewed as a high-quality example of qualitative research, researchers cite the Case of Benny in order to make a case for the value of qualitative research in general and, again, case studies in particular. Stories like the Case of Benny "can provide compelling evidence about the educational process" (Cooney, 1999, p. 1); and "the use of a single case study to highlight students' understanding of mathematics programs has made significant contributions in the field of mathematics education" (Strickland, 2011, p. 90). Indeed Erlwanger's work is seen as representative of a "respectability gained by ethnographic research" (Clements, 1993, p. 24).

4.2.5. Students as sense-makers in learning mathematics

Finally, a number of the instances of citation (30 or 10%) used the Case of Benny to support the claim that students are always trying to make sense of their environment, necessarily developing conceptions (more or less "mathematical" depending on that environment) as they try to make sense of their experiences. Erlwanger (1973) explained that Benny

Table 1
Summary of the primary reasons for citing the Case of Benny.

Reason	Percent (n)
Students' conceptions of mathematics	36% (n = 112)
Correct answers do not imply understanding	25% (n = 79)
The limitations of a behaviorist-based curriculum	13% (n = 40)
The value of qualitative research	12% (n = 38)
Students as sense-makers in learning mathematics	10% (n = 30)

had "developed consistent methods for different operations which he can explain and justify to his own satisfaction" (p. 12). Here researchers appeal to the Case of Benny to support particular views of student learning. Although this theme is related to the first theme discussed—that students have idiosyncratic conceptions that influence and are influenced by their experiences—it differs in that the focus here is not on the conceptions themselves, but on the learning process itself and students' "intention to make sense" (Wheatley, 1992, p. 533). We thus see Benny characterized as having "arrived at his conception of mathematics by reflecting on, and attempting to make sense of, his past experiences of doing mathematics" (Cobb & Steffe, 1983, p. 84). All students' conceptions (including Benny's) can be seen as "quite rational and consistent abstractions from their past experiences" (Hoyles & Noss, 1987, p. 590).

5. Discussion

The Case of Benny is actually seen as a case of many things. It is a case of one 6th grade student who had conceptions of mathematics that differed significantly from those intended by his curriculum and his teacher. He developed these conceptions as he quite naturally (and enthusiastically) tried to make sense of the mathematical activities presented to him. Because the curriculum, its instructional model and its assessment, privileged procedural answers over relational understanding, Benny is a compelling case of a student who was excelling in his class while languishing in his mathematical understanding. The Case of Benny thus gets at the heart of research in mathematics education—what students know, how they come to know it, and how curriculum and instruction can attend to what students know and facilitate their further learning. The Case of Benny speaks powerfully to this entire enterprise. It conveys the important message that learning and teaching mathematics is a complicated and complex endeavor. At the same time, qualitative research such as that exemplified in the Case of Benny has played an ever increasing and ever more important role in mathematics education research.

Table 1 summarizes the five primary reasons for citing Benny. One might wonder to what extent the prevalence of these themes has changed over the past 40 years. These five reasons had all been used by 1976, within the first 8 citation instances, and they continued to be used consistently throughout the ensuing decades. Although the rate of use varied from year to year, the trend is constant. That is, the line of best fit is basically a horizontal line through the average number of citations per year. And most recently, all five reasons were used between 2011 and 2012. Thus we see the consistent and enduring nature of these themes.

Four of the five reasons summarized in Table 1 are points that Erlwanger himself explicitly made. That the Case of Benny is seen as an exemplar of qualitative research is somewhat different in that it is not a result of the study but an assessment by the research community of the value of the means for attaining those results—results that came about because of the use of qualitative case study methodology. Certainly Erlwanger thought that his methods were extremely useful—he advocated them and provided a clear rationale for how they had the potential to overcome the limitations of the primarily quantitative methods of the day. But it is actually the uptake by the research community over the ensuing 40 years that lends credence to his claim. The research community came to value qualitative research because of research like the Case of Benny, and the Case of Benny is held up as a prime example because it was early, timely, and compelling.

An interesting parallel exists between (a) the qualitative methods Erlwanger employed in order to counter the limitations of quantitative methods in studying student mathematical behavior; and (b) Benny's lack of conceptual understanding that demonstrated the limitations of behaviorist models of assessing learning and understanding. One fascinating and compelling aspect of the Case of Benny is the way it simultaneously critiques the limitations of behaviorist theories of learning and positivistic theories of research. Neither the prevailing research paradigms of the time nor the individually prescribed curriculum of Benny's classroom accounted for and valued Benny's ways of making sense of the world around him. It took qualitative methods that valued such individual meanings and that capitalized on (rather than tried to minimize) the researcher as instrument in order to illustrate the needs for a teacher to do the same thing. As Erlwanger (1975) stated,

> If in the course of learning mathematics a child gradually develops his own ideas, views and beliefs—his conception of mathematics and learning mathematics—then this conception has to be understood first before his external mathematical behavior can be interpreted and explained (p. 277).

6. Conclusion

This paper has described the primary reasons *why* the field of mathematics education values the Case of Benny. But this overall argument would be incomplete if we did not consider the extent to which this value is appropriately placed. In other words, *should* the mathematic education research community value the Case of Benny for the reasons it does? Does the case deserve to have had so much influence? As we have already argued, four of the five primary purposes for citing the Case of Benny are indeed related to claims made in that case. Thus, with respect to those purposes, the question of whether the mathematics education research community *should* be significantly influenced by the Case of Benny is in part a question about the quality of the research study itself, which, it turns out, is directly related to the remaining primary purpose for citing the case. If the Case of Benny is indeed exemplary research, and if the findings of that research relate to important themes in mathematics education, then one could argue the influence of the case would be legitimate. Therefore, we briefly consider whether the Case of Benny truly is an exemplary case study. Schoenfeld (2007) argued for evaluating the quality of empirical research with respect to trustworthiness, generality, and importance. We consider each of these dimensions in turn with respect to the Case of Benny, using (as was done throughout this paper) the citation instances to support the arguments.

One can measure the trustworthiness of the Case of Benny by assessing "the degree of believability of the claims made" (Schoenfeld, 2007, p. 93). That the mathematics education research community has continued to appeal to those claims, in no small part because of the richly descriptive, "in-depth empirical" (Niss, 2004, p. 54) nature of the evidence presented, speaks to the believability of the claims. Interestingly, while the case itself exemplifies the power of qualitative research, the most cited 1973 version does not take the same "form" one would see today when reporting such research. The 1973 version begins with a three-paragraph introduction that provides a bit of background and rationale for the study. It then launches directly into the case itself. There is no connection to the literature and there is no methods section. (These elements are dealt with in significant ways, however, in the 1974 dissertation.) Nevertheless, the case provides "the depth of observation and analysis that enables readers to understand a connection or phenomenon clearly" (Boaler, 2008, p. 592) and thus has descriptive power.

With respect to generality, Ball (2000) argued that the Case of Benny, although focused on a single student, is not a "an isolated 'nonrepresentative case,'" but rather "about problems of learning and assessment. . .. The claims are only in part about Benny" (p. 375). Furthermore, according to Boaler (2008),

> the degree of generalizability rests not only with the number of cases consulted or the randomization of subjects, but with the power of the observation and analysis produced within a study. Erlwanger's analysis was powerful, and his findings have contributed to an improved understanding in our field (p. 592).

For these reasons the Case of Benny provides useful and "warranted" (Schoenfeld, 2007) generality.

In many ways the importance of the Case of Benny is what this paper is all about, as we have documented the numerous ways mathematics education researchers have found the Case of Benny to be important to their work. The introduction for this paper cited claims that the Case of Benny is "classic" (Shulman, 1985, p. 442) and "seminal" (Ernest, 1996, p. 805). In fact, in conjunction with using the Case of Benny to support their claims, numerous articles (75 or 30% of the articles from this study) lend further credence to this support by claiming that the Case of Benny holds a special place in the mathematics education literature—that it is important. The Case of Benny has been referred to as brilliant, celebrated, classic, crucial, both famous and infamous, important, influential, landmark, notable, notorious, pioneering, prototypical, seminal, striking and well-known. Schoenfeld (2007) claimed that importance is a value judgment, and it is clear from the literature that the field of mathematics education values the Case of Benny as "one of the most influential and important research studies in mathematics education" (Boaler, 2008, p. 592).

There is thus substantial evidence that the Case of Benny measures up well with respect to the quality dimensions of trustworthiness, generality and importance, and is indeed exemplary research. Beyond the issue of whether the Case of Benny is exemplary, however, the question of whether the mathematics education research community *should* be significantly influenced by the Case of Benny is also in part a question about the quality of the ways in which the case is cited. That is, if the Case of Benny is being used to support claims that it does not actually support, then the legitimacy of the influence could also be questioned. The primary purposes reported in this paper align with claims that Erlwanger did indeed make, but this paper does not report the extent to which citation instances "accurately" referenced those claims. Elsewhere (Leatham, 2014) we reported on an analysis of this accuracy and documented that a number of citation instances do actually misrepresent the Case of Benny through the reporting of incorrect details and the misrepresentation or overgeneralization of Erlwanger's claims. In instances such as these, Erwlanger's "influence" is not completely warranted because, in essence, the Case of Benny does not provide the claimed support. Despite the various misrepresentations of the Case of Benny, however, we argue that the Case of Benny does indeed warrant its substantial influence. It explored questions that were and continue to be central to the field of mathematics education, and subsequent research has built on Erlwanger's initial answers to those questions as well as provided substantial additional evidence to support those inferences. The Case of Benny is a compelling illustration of how a case of one can indeed make a difference.

This review of the influence of the Case of Benny provides insights into the history of the mathematics education research community over the past 40 years. It highlights the centrality of students' mathematics when studying both learning and teaching mathematics and it exemplifies the power in qualitative research methods to advance such research. That said, this review of the influence of this case also illuminates challenges still facing that community. Although the evidence that correct answers to procedural questions do not imply understanding has supported focus within the mathematics education community on attending to, valuing and building on student mathematical thinking, high-stakes and procedure-driven testing dominate the ways that the performance of mathematics students (and their teachers) is assessed (see Au, 2011). Much has been learned about students' conceptions of mathematics, particularly with respect to mathematical understanding, but this knowledge still has minimal impact on students' day-to-day mathematical experiences (Hiebert, 2013). This lack of impact exists, in part because, although reform-oriented curricula have been developed, revised and proven through ongoing research (e.g., Senk & Thompson, 2003), such curricula are in the minority and behaviorist-based curricula and teaching approaches continue to govern public and political discourse (Stigler & Hiebert, 2009). Although the mathematics education research community embraces qualitative research methodologies and has developed a convincing associated knowledge base, that community is more convinced than practicing teachers and far more convinced than policy makers (Boaler, 2008). The recognition of students as sense-makers is encouraging, providing hope in the abilities of all students to develop powerful and productive mathematics, yet discouraging as so few students find themselves in environments worth making mathematical sense of (cf. Speiser & Walter, 2004).

Over the next 40 years of mathematics education research, the themes encompassing the primary purposes for citing the Case of Benny are likely to continue to drive the questions that are asked and the means through which they are answered. Hopefully this contemplation on the influence of Erlwanger's (1973) study will further lead to ways to transform the answers to these questions into changes in educational policies, improvements in teaching practices and, ultimately, students' conceptions of mathematics that are both robust and empowering. We owe it to Benny.

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