A person deciding on a career, a wife, or a place to live bases his choice on two factors: (1) How much do I like each of the available alternatives? and (2) What are the chances for a successful outcome of each alternative? These two factors comprise the *utility* of each outcome for the person making the choice. This notion of utility is fundamental to most current theories of decision behavior. According to the expected utility hypothesis, if we could know the utility function of a person, we could predict his choice from among any set of actions or objects. But the utility function of a given subject is almost impossible to measure directly. To circumvent this difficulty, stochastic models of choice behavior have been formulated which do not predict the subject's choices but make statements about the probabilities that the subject will choose a given action. This paper reports an experiment to measure utility and to test one stochastic model of choice behavior.

MEASURING UTILITY BY A SINGLE-RESPONSE SEQUENTIAL METHOD

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THE purpose of this paper is to describe a sequential experiment that provides, at each stage in the sequence, an estimate of the utility to the subject of some amount of a commodity (e.g., money), and to present a few experimental results obtained with the method. The procedure is based upon the following well-known "expected utility hypothesis." For each person there exist numerical *constants*, called *utilities*, associated with the various possible outcomes of his actions, given the external events not under his control. If, for a given subject, we could know the values of these constants and the ("personal") probabilities he assigns to the various external events we could, according to this model, predict his choice from among any available set of actions. He will choose an action with the highest expected utility; i.e., with the highest average of utilities of outcomes, weighted by the probabilities he assigns to the corresponding events. He will be indifferent between any two actions with equal expected utilities. Note that (by the nature of weighted averages) the comparison between expected utilities does not depend on which two particular outcomes are regarded as having zero-utility and unit-utility.

Other models of choice behavior, called *stochastic models*, do not predict the actual choices of a subject from each given set of available actions but rather they make statements about the *probabilities* that the scientist might assign to the various actions being chosen by the subject. It is assumed that these probabilities do not change during the time period under consideration, thus precluding learning or any systematic change of behavior. Relations between these probabilities of choice and the expected utilities described above are postulated.

One such postulate (associated with the name of Fechner) specifies that, for a given subject, action A has a larger expected utility than action B if and only if, when forced to choose between A and B, the probability that he chooses A is larger than the probability that he chooses B. It follows that if a choice between A and B is made many times under identical conditions, the person will choose the action with the larger expected utility more than half of the time. If he is indifferent he will choose each action 50 per cent of the time.

Mosteller and Nogee (1951), in what was perhaps the first laboratory measurement of utility, based their experiment on the Fechner postulate. They offered a subject choices of the following type: either accept a wager $(a,p,-b)$ in which you will win a dollars with probability p and you will win $-b$ dollars (i.e., lose b dollars) with probability $1 - p$, or do not bet at all. They repeated the same offer several times, thereby obtaining the proportion of times that the subject decided to accept the wager. By holding p and b constant and varying a they were able to estimate the amount of money a_o at which this proportion was 50 per cent. Then by assumption the subject was indifferent between accepting the wager $(a_o, p, -b)$ and not betting at all. Hence, these two actions have equal expected utilities. Therefore, denoting by $u(x)$ the utility of gaining x dollars,

$$u(0) = pu(a_o) + (1 - p)u(-b).$$

As stated above, one can arbitrarily fix $u(0) = 0$ and $u(-b) = -1$. Then

$$u(a_o) = (1 - p)/p.$$

By keeping b constant and using the above technique for seven different values of p, Mosteller and Nogee estimated seven points on the subject's "money-gain utility curve (fuction)," which represents the relation between money gains and their utilities.

The experiment just described depends heavily on the assumption that the subject's probabilities of choice remain constant throughout the many times that he is choosing from the same available set of actions, and also on the assumption that each of the seven values of p used in the experiment is "understood" by the subject: i.e., that his personal probability of winning a dollars in a given wager is in fact p.

The procedure to be presented here differs from that of Mosteller and Nogee in several respects. No choice is repeated, but a check on the subject's consistency, or on his learning process, is provided. This is achieved by letting each set of available actions depend on the subject's previous responses in a manner that leads to repeated estimates of the same points on his utility curve. Some of these checks for consistency would be applicable even if the personal probabilities p of the subject were not known to the experimenter. However, the only odds used in our experiment were 1:1 and 3:1, and it did not seem unreasonable to assume that the simple probabilities 1/2 and (to a lesser extent) 3/4 were "understood" by the subject.

THE SEQUENTIAL PROCEDURE

Let p be the probability of an event E, and let (y,p,z) be a wager in which one wins the amount y if the event E occurs and one wins the amount z if E does not occur. If a subject is indifferent between accepting the wager (y,p,z) and accepting a certain monetary gain of amount x, we shall call x his cash-equivalent of (y,p,z). Thus, $u(x) = pu(y) + (1 - p)u(z)$.

Let a and b, with $a < b$, be two convenient amounts of money and fix arbitrarily $u(a) = 0$, $u(b) = 1$. In the next section we shall describe a method for determining a subject's cash-equivalent of any given wager. For the moment let us assume that we know how to do this. Then, if x_1 is the cash-equivalent of the wager (a,p,b), we have

$$u(x_1) = pu(a) + (1 - p)u(b) = 1 - p.$$

Similarly, if x_2 is the cash-equivalent of (b,p,a), then

$$u(x_2) = pu(b) + (1 - p)u(a) = p.$$

The amounts x_1, x_2, a, and b can now be used in later stages of a sequential experiment to form ten new wagers, (a,p,x_1), (x_2,p,b), etc., and the cash-equivalents of each of these wagers can also be determined.

If we define $u^*(y,p,z) = pu(y) + (1 - p)u(z)$ to be the utility of the wager (y,p,z) then it is easily verified that

$$\begin{aligned}
u^*(x_1,p,a) &= u^*(a,p,x_2) = p(1 - p),\\
u^*(x_2,p,b) &= u^*(b,p,x_1) = 1 - p + p^2,\\
u^*(a,p,x_1) &= (1 - p)^2,\\
u^*(x_1,p,b) &= 1 - p^2,\\
u^*(x_2,p,a) &= p^2,\\
u^*(b,p,x_2) &= p(2 - p),\\
u^*(x_1,p,x_2) &= 2p(1 - p),\\
u^*(x_2,p,x_1) &= 1 - 2p + 2p^2.
\end{aligned}$$

It is seen from these equations that, regardless of the value of p, the wagers (x_1,p,a) and (a,p,x_2) have the same utility and consequently they also have the same cash-equivalent. Similarly, (x_2,p,b) and (b,p,x_1) have the same cash-equivalent. Thus the determination of a subject's cash-equivalents of the wagers (x_1,p,a) and (a,p,x_2), say, provides a check on whether the subject is behaving in a manner consistent with a well-defined utility function as specified by the utility model. If p is known, further checks of this kind can be established, as will be seen later.

The same general procedure can be followed throughout the experiment, using at each stage a different probability p' selected so that some of the wagers formed with p' will have the same utilities as some of the wagers formed with p.

DETERMINING THE CASH-EQUIVALENT OF A WAGER

The following method can be used to determine the cash-equivalent of a wager (y,p,z) for a given subject. The subject is told that he will be rewarded from the wager (y,p,z); i.e., that he will receive the amount y if the event E of probability p occurs and he will receive the amount z otherwise.

The subject is then told that as an alternative to receiving this random reward from the wager he has the privilege of trying to sell the wager for cash. Accordingly, he is asked to state the smallest amount s that he will accept (his *selling price*) in lieu of being rewarded from the wager. The understanding is that if a buyer can be found who is willing to pay an amount $b \geq s$ then the subject will receive b. If no buyer can be found who is willing to pay at least s then the subject retains the wager and receives the random reward, either y or z, as specified by the wager.

Let s be the subject's selling price and let e be his cash-equivalent of the wager. Let b be the maximum amount that any buyer is willing to pay. It is assumed that b does not depend on s, but the method of generating b is otherwise irrelevant. If $b \geq s$, the subject will receive the amount b. If $b < s$, the subject will receive a random reward as specified by the wager.

We claim that it is to the subject's advantage for his selling price to be precisely his cash-equivalent of the wager; that is, to have $s = e$. This can be seen as follows.

From the definition of e, $u(e) = u^*(y,p,z)$ and, hence, receiving the certain cash amount e is equivalent to the subject to receiving a random reward from the wager. Now suppose $s > e$. If $b < e$ or $b \geq s$, the subject's fortune changes just as it would had his selling price been $s = e$. However, if b is such that $e \leq b < s$, the subject does not sell the wager and he receives a random reward whose cash-equivalent is e. Had his selling price been $s = e$, then for the same value of b he would have received the amount $b \geq e$. Thus, for all possible values of b, the expected utility of the subject's reward is at least as large when his selling price is $s = e$ as it is when $s > e$, and for some values of b it is strictly larger.

To complete the argument, suppose $s < e$. If $b < s$ or $b \geq e$, the subject's fortune changes just as it would had his selling price been $s = e$. However, if $s \leq b < e$, the subject receives the amount b whereas had his selling price been $s = e$ he would not sell the wager and would receive a random reward with cash-equivalent $e > b$. Thus, again, for all values of b the expected utility of the reward is at least as much when $s = e$ as it is when $s < e$, and for some values of b it is strictly larger. This demonstrates that the subject's optimal selling price is $s = e$. It brings him the highest expected utility.

EXPERIMENTAL PROCEDURE

At the ith stage ($i = 1, 2, \cdots, 24$) of the sequential experiment, the subject was presented with a wager from which he would receive the amount A_i if a number X_i selected at random from a rotating bingo basket containing balls with the integers 1 through 100 was less than or equal to C_i, and he would receive the amount B_i if X_i was greater than C_i. (In the notation of the preceding sections, this is the wager $(A_i, C_i/100, B_i)$ if we make the assumption that all integers between 1 and 100 have the same probability of being selected.) After each selection the ball was put back into the basket.

Before X_i was selected, the subject named his selling price s_i for the wager. A random integer Y_i was then selected from the basket. If $Y_i \geq s_i$, the subject received the amount Y_i. In effect, he had sold the wager for the amount Y_i. If $Y_i \leq s_i$, the subject did not sell the wager, X_i was observed, and the subject received either A_i or B_i, according as $X_i \leq C_i$ or $X_i > C_i$.

The amounts A_i, B_i, C_i used at each of the 24 stages of the sequential experiment are shown in Table 1 together with the utilities of the wagers under the arbitrary assignment of the two values $u(0) = 0$ and $u(100) = 1$. It should be noted that several of the 24 wagers have the same utility; e.g., $u(s_3) = u(s_4) = u(s_{18}) = u(s_{22}) = 3/4$.

Moreover, even if the subject does not feel that all integers have the same probability of being selected, it is still true that

$$\begin{aligned} u(s_3) &= u(s_{22}), \\ u(s_7) &= u(s_{19}), \\ u(s_9) &= u(s_{21}), \\ u(s_{10}) &= u(s_{23}). \end{aligned}$$

To see why this is so, we will prove, for example, that $u(s_3) = u(s_{22})$.

Let p be the probability that the number selected at random will be at most 25. Then it is seen from Table 1 that $u(s_{22}) = u^* (s_{11},p,s_8) = pu(s_{11}) + (1 - p)u(s_8)$. But it is also seen from Table 1 that $u(s_{11}) = u^*(0,p,s_3) = (1 - p)u(s_3)$ and $u(s_8) = u^*(100,p,s_3) = p + (1 - p)u(s_3)$. Furthermore, $u(s_3) = u^*(0,p,100) = 1 - p$. Thus, $u(s_{22}) = p(1 - p)u(s_3) + p(1 - p) + (1 - p)^2u(s_3) = 1 - p = u(s_3)$.

The selling price s_i at a given stage is sometimes used as one of the rewards in the wagers presented at later stages. It should be noted that the subject can increase the utility of wagers presented at later stages by naming a higher selling price for wagers presented at earlier stages. In order to prevent the subject from recognizing that he had such control, his earlier selling prices were not used until several stages later.

In order to avoid changing the total capital of the subject during the sequence, the subject was not paid on any trial during the experiment and in fact was told that he would be paid on only one of the 24 trials. This trial was determined by drawing a number between 1 and 24 at random at the end of the session.

TABLE 1
EXPERIMENTAL DESIGN

Stage, i	A_i	B_i	C_i	s_i	$u(s_i)$
1	0	100	50	s_1	1/2
2	0	100	75	s_2	1/4
3	0	100	25	s_3	3/4
4	s_1	100	50	s_4	3/4
5	0	s_1	50	s_5	1/4
6	s_2	s_3	75	s_6	3/8
7	s_2	s_3	25	s_7	5/8
8	100	s_3	25	s_8	13/16
9	s_5	100	50	s_9	5/8
10	s_5	s_1	50	s_{10}	3/8
11	0	s_3	25	s_{11}	9/16
12	s_2	100	75	s_{12}	7/16
13	s_4	100	50	s_{13}	7/8
14	s_7	s_8	75	s_{14}	43/64
15	s_5	s_4	50	s_{15}	1/2
16	s_9	100	50	s_{16}	13/16
17	s_6	s_7	75	s_{17}	7/16
18	s_9	s_{13}	50	s_{18}	3/4
19	s_{11}	s_8	75	s_{19}	5/8
20	0	s_{13}	50	s_{20}	7/16
21	s_1	s_4	50	s_{21}	5/8
22	s_{11}	s_8	25	s_{22}	3/4
23	0	s_4	50	s_{23}	3/8
24	s_{11}	100	75	s_{24}	43/64

Instructions

The subjects were instructed as follows:

Stage 1 only. "In this game we will draw a number between 1 and 100. If the number is equal to or less than 50, you will win nothing. If the number is greater than 50 you will win 100 cents. How much are you willing to accept instead of playing the game? After you tell me how much you are willing to take for the game, I will draw a number between 1 and 100. If the number I draw is equal to or greater than the price you asked, I will pay you whatever number I drew and you will not play the game. If the number I draw is less than the price you ask, you will play the game and win nothing if the next number I draw is equal to or less than 50, or you will win 100 cents if that number is greater than 50.

"Do you have any questions?

"What is the lowest amount you are willing to take for the game?"

Stages 2 through 24. "This time you will win the amount A_i if the number is equal to or less than C_i or you will win B_i if the number is larger than C_i.

"Do you have any questions?

"What is your lowest price for the game?"

Subjects

Two male students were obtained through the Yale Student Placement Office to serve as subjects in an economic experiment being conducted at the Cowles Foundation for Economic Research, Yale University. Subject 1 was an undergraduate in the Department of Psychology. Subject 2 was a graduate theology student. Each subject was guaranteed $1.25 per session (i.e., for his responses to the full sequence of 24 stages) and each subject participated in three sessions. In addition the subject received a bonus each session consisting of his winnings at one of the stages in the sequence. As already described, the subject was not told which stage would be used to determine his bonus until the end of the session.

RESULTS

Since each subject participated in three sessions the data provide an opportunity for the study of the change in the subjects' behavior as they grew familiar with the task.

TABLE 2
SELLING PRICES OF THE WAGERS

Stage	Subject 1 Session 1	Subject 1 Session 2	Subject 1 Session 3	Subject 2 Session 1	Subject 2 Session 2	Subject 2 Session 3
1	65 (50)	45 (50)	50 (50)	60 (50)	80 (50)	65 (50)
2	40 (25)	25 (25)	25 (25)	50 (25)	55 (25)	45 (25)
3	80 (75)	80 (75)	75 (75)	80 (75)	90 (75)	75 (75)
4	70 (83)	65 (73)	65 (75)	100 (80)	92 (90)	80 (83)
5	30 (33)	25 (23)	30 (25)	24 (30)	65 (40)	45 (33)
6	40 (50)	25 (39)	50 (38)	65 (58)	75 (64)	79 (53)
7	60 (70)	65 (66)	60 (63)	75 (73)	80 (81)	65 (68)
8	75 (85)	90 (85)	75 (81)	95 (85)	98 (83)	87 (81)
9	45 (65)	40 (63)	45 (65)	75 (62)	80 (83)	75 (73)
10	40 (48)	30 (35)	40 (40)	40 (42)	70 (73)	55 (55)
11	60 (60)	65 (60)	50 (56)	75 (60)	80 (68)	60 (56)
12	50 (55)	40 (44)	50 (44)	80 (63)	75 (66)	70 (59)
13	65 (85)	65 (83)	75 (83)	100 (100)	99 (96)	95 (90)
14	65 (64)	75 (71)	60 (64)	90 (80)	95 (85)	82 (71)
15	40 (50)	40 (45)	50 (48)	60 (62)	78 (79)	70 (63)
16	55 (73)	60 (70)	65 (73)	90 (88)	95 (90)	90 (88)
17	45 (45)	45 (35)	55 (53)	75 (70)	79 (76)	70 (76)
18	50 (55)	55 (53)	60 (60)	95 (88)	98 (90)	93 (85)
19	65 (64)	70 (71)	60 (56)	85 (80)	95 (85)	75 (67)
20	25 (33)	45 (33)	50 (38)	50 (50)	60 (50)	63 (48)
21	65 (68)	55 (55)	60 (58)	75 (80)	89 (86)	78 (73)
22	65 (71)	80 (84)	70 (69)	90 (90)	97 (94)	82 (80)
23	25 (35)	45 (33)	45 (33)	50 (50)	55 (46)	55 (40)
24	65 (70)	65 (74)	60 (63)	90 (81)	98 (85)	80 (70)

Note: The figures given in parentheses are the actuarial values of the wagers.

TABLE 3
DIFFERENCES IN SELLING PRICES OF WAGERS OF EQUAL UTILITY

	Subject 1 Session 1	Subject 1 Session 2	Subject 1 Session 3	Subject 2 Session 1	Subject 2 Session 2	Subject 2 Session 3
$s_{22} - s_3$	−15	0	−5	10	7	7
$s_{19} - s_7$	5	5	0	10	15	10
$s_{21} - s_9$	20	15	15	0	9	3
$s_{23} - s_{10}$	−15	15	5	10	−15	0

Their selling prices for the wagers at each stage of each session, and the actuarial value of each wager (i.e., the expected monetary reward $(C_i/100)A_i + (1 - C_i/100)B_i$) are shown in Table 2. (Note that the selling prices named by Subject 1 were always multiples of 5 cents. Although such rounding violates the assumption of a strictly monotonic utility function, the effects of the violation are negligible here.)

As discussed earlier, $u(s_3) = u(s_{22})$, $u(s_7) = u(s_{19})$, $u(s_9) = u(s_{21})$, and $u(s_{10}) = u(s_{23})$, regardless of the value of p used in computing the utilities of the wagers. Thus, if the subjects' behavior is consistent with the expected utility model their selling prices should be such that $s_3 = s_{22}$, $s_7 = s_{19}$, $s_9 = s_{21}$, and $s_{10} = s_{23}$. The observed differences $s_{22} - s_3$, etc., are shown in Table 3. Since most of these differences are nonzero the data are not consistent with an expected utility model.

It should also be noted, however, that the differences in prices decrease, on the average, from session to session, indicating that behavior does become, in some sense, more consistent with an expected utility model as the subject becomes more familiar with the task. Thus, despite the fact that the model does not precisely fit the behavior of the subjects, there is some indication that it approximates such behavior and that the model becomes more appropriate as the subject becomes more familiar with the experiment.

Now let us again assume that the values $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{3}{4}$ are in fact the subjects' personal probabilities of the relevant events (i.e., we assume that, for the subjects, all

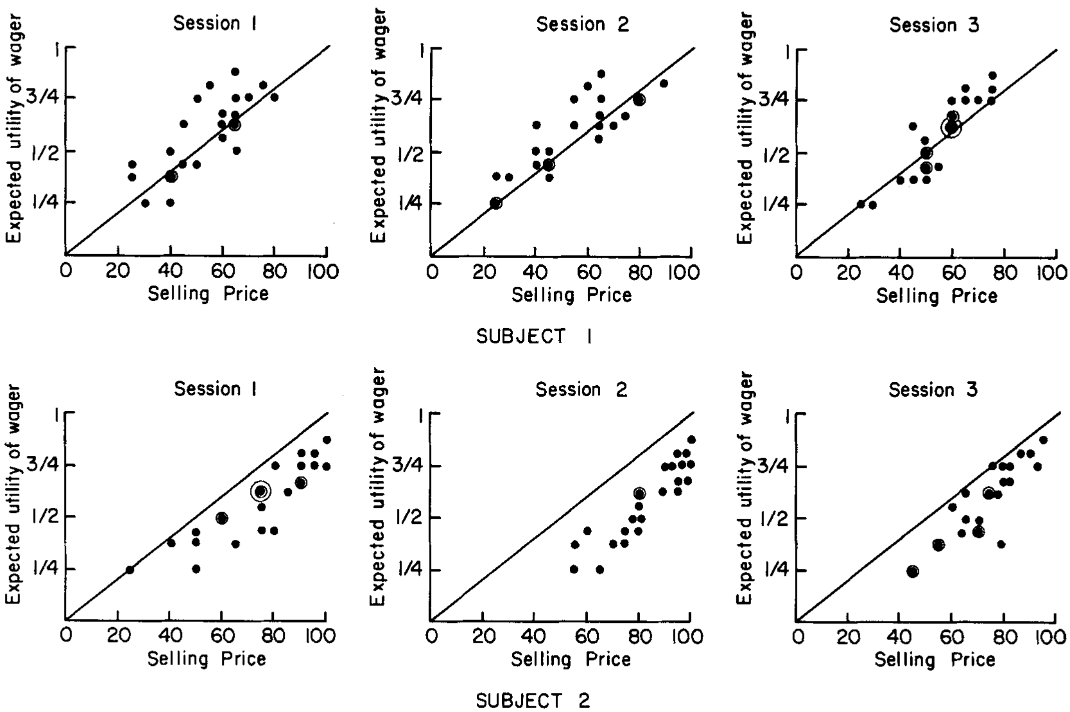

FIG. 1. Scatter Diagram of Subjects' Selling Prices Plotted Against the Utilities of the Wagers. A circled dot indicates two coincident points. A twice-circled dot indicates three coincident points.

integers are equally probable of being selected). The utilities of the wagers used in the experiment are given in Table 1 and, since the optimal selling price of a wager is its cash-equivalent, the selling prices named by a subject provide estimates of the amounts of money having these utilities. Thus, a scatter diagram of the selling prices named by a subject plotted against the utilities of the wagers should provide some suggestion of the subject's utility curve. These diagrams are given in Figure 1.

When viewing these diagrams it should be kept in mind that at each stage the utility of the wager was fixed and the selling price was the observed variable. Hence, the horizontal spread of points with the same utility is indicative of the inconsistency of the subjects' responses. In attempting to fit utility curves to these points, either "by eye" or by some more refined method, the horizontal distances should perhaps be given primary consideration. Nevertheless, this discussion is not intended to minimize the relevance of the vertical spread of the utilities of wagers with the same selling price.

Because of the sequential nature of the experiment there is a dependence among the observations that makes a precise statistical analysis along traditional lines impractical. Accordingly, we content ourselves here with giving just a few general comments.

For both subjects, the horizontal spread of the points obtained in Session 3 at the fixed utility levels is relatively small. Thus, there is a trend toward the adoption of response patterns consistent with a set of constant utilities.

For a given wager, the larger the selling price named by a subject, the more willing he is to take risks. The selling prices named by Subject 1 were, in general, largest in Session 3. The linear (or actuarial) utility curve provides a not unreasonable fit to the observed points for this subject, although he is slightly more willing to take risks with small amounts and less willing with large amounts.

The observed points for Subject 2 in Session 3 lie below the linear utility curve, indicating the subject's willingness to take risks.

For each subject, it is felt, a reasonable estimate of his utility curve can be sketched.

CONCLUSIONS

An attempt has been made to measure the utility of money. Estimates of the amounts of money having been given, preselected utilities were obtained in a sequential procedure. At each stage of the procedure the subject stated the lowest price he would accept in lieu of a wager in his possession. It was shown that under the utility model this selling price will be the subject's cash-equivalent of the wager.

The inconsistency of the responses of two subjects led to the rejection of the model. However, as the subjects became more familiar with the task their experience appeared to lead to more consistent behavior and less deviation from the results specified by the utility model. Thus, despite the rejection of the model in the strict sense, it is felt that it does, at least, approximate the observed behavior.

It is also our feeling that the procedure used here might provide useful data for the study of specific stochastic models of choice behavior, such as those given by Becker, DeGroot, and Marschak (1963), and that it might be possible to use this procedure, alone or in combination with others, to estimate both personal probabilities and utilities of an experienced subject.

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At the very best, we admit, each time you scrutinize a concept of substance, it dissolves into thin air. But conversely, the moment you relax your gaze a bit, it re-forms again. For things *do* have intrinsic natures, whatever may be the quandaries that crowd upon us as soon as we attempt to decide definitively what these intrinsic natures are. If you will, call the category of substance sheer error. Yet it is so fertile a source of error, that only by learning to recognize its nature from within can we hope to detect its many disguises from without.

KENNETH BURKE, *A Grammar of Motives*